Lecture 2 (part B)
My Not-So-Fair Casino

- we play one game only:
  - heads / tails

- we have both fair and biased coins
- at the beginning of the day my croupier picks a coin and heads to his or her table

- can you win?
- can you loose?
- can you tell *which* coin the croupier has selected?
My Not-So-Fair Casino (cont.)

- what do you need to do in order to be able to tell which coin is in use?

- how can you tell which coin is in use?

- Is the following information useful?
  - \( \Pr(\text{head}/\text{coin is fair}) = \Pr(\text{tail}/\text{coin is fair}) = \frac{1}{2} \)
  - \( \Pr(\text{head}/\text{coin is biased}) = \frac{3}{4} \)
  - \( \Pr(\text{tail}/\text{coin is biased}) = \frac{1}{4} \)
Finite Automata

- a finite automaton comprises
  - states
  - transitions occurring on input symbols that come from an alphabet $\Sigma$
- a special "begin" state / one or more special "end" states
- a (directed) transition diagram is associated with an fsa

- a f.a. accepts a string $x$ iff the string $x$ leads to a sequence of transitions from "begin" to "end"
  (i.e. output of an f.a. is "accept" or "don't accept")
Finite Automata (cont.)
Finite Automata (cont.)

- the set of all strings accepted by a f.a. form the language accepted by the f.a.
- a language is regular iff it is accepted by an f.a.

- an f.a. can be
  - deterministic
  - non-deterministic (allows *multiple* transitions out of a state on the *same* symbol)

- d-f.a. and non-d-f.a. are equivalent!
Finite Automata (cont.)

example of a non-deterministic finite automaton
Regular Expressions

- Simple expressions defined in conjunction with an alphabet Σ and describing regular languages
- Definition:
  - Ø is a r.e. and denotes {} 
  - ε is a r.e. that denotes {ε} 
  - For each a in Σ, a is a r.e. that denotes {a} 
  - If e₁ and e₂ are r.e. that denote the languages E₁ and E₂ respectively, then e₁ + e₂, e₁ e₂ and e₁* denote languages E₁ + E₂, E₁ E₂ and E₁* respectively
let's derive the regular expression for this f.a. ...
Finite Automata with Output

- two choices:
  - attach output to the state that is entered (Moore machines)
  - attach output to the transition that is traversed (Mealy machines)

- Moore and Mealy machines are equivalent!
Trivial Extensions

\[ \text{begin} \rightarrow \varepsilon \rightarrow 0 \rightarrow \varepsilon \rightarrow \text{end} \]

Probabilities, Dynamic Programming

Date: February 11, 2003
More Extensions

- a f.a. that "accepts" all DNA inputs
  (not all transitions are labelled)

- what is the limitation of this state diagram?
My Not-So-Fair Casino / Part 2

- what if my croupier could switch coins on-the-fly while at his/her table?
  - can you win?
  - can you lose?

- can you tell?

- how is this different from the previous situation?

- Markov Chain vs. Hidden Markov Model