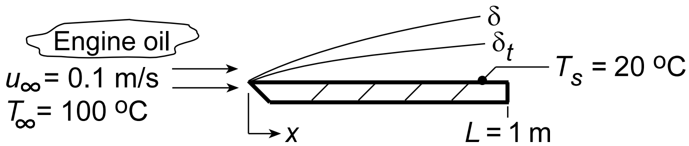
PROBLEM 7.2

KNOWN: Temperature and velocity of engine oil. Temperature and length of flat plate.

FIND: (a) Velocity and thermal boundary layer thickness at trailing edge, (b) Heat flux and surface shear stress at trailing edge, (c) Total drag force and heat transfer per unit plate width, and (d) Plot the boundary layer thickness and local values of the shear stress, convection coefficient, and heat flux as a function of x for 0 ≤ x ≤ 1 m.

SCHEMATIC:



ASSUMPTIONS: (1) Critical Reynolds number is 5×10^5, (2) Flow over top and bottom surfaces.

PROPERTIES: *Table A.5*, Engine Oil (T_f = 333 K): $\rho = 864\ \text{kg/m}^3$, $\nu = 86.1 \times 10^{-6}\ \text{m}^2/\text{s}$, k = 0.140 W/m·K, Pr = 1081.

ANALYSIS: (a) Calculate the Reynolds number to determine nature of the flow,

$$Re_L = \frac{u_\infty L}{\nu} = \frac{0.1\,\text{m/s} \times 1\,\text{m}}{86.1 \times 10^{-6}\,\text{m}^2/\text{s}} = 1161$$

Hence the flow is laminar at x = L, from Eqs. 7.19 and 7.24, and

$$\delta = 5L\,Re_L^{-1/2} = 5(1\,\text{m})(1161)^{-1/2} = 0.147\,\text{m} \qquad <$$

$$\delta_t = \delta\,Pr^{-1/3} = 0.147\,\text{m}(1081)^{-1/3} = 0.0143\,\text{m} \qquad <$$

(b) The local convection coefficient, Eq. 7.23, and heat flux at x = L are

$$h_L = \frac{k}{L} 0.332\,Re_L^{1/2}\,Pr^{1/3} = \frac{0.140\,\text{W/m}\cdot\text{K}}{1\text{m}} 0.332(1161)^{1/2}(1081)^{1/3} = 16.25\,\text{W/m}^2\cdot\text{K}$$

$$q''_x = h_L(T_s - T_\infty) = 16.25\,\text{W/m}^2\cdot\text{K}(20-100)^\circ\text{C} = -1300\,\text{W/m}^2 \qquad <$$

Also, the local shear stress is, from Eq. 7.20,

$$\tau_{s,L} = \frac{\rho u_\infty^2}{2} 0.664\,Re_L^{-1/2} = \frac{864\,\text{kg/m}^3}{2}(0.1\,\text{m/s})^2\,0.664(1161)^{-1/2}$$

$$\tau_{s,L} = 0.0842\,\text{kg/m}\cdot\text{s}^2 = 0.0842\,\text{N/m}^2 \qquad <$$

(c) With the drag force per unit width given by $D' = 2L\bar{\tau}_{s,L}$ where the factor of 2 is included to account for both sides of the plate, it follows that

$$D' = 2L\left(\rho u_\infty^2/2\right)1.328\,Re_L^{-1/2} = (1\,\text{m})864\,\text{kg/m}^3(0.1\,\text{m/s})^2/2\ 1.328(1161)^{-1/2} = 0.337\,\text{N/m} \qquad <$$

For laminar flow, the average value $\bar{h}_L$ over the distance 0 to L is twice the local value, h_L,

$$\bar{h}_L = 2h_L = 32.5\,\text{W/m}^2\cdot\text{K}$$

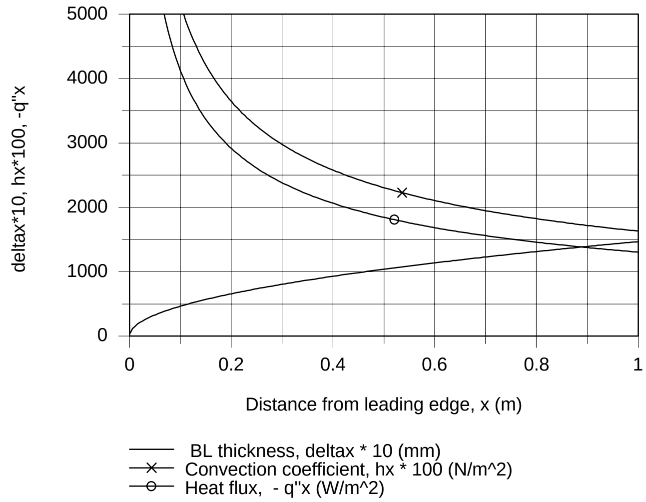
The total heat transfer rate per unit width of the plate is

$$q' = 2L\bar{h}_L(T_s - T_\infty) = 2(1\,\text{m})32.5\,\text{W/m}^2\cdot\text{K}(20-100)^\circ\text{C} = -5200\,\text{W/m} \qquad <$$

Continued...

PROBLEM 7.2 (Cont.)

(c) Using IHT with the foregoing equations, the boundary layer thickness, and local values of the convection coefficient and heat flux were calculated and plotted as a function of x.



COMMENTS: (1) Note that since Pr >> 1, $\delta >> \delta_t$. That is, for the high Prandtl liquids, the velocity boundary layer will be much thicker than the thermal boundary layer.

(2) A copy of the *IHT Workspace* used to generate the above plot is shown below.

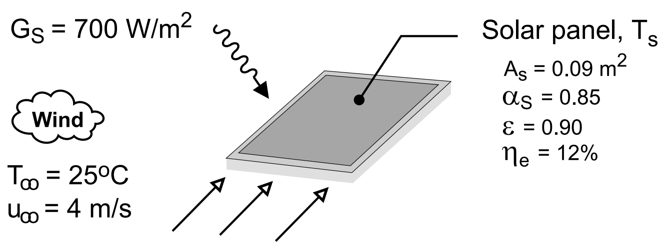
```
// Boundary layer thickness, delta
delta = 5 * x * Rex ^-0.5
delta_mm = delta * 1000
delta_plot = delta_mm * 10          // Scaling parameter for convenience in  plotting

// Convection coefficient and heat flux, q''x
q''x = hx * (Ts - Tinf)
Nux = 0.332 * Rex^0.5 * Pr^(1/3)
Nux = hx * x / k
hx_plot = 100 * hx                  // Scaling parameter for convenience in plotting
q''x_plot = ( -1 ) * q''x           // Scaling parameter for convenience in plotting

// Reynolds number
Rex = uinf * x / nu

// Properties Tool: Engine oil
// Engine Oil property functions : From Table A.5
// Units: T(K)
rho = rho_T("Engine Oil",Tf)        // Density, kg/m^3
cp = cp_T("Engine Oil",Tf)          // Specific heat, J/kg·K
nu = nu_T("Engine Oil",Tf)          // Kinematic viscosity, m^2/s
k = k_T("Engine Oil",Tf)            // Thermal conductivity, W/m·K
Pr = Pr_T("Engine Oil",Tf)          // Prandtl number


// Assigned variables
Tf = (Ts + Tinf) / 2                // Film temperature, K
Tinf = 100 + 273                    // Freestream temperature, K
Ts = 20 + 273                       // Surface temperature, K
uinf = 0.1                          // Freestream velocity, m/s
x = 1                               // Plate length, m
```

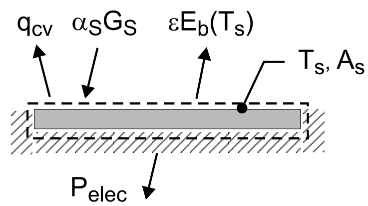
PROBLEM 7.18

KNOWN: Square solar panel with an area of 0.09 m^2 has solar-to-electrical power conversion efficiency of 12%, solar absorptivity of 0.85, and emissivity of 0.90. Panel experiences a 4 m/s breeze with an air temperature of 25°C and solar insolation of 700 W/m^2.

FIND: Estimate the temperature of the solar panel for: (a) The operating condition *(on)* described above when the panel is producing power, and (b) The *off* condition when the solar array is inoperative. Will the panel temperature increase, remain the same or decrease, all other conditions remaining the same?

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) The backside of the panel experiences no heat transfer, (3) Sky irradiation is negligible, and (4) Wind is in parallel, fully turbulent flow over the panel.

PROPERTIES: *Table A-4*, Air (Assume T_f = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: (a) Perform an energy balance on the panel as represented in the schematic above considering convection, absorbed insolation, emission and generated electrical power.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_{cv} + \left[\alpha_S G_S - \varepsilon\sigma T_s^4\right] A_s - P_{elec} = 0 \qquad (1)$$

Using the convection rate equation and power conversion efficiency,

$$q_{cv} = \bar{h}_L A_s \left(T_s - T_\infty\right) \qquad P_{elec} = \eta_e \alpha_S G_S A_s \qquad (2,3)$$

The average convection coefficient for fully turbulent conditions is

$$\overline{Nu}_L = \bar{h}L / k = 0.037\ Re_L^{4/5}\ Pr^{1/3}$$

$$Re_L = u_\infty L / \nu = 4\,m/s \times 0.3\,m / 15.89 \times 10^{-6} m^2/s = 7.49 \times 10^4$$

$$\bar{h}_L = \left(0.0263\ W/m \cdot K / 0.3m\right) \times 0.037 \times \left(7.49 \times 10^4\right)^{4/5} (0.707)^{1/3}$$

$$\bar{h}_L = 23.0\ W/m^2 \cdot K$$

Substituting numerical values in Eq. (1) using Eqs. (2 and 3) and dividing through by A_s, find T_s.

Continued …..

PROBLEM 7.18 (Cont.)

$$23\ \mathrm{W/m^2\cdot K}\left(T_s-298\right)\mathrm{K}+0.85\times 700\ \mathrm{W/m^2}-0.90\times 5.67\times 10^{-8}\mathrm{W/m^2\cdot K^4}\,T_s^4$$
$$-0.12\left[0.85\times 700\ \mathrm{W/m^2}\right]=0 \qquad (4)$$

$$T_s = 302.2\ \mathrm{K} = 29.2°\mathrm{C} \qquad <$$

(b) If the solar array becomes inoperable *(off)* for reason of wire bond failures or the electrical circuit to the battery is opened, the P_{elec} term in the energy balance of Eq. (1) is zero. Using Eq. (4) with $\eta_e = 0$, find

$$T_s = 31.7°\mathrm{C} \qquad <$$

COMMENTS: (1) Note how the electrical power P_{elec} is represented by the $\dot{E}_{gen}$ term in the energy balance. Recall from Section 1.2 that $\dot{E}_{gen}$ is associated with conversion *from* some form of energy *to* thermal energy. Hence, the solar-to-electrical power conversion (P_{elec}) will have a negative sign in Eq. (1).

(2) It follows that when the solar array is *on*, a fraction (η_e) of the absorbed solar power (thermal energy) is converted to electrical energy. As such, the array surface temperature will be higher in the *off* condition than in the *on* condition.

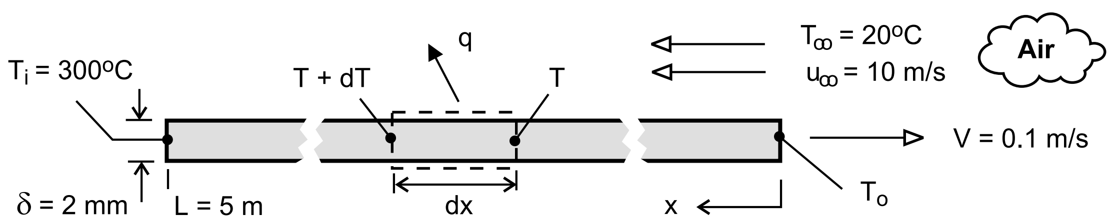
(3) Note that the assumed value for T_f at which to evaluate the properties was reasonable.

PROBLEM 7.26

KNOWN: Velocity, initial temperature, and dimensions of aluminum strip on a production line. Velocity and temperature of air in counter flow over top surface of strip.

FIND: (a) Differential equation governing temperature distribution along the strip and expression for outlet temperature, (b) Value of outlet temperature for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible variation of sheet temperature across its thickness, (2) Negligible effect of conduction along length (x) of sheet, (3) Negligible radiation, (4) Turbulent flow over entire top surface, (5) Negligible effect of sheet velocity on boundary layer development, (6) Negligible heat transfer from bottom surface and sides, (7) Constant properties.

PROPERTIES: *Table A-1*, Aluminum, 2024-T6 $\left(\overline{T}_{AL} \approx 500\text{K}\right)$: $\rho = 2770\,\text{kg/m}^3$, $c_p = 983\,\text{J/kg}\cdot\text{K}$, k=186 W/m·K. *Table A-4*, Air $\left(p = 1\,\text{atm},\ T_f \approx 400\text{K}\right)$: $\nu = 26.4\times10^{-6}\,\text{m}^2/\text{s}$, $k = 0.0338\,\text{W/m}\cdot\text{K}$, $\text{Pr} = 0.69$

ANALYSIS: (a) Applying conservation of energy to a stationary control surface, through which the sheet moves, steady-state conditions exist and $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, with *inflow* due to *advection* and outflow due to *advection* and *convection*,

$$\rho V A_c c_p \left(T + dT\right) - \rho V A_c c_p T - dq = 0$$

$$+\rho V \delta W c_p dT - h_x \left(dx \cdot W\right)\left(T - T_\infty\right) = 0$$

$$\frac{dT}{dx} = +\frac{h_x}{\rho V \delta c_p}\left(T - T_\infty\right) \qquad (1) \quad <$$

Alternatively, if the control surface is fixed to the sheet, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-h_x\left(dx \cdot W\right)\left(T - T_\infty\right) = \rho\left(dx \cdot W \cdot \delta\right)c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{h_x}{\rho \delta c_p}\left(T - T_\infty\right)$$

Dividing the left- and right-hand sides of the equation by dx/dt and dx/dt = - V, respectively, equation (1) is obtained. The equation may be integrated from x = 0 to x = L to obtain

$$\int_{T_o}^{T_i} \frac{dT}{T - T_\infty} = \frac{L}{\rho V \delta c_p}\left[\frac{1}{L}\int_0^L h_x\, dx\right]$$

Continued …..

where $h_x = (k/x)0.0296\,Re_x^{4/5}\,Pr^{1/3}$ and the bracketed term on the right-hand side of the equation reduces to $\overline{h}_L = (k/L)0.037\,Re_L^{4/5}\,Pr^{1/3}$.

Hence,

$$\ln\left(\frac{T_i - T_\infty}{T_o - T_\infty}\right) = \frac{L\,\overline{h}_L}{\rho V \delta c_p}$$

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{L\,\overline{h}_L}{\rho V \delta c_p}\right) \qquad <$$

(b) For the prescribed conditions, $Re_L \approx u_\infty L/\nu = 20\,m/s \times 5m / 26.4\times10^{-6} m^2/s = 3.79\times10^6$ and

$$\overline{h}_L = \left(\frac{0.0338\,W/m\cdot K}{5m}\right)0.037\left(3.79\times10^6\right)^{4/5}(0.69)^{1/3} = 40.5\,W/m^2\cdot K$$

$$T_o = 20°C + (280°C)\exp\left(-\frac{5m\times 40.5\,W/m^2\cdot K}{2770\,kg/m^3 \times 0.1\,m/s \times 0.002m \times 983\,J/kg\cdot K}\right) = 213°C \qquad <$$

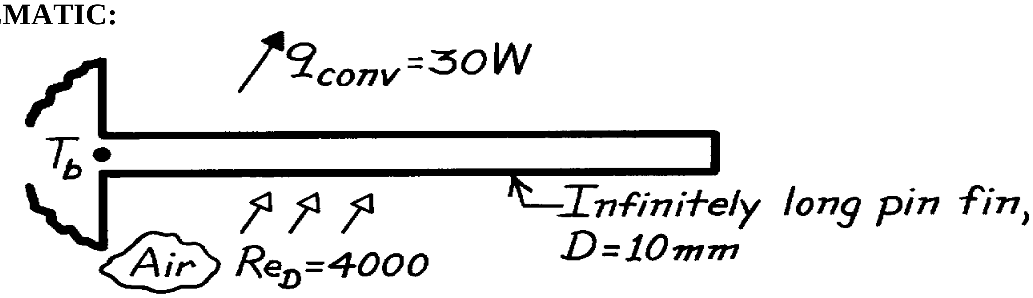
COMMENTS: (1) With $T_o = 213°C$, $\overline{T}_{Al} = 530K$ and $T_f = 411K$ are close to values used to determine the material properties, and iteration is not needed. (2) For a representative emissivity of $\varepsilon = 0.2$ and $T_{sur} = T_\infty$, the maximum value of the radiation coefficient is $h_r = \varepsilon\sigma(T_i + T_{sur})\left(T_i^2 + T_{sur}^2\right) = 4.1\,W/m^2\cdot K << \overline{h}_L$. Hence, the assumption of negligible radiation is appropriate.

PROBLEM 7.45

KNOWN: Pin fin of 10 mm diameter dissipates 30 W by forced convection in cross-flow of air with $Re_D = 4000$.

FIND: Fin heat rate if diameter is doubled while all conditions remain the same.

SCHEMATIC:



ASSUMPTIONS: (1) Pin behaves as infinitely long fin, (2) Conditions of flow, as well as base and air temperatures, remain the same for both situations, (3) Negligible radiation heat transfer.

ANALYSIS: For an infinitely long pin fin, the fin heat rate is

$$q_f = q_{conv} = \left(\bar{h}PkA_c\right)^{1/2}\theta_b$$

where $P = \pi D$ and $A_c = \pi D^2/4$. Hence,

$$q_{conv} \sim \left(\bar{h}\cdot D\cdot D^2\right)^{1/2}.$$

For forced convection cross-flow over a cylinder, an appropriate correlation for estimating the dependence of $\bar{h}$ on the diameter is

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = CRe_D^m\ Pr^{1/3} = C\left(\frac{VD}{\nu}\right)^m Pr^{1/3}.$$

From Table 7.2 for $Re_D = 4000$, find m = 0.466 and

$$\bar{h} \sim D^{-1}(D)^{0.466} = D^{-0.534}.$$

It follows that

$$q_{conv} \sim \left(D^{-0.534}\cdot D\cdot D^2\right)^{1/2} = D^{1.23}.$$

Hence, with $q_1 \rightarrow D_1$ (10 mm) and $q_2 \rightarrow D_2$ (20 mm), find

$$q_2 = q_1\left(\frac{D_2}{D_1}\right)^{1.23} = 30\text{ W}\left(\frac{20}{10}\right)^{1.23} = 70.4\text{ W}.$$ <

COMMENTS: The effect of doubling the diameter, with all other conditions remaining the same, is to increase the fin heat rate by a factor of 2.35. The effect is nearly linear, with enhancements due to the increase in surface and cross-sectional areas ($D^{1.5}$) exceeding the attenuation due to a decrease in the heat transfer coefficient ($D^{-0.267}$). Note that, with increasing Reynolds number, the exponent m increases and there is greater heat transfer enhancement due to increasing the diameter.

Addendum to Problem 7.45

Ignore everything after the average nusselt number expression in the book's solution. Instead, either of the two following methods are correct.

Correct Method #1

For Re_{D1} = 4000, From Table 7.2, C_1 = .193, m_1 = .618
For Re_{D2} = 8000, From Table 7.2, C_2 = .193, m_2 = .618

$q_{conv1} = q_{conv2} \sim (D^{-1}D^{.618}DD^2)^{1/2} = D^{1.309}$

$q_{conv2} = q_{conv1}\ (D_2/D_1)^{1.309} = 30W\ (20/10)^{1.309} = 74.3\ W$

Correct Method #2

For Re_{D1} = 4000, From Table 7.2, C_1 = .683, m_1 = .466

$h_{overbar1} \sim D_1^{-1}C_1D_1^{.466} = .683D_1^{-.534}$

$q_{conv1} \sim (.683D_1^{-.534}D_1D_1^2)^{1/2} = .826D_1^{1.23}$

For $D_2 = 2D_1$, $Re_{D2} = 2Re_{D1}$ = 8000, now from Table 7.2 C, C_2 = .193, m_2 = .618

$h_{overbar2} \sim D_2^{-1}C_2D_2^{.618} = .193D_2^{-.382}$

$q_{conv1} \sim (.193D_2^{-.382}D_2D_2^2)^{1/2} = .439D_2^{1.309}$

$q_{conv2}/q_{conv1} = (\ .439D_2^{1.309})\ /(.826D_1^{1.23}\ ) = (.439(20^{1.309}))/(.862(10^{1.23}) = 1.51$

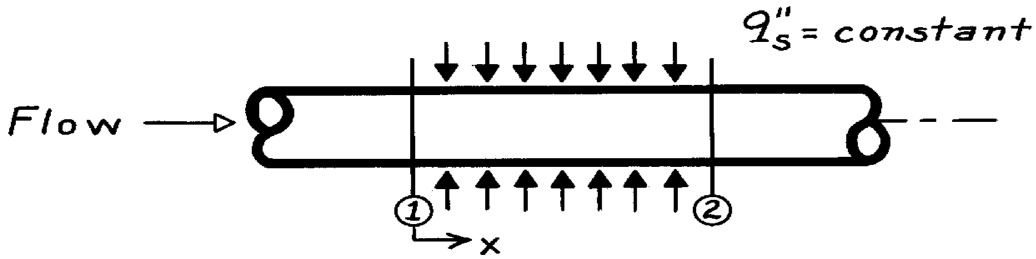
$q_{conv2}\ = 30\ W\ (1.51) = 45.4\ W$

PROBLEM 8.12

KNOWN: Internal flow with constant surface heat flux, q_s''.

FIND: (a) Qualitative temperature distributions, T(x), under developing and fully-developed flow, (b) Exit mean temperature for both situations.

SCHEMATIC:



ASSUMPTIONS: (a) Steady-state conditions, (b) Constant properties, (c) Incompressible flow.

ANALYSIS: Based upon the analysis leading to Eq. 8.40, note for the case of constant surface heat flux conditions,

$$\frac{dT_m}{dx} = \text{constant.}$$

Hence, regardless of whether the hydrodynamic or thermal boundary layer is fully developed, it follows that

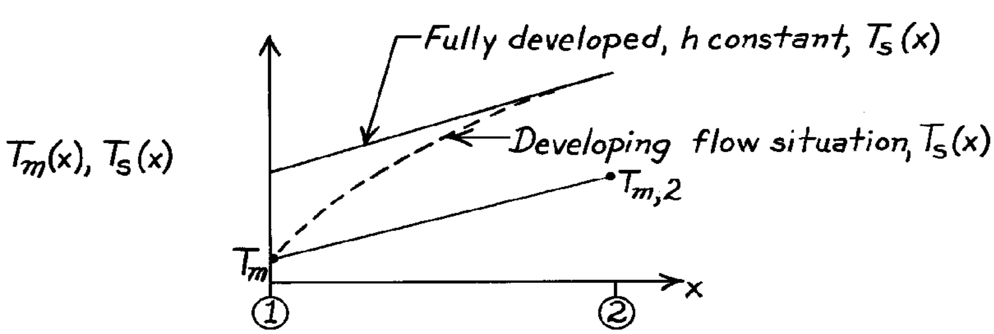
$T_m(x)$ is linear and

$T_{m,2}$ will be the same for all flow conditions. <

The surface heat flux can also be written, using Eq. 8.28, as

$$q_s'' = h\left[T_s(x) - T_m(x)\right].$$

Under fully-developed flow and thermal conditions, $h = h_{fd}$ is a constant. When flow is developing $h > h_{fd}$. Hence, the temperature distributions appear as below.



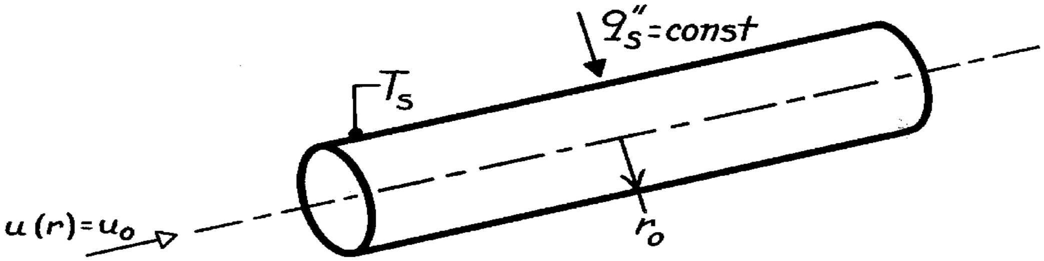
<

PROBLEM 8.18

KNOWN: Laminar, slug flow in a circular tube with uniform surface heat flux.

FIND: Temperature distribution and Nusselt number.

SCHEMATIC:



ASSUMPTIONS: (1) Steady, incompressible flow, (2) Constant properties, (3) Fully developed, laminar flow, (4) Uniform surface heat flux.

ANALYSIS: With v = 0 for fully developed flow and $\partial T/\partial x = dT_m/dx$ = const, from Eqs. 8.33 and 8.40, the energy equation, Eq. 8.48, reduces to

$$u_o \frac{dT_m}{dx} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right).$$

Integrating twice, it follows that

$$T(r) = \frac{u_o}{\alpha}\frac{dT_m}{dx}\frac{r^2}{4} + C_1 \ell n(r) + C_2.$$

Since T(0) must remain finite, $C_1 = 0$. Hence, with $T(r_o) = T_s$

$$C_2 = T_s - \frac{u_o}{\alpha}\frac{dT_m}{dx}\frac{r_o^2}{4} \qquad T(r) = T_s - \frac{u_o}{4\alpha}\frac{dT_m}{dx}\left(r_o^2 - r^2\right). \qquad <$$

From Eq. 8.27, with $u_m = u_o$,

$$T_m = \frac{2}{r_o^2}\int_0^{r_o} Tr\,dr = \frac{2}{r_o^2}\int_0^{r_o}\left[T_s r - \frac{u_o}{4\alpha}\frac{dT_m}{dx}\left(rr_o^2 - r^3\right)\right]dr$$

$$T_m = \frac{2}{r_o^2}\left[T_s\frac{r_o^2}{2} - \frac{u_o}{4\alpha}\frac{dT_m}{dx}\left(\frac{r_o^4}{2} - \frac{r_o^4}{4}\right)\right] = T_s - \frac{u_o r_o^2}{8\alpha}\frac{dT_m}{dx}.$$

From Eq. 8.28 and Fourier's law,

$$h = \frac{q_s''}{T_s - T_m} = \frac{k\left.\frac{\partial T}{\partial r}\right|_{r_o}}{T_s - T_m}$$

hence,

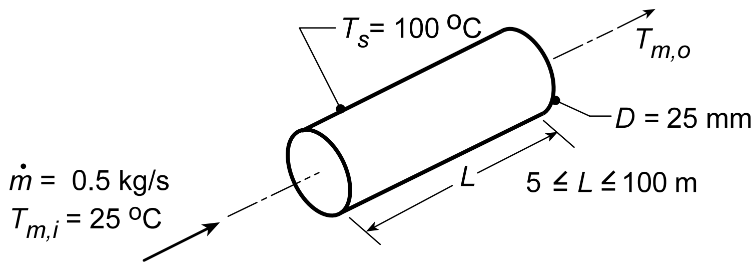
$$h = \frac{k\left(\frac{u_o r_o}{2\alpha}\right)\frac{dT_m}{dx}}{\frac{u_o r_o^2}{8\alpha}\frac{dT_m}{dx}} = \frac{4k}{r_o} = \frac{8k}{D} \qquad \overline{Nu}_D = \frac{hD}{k} = 8. \qquad <$$

PROBLEM 8.24

KNOWN: Inlet temperature and flowrate of oil moving through a tube of prescribed diameter and surface temperature.

FIND: (a) Oil outlet temperature $T_{m,o}$ for two tube lengths, 5 m and 100 m, and log mean and arithmetic mean temperature differences, (b) Effect of L on $T_{m,o}$ and $\overline{Nu}_D$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic energy, potential energy and flow work changes, (3) Constant properties.

PROPERTIES: *Table A.4*, Oil (330 K): c_p = 2035 J/kg·K, μ = 0.0836 N·s/m^2, k = 0.141 W/m·K, Pr = 1205.

ANALYSIS: (a) Using Eqs. 8.42b and 8.6

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{\pi DL}{\dot{m}c_p}\bar{h}\right)$$

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.5\,\text{kg/s}}{\pi\times 0.025\,\text{m}\times 0.0836\,\text{N}\cdot\text{s/m}^2} = 304.6$$

Since entry length effects will be significant, use Eq. 8.56

$$\bar{h} = \frac{k}{D}\left[3.66 + \frac{0.0688(D/L)Re_D\,Pr}{1+0.04\left[(D/L)Re_D\,Pr\right]^{2/3}}\right] = \frac{0.141\,\text{W/m}\cdot\text{K}}{0.025\,\text{m}}\left[3.66 + \frac{2.45\times 10^4\,D/L}{1+205(D/L)^{2/3}}\right]$$

For L = 5 m, $\bar{h} = 5.64(3.66+17.51) = 119\,\text{W/m}^2\cdot\text{K}$, hence

$$T_{m,o} = 100^\circ\text{C} - \left(75^\circ\text{C}\right)\exp\left(-\frac{\pi\times 0.025\,\text{m}\times 5\,\text{m}\times 119\,\text{W/m}^2\cdot\text{K}}{0.5\,\text{kg/s}\times 2035\,\text{J/kg}\cdot\text{K}}\right) = 28.4^\circ\text{C} \quad <$$

For L = 100 m, $\bar{h} = 5.64(3.66+3.38) = 40\,\text{W/m}^2\cdot\text{K}$, $T_{m,o}$ = 44.9°C. <

Also, *for* L = 5 m,

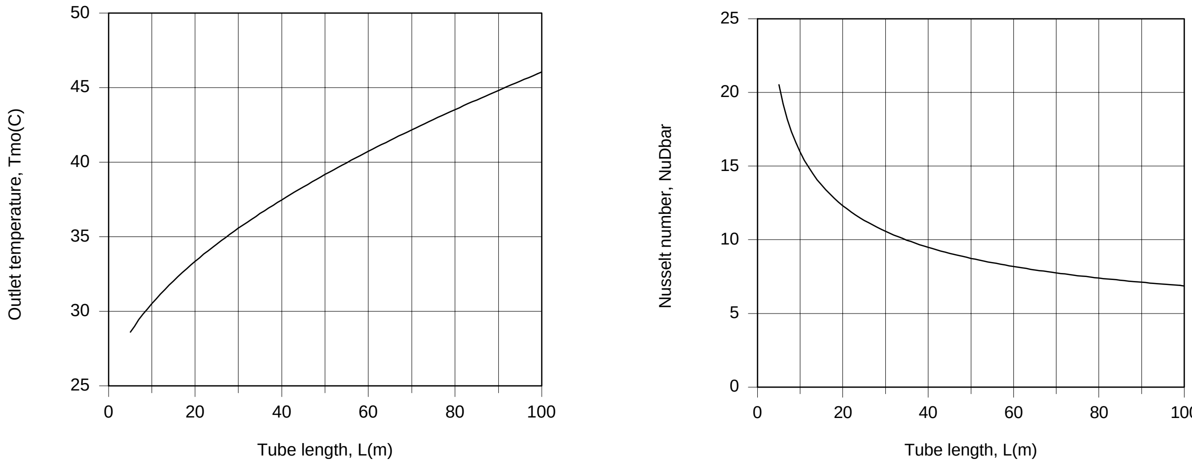
$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ell n\left(\Delta T_o/\Delta T_i\right)} = \frac{71.6-75}{\ell n(71.6/75)} = 73.3^\circ\text{C} \qquad \Delta T_{am} = \left(\Delta T_o + \Delta T_i\right)/2 = 73.3^\circ\text{C} \quad <$$

For L = 100 m, $\Delta T_{\ell m} = 64.5^\circ\text{C}$, $\Delta T_{am} = 65.1^\circ\text{C}$ <

(b) The effect of tube length on the outlet temperature and Nusselt number was determined by using the *Correlations* and *Properties* Toolpads of IHT.

Continued...

PROBLEM 8.24 (Cont.)



The outlet temperature approaches the surface temperature with increasing L, but even for L = 100 m, $T_{m,o}$ is well below T_s. Although $\overline{Nu}_D$ decays with increasing L, it is still well above the fully developed value of $Nu_{D,fd} = 3.66$.

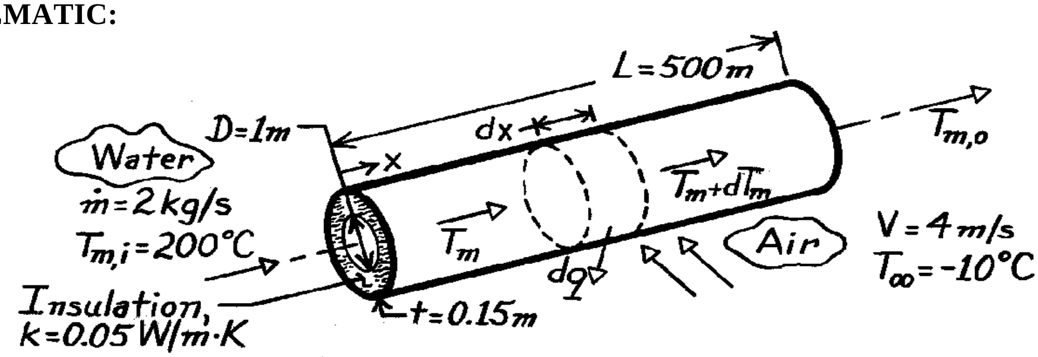
COMMENTS: (1) The average, mean temperature, $\overline{\overline{T}}_m$ = 330 K, was significantly overestimated in part (a). The accuracy may be improved by evaluating the properties at a lower temperature. (2) Use of ΔT_{am} instead of $\Delta T_{\ell m}$ is reasonable for small to moderate values of $(T_{m,i} - T_{m,o})$. For large values of $(T_{m,i} - T_{m,o})$, $\Delta T_{\ell m}$ should be used.

PROBLEM 8.69

KNOWN: Flow conditions associated with water passing through a pipe and air flowing over the pipe.

FIND: (a) Differential equation which determines the variation of the mixed-mean temperature of the water, (b) Heat transfer per unit length of pipe at the inlet and outlet temperature of the water.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature drop across the pipe wall, (2) Negligible radiation exchange between outer surface of insulation and surroundings, (3) Fully developed flow throughout pipe, (4) Negligible potential and kinetic energy and flow work effects.

PROPERTIES: *Table A-6*, Water ($T_{m,i}$ = 200°C): $c_{p,w}$ = 4500 J/kg·K, $\mu_w = 134 \times 10^{-6}$ N·s/m^2, k_w = 0.665 W/m·K, Pr_w = 0.91; *Table A-4*, Air (T_∞ = - 10°C): $\nu_a = 12.6 \times 10^{-6}$ m^2/s, k_a = 0.023 W/m·K, Pr_a = 0.71, $Pr_s \approx 0.7$.

ANALYSIS: (a) Following the development of Section 8.3.1 and applying an energy balance to a differential element in the water, we obtain

$$\dot{m}\, c_{p,w}\, T_m - dq - \dot{m}\, c_{p,w}\left(T_m + dT_m\right) = 0.$$

Hence $\qquad dq = -\dot{m}\, c_{p,w}\, dT_m$

where $\qquad dq = U_i dA_i\left(T_m - T_\infty\right) = U_i \pi D\, dx\left(T_m - T_\infty\right).$

Substituting into the energy balance, it follows that

$$\frac{d\,T_m}{dx} = -\frac{U_i \pi D}{\dot{m}\, c_p}\left(T_m - T_\infty\right). \qquad (1) \quad <$$

The overall heat transfer coefficient based on the inside surface area may be evaluated from Eq. 3.30 which, for the present conditions, reduces to

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{D}{2k}\ell n\left(\dfrac{D+2t}{D}\right) + \dfrac{D}{D+2t}\,\dfrac{1}{h_o}}. \qquad (2)$$

For the *inner water flow*, Eq. 8.6 gives

$$Re_D = \frac{4\,\dot{m}}{\pi D \mu_w} = \frac{4\times 2\text{ kg/s}}{\pi\left(1\text{ m}\right)\times 134\times 10^{-6}\text{ kg/s}\cdot\text{m}} = 19{,}004.$$

Continued …..

PROBLEM 8.69 (Cont.)

Hence, the flow is turbulent. With the assumption of fully developed conditions, it follows from Eq. 8.60 that

$$h_i = \frac{k_w}{D} \times 0.023\, Re_D^{4/5}\; Pr_w^{0.3}. \tag{3}$$

For the *external air flow*

$$Re_D = \frac{V(D+2t)}{n} = \frac{4\text{ m/s}\,(1.3\text{m})}{12.6\times10^{-6}\text{ m}^2/\text{s}} = 4.13\times10^5.$$

Using Eq. 7.31 to obtain the outside convection coefficient,

$$h_o = \frac{k_a}{(D+2t)} \times 0.076\, Re_D^{0.7}\; Pr_a^{0.37}\; (Pr_a/Pr_s)^{1/4}. \tag{4}$$

(b) The heat transfer per unit length of pipe at the inlet is

$$q' = p\; D\, U_i\left(T_{m,i} - T_\infty\right). \tag{5}$$

From Eqs. (3 and 4),

$$h_i = \frac{0.665\text{ W/m}\cdot\text{K}}{1\text{ m}} \times 0.023(19{,}004)^{4/5}\;(0.91)^{0.3} = 39.4\text{ W/m}^2\cdot\text{K}$$

$$h_o = \frac{0.023\text{ W/m}\cdot\text{K}}{(1.3\text{ m})} \times 0.076\left(4.13\times10^5\right)^{0.7}(0.71)^{0.37}(1)^{1/4} = 10.1\text{ W/m}^2\cdot\text{K}.$$

Hence, from Eq. (2)

$$U_i = \left[\frac{1}{39.4\text{ W/m}^2\cdot\text{K}} + \frac{1\text{ m}}{0.1\text{ W/m}\cdot\text{K}}\,\ell n\left(\frac{1.3}{1}\right) + \frac{1}{1.3}\times\frac{1}{10.1\text{ W/m}^2\cdot\text{K}}\right]^{-1} = 0.37\text{ W/m}^2\cdot\text{K}$$

and from Eq. (5)

$$q' = p\,(1\text{ m})\;\left(0.37\text{ W/m}^2\cdot\text{K}\right)\;(200+10)^\circ\text{C} = 244\text{ W/m}.$$ <

Since U_i is a constant, independent of x, Eq. (1) may be integrated from x = 0 to x = L. The result is analogous to Eq. 8.42b and may be expressed as

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{p\;DL}{\dot m\, c_{p,w}}U_i\right) = \exp\left(-\frac{p\times1\text{m}\times500\text{m}}{2\text{ kg/s}\times4500\text{ J/kg}\cdot\text{K}}\times0.37\text{ W/m}^2\cdot\text{K}\right)$$

Hence

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = 0.937.$$

$$T_{m,o} = T_\infty + 0.937\left(T_{m,i} - T_\infty\right) = 187^\circ\text{C}.$$ <

COMMENTS: The largest contribution to the denominator on the right-hand side of Eq. (2) is made by the conduction term (the insulation provides 96% of the total resistance to heat transfer). For this reason the assumption of fully developed conditions throughout the pipe has a negligible effect on the calculations. Since the reduction in T_m is small (13°C), little error is incurred by evaluating all properties of water at $T_{m,i}$.