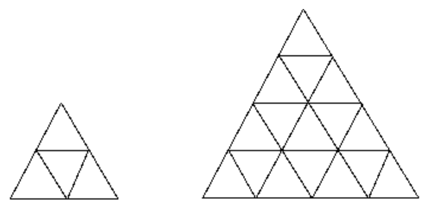
10.420 / 10.520
Problem Set #4

1. Consider a molecule with a geometrical shape approximating a flat triangle.

   a) Determine the number of actual and potential favorable packing forces for the illustrated 4-mer and 16-mer. Assume that each edge-wise interaction with another molecule is worth $\alpha kT/3$ per molecule.



   b) Provide the equations that generalize these forces for an n-mer.

   c) For the n-mer, derive and expression for μ_0^N that is a function of μ_0^∞ and N.

2. Consider a molecule that "self-assembles" into a rod at 25 °C with an interaction energy between subunits of αkT = 24.75 kJ/mol (10 kT).

   a) Produce plots of X_N vs. N for total concentrations, C, of 0.01 mM, 1 mM, and 10 mM for N = 1 to N = 25. (hint: you will need eqns 16.18 and 16.19 in your Israelachvili handout and/or lecture notes).

   b) Determine the nalue of N where X_N reaches a maximum for total concentrations, C, of 0.1 mM, 1 mM, 10 mM, 100 mM, and 1 M.

   c) N_{max}, the value of N where X_N reaches a maximum, is a function of the concentration of C. Plot log(N_{max}) vs. log(C) and determine the relationship between these two parameters. In your analysis, plot all values, but only use values of $N_{max} > 3$

3. Provide a physical (P) and/or molecular (M) reason for the following observations. Molecular (M) rationales should include a rough figure.

   a) Hexadecane wets glass (P).

   b) Hexadecane containing stearic acid, $CH_3(CH_2)_{16}CO_2H$, does not wet glass (M).

   c) Addition of small amounts of sodium dodecyl sulfate (SDS) increases the spreading of water on polyethylene (P/M).

   d) Continued adition of SDS to water begins to have no effect on the spreading nature of water on polyethylene (M).

   e) The contact angle for a drop of rainwater on the hood of a car is greater during a falling rain than afterwards (P).

4. One method for measuring surface tension is to use a U-tube with a smaller radius on one side and a larger radius on the other. In such a system, a liquid that wets glass was determined to have a Δh of 19 mm between the levels of the two meniscuses in the U-tube.

   a) Draw the U-tube and liquid identifying Δh and the two radii.

   b) If the radii are 1 mm and 10 mm, and ρ is 950 kg/m^3, determine the surface tension of the liquid.

5. a) Given the surface tensions of heptane (20.14 dyn/cm) and diethylene glycol (30.9 dyn/cm), calculate the works of cohesions for these solvents. The work of cohesion is the energy (erg/cm^2) required to separate one body of liquid/material into two. If two 1 μL drops of heptane that are suspended in air combine to form one 2 μL drop, estimate the energy gain (erg/cm^3) for this process and a temperature rise for the heptane.

   b) Given the interfacial tension of heptane-diethylene glycol (10.6 dyn/cm), calculate the work of adhesion for the heptane-diethylene glycol interface.

6. a) A fabric is made of wool fibers of individual diameter 20 μm and density of 1.3 g/cm^3. The contact angle for water on a single fiber is 120°. Calculate the contact angle of water on fabric woven so that its bulk density is 0.8 g/cm^3.

   b) If the fibers are chemically modified so that the contact angle of water on the individual fiber is 60°, what would be the contact angle on the above woven fabric?