10.555 Bioinformatics
Spring 2003

Lecture 2 (part B)

My Not-So-Fair Casino

- we play one game only:
  - heads / tails

- we have both fair and biased coins
- at the beginning of the day my croupier picks a coin and heads to his or her table

- can you win?
- can you loose?
- can you tell *which* coin the croupier has selected?

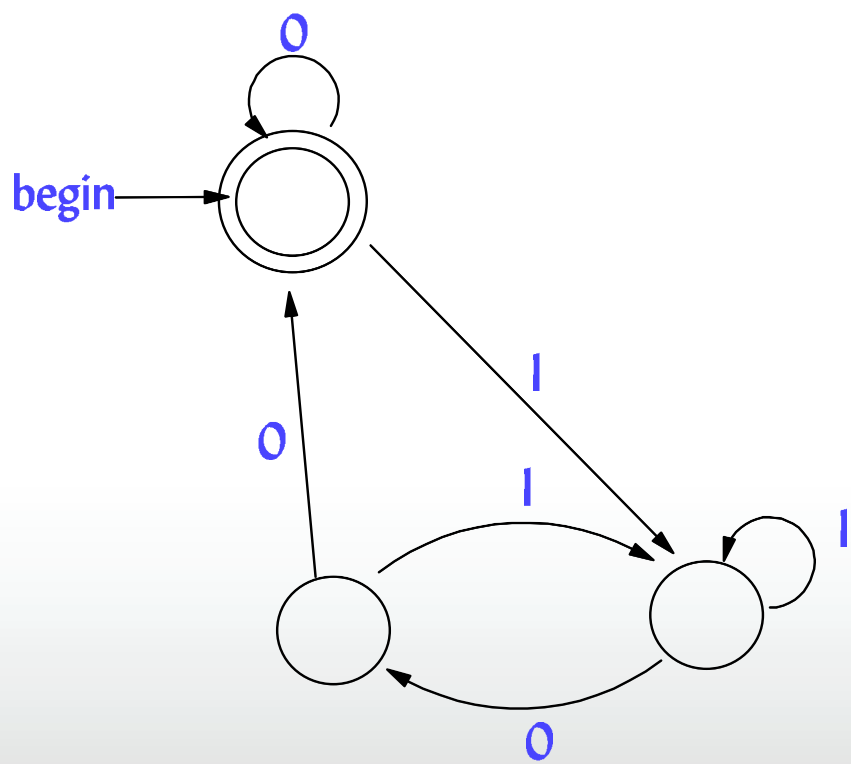
My Not-So-Fair Casino (cont.)

- what do you need to do in order to be able to tell which coin is in use?
- how can you tell which coin is in use?
- Is the following information useful?
  - Pr(head/coin is fair)=Pr(tail/coin is fair)=1/2
  - Pr(head/coin is biased)= 3/4
  - Pr(tail/coin is biased)=1/4

Finite Automata

- a finite automaton comprises
  - *states*
  - *transitions* occurring on input symbols that come from an alphabet Σ
- a special "begin" state / one or more special "end" states
- a (directed) *transition diagram* is associated with an fsa
- a f.a. *accepts* a string x iff the string x leads to a sequence of transitions from "begin" to "end" (i.e. output of an f.a. is "accept" or "don't accept")

Finite Automata (cont.)



Finite Automata (cont.)

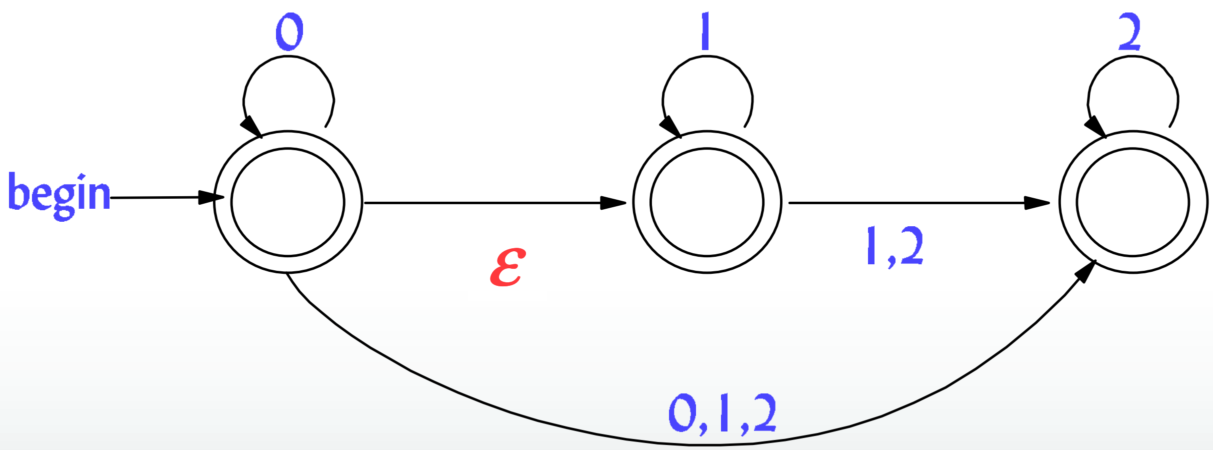
- the set of all strings accepted by a f.a. form the *language* accepted by the f.a.
- a language is *regular* iff it is accepted by an f.a.

- an f.a. can be
  - deterministic
  - non-deterministic (allows *multiple* transitions out of a state on the *same* symbol)

- d-f.a. and non-d-f.a. are equivalent!

Finite Automata (cont.)

- example of a non-deterministic finite automaton

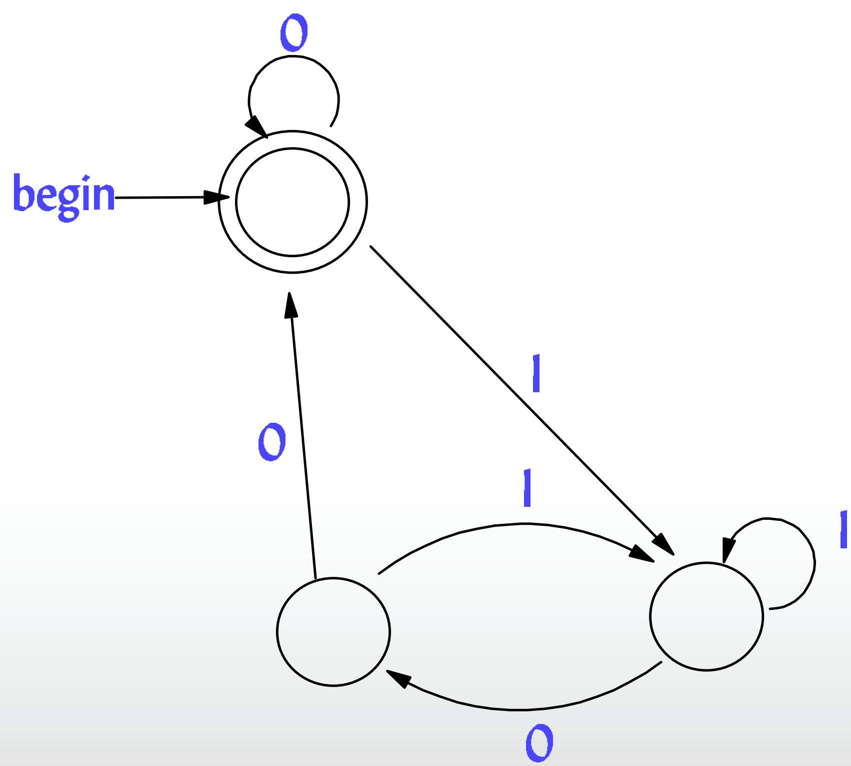


Regular Expressions

- simple expressions defined in conjunction with an alphabet Σ and describing regular languages
- definition:
  - $\varnothing$ is a r.e. and denotes { }
  - ε is a r.e. that denotes $\{\varepsilon\}$
  - for each a in Σ, a is a r.e. that denotes {a}
  - if e_1 and e_2 are r.e. that denote the languages E_1 and E_2 respectively, then e_1+e_2, e_1e_2 and e_1^* denote languages E_1+E_2, E_1E_2 and E_1^* respectively

Regular Expressions (cont.)

- let's derive the regular expression for this f.a. ...

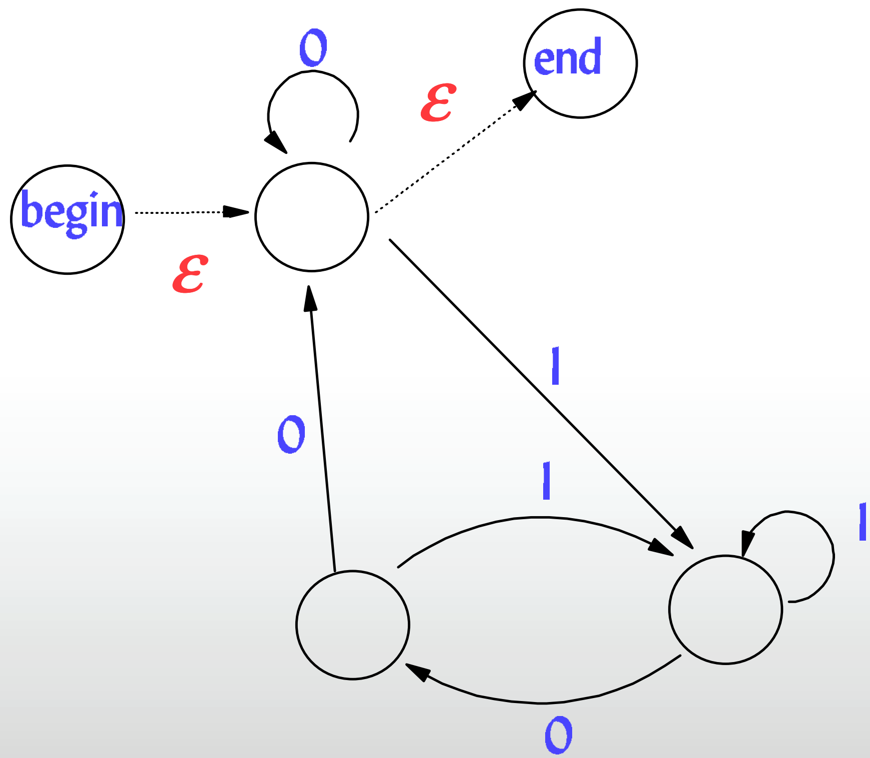


Finite Automata with Output

- two choices:
  - attach output to the state that is entered (Moore machines)
  - attach output to the transition that is traversed (Mealy machines)

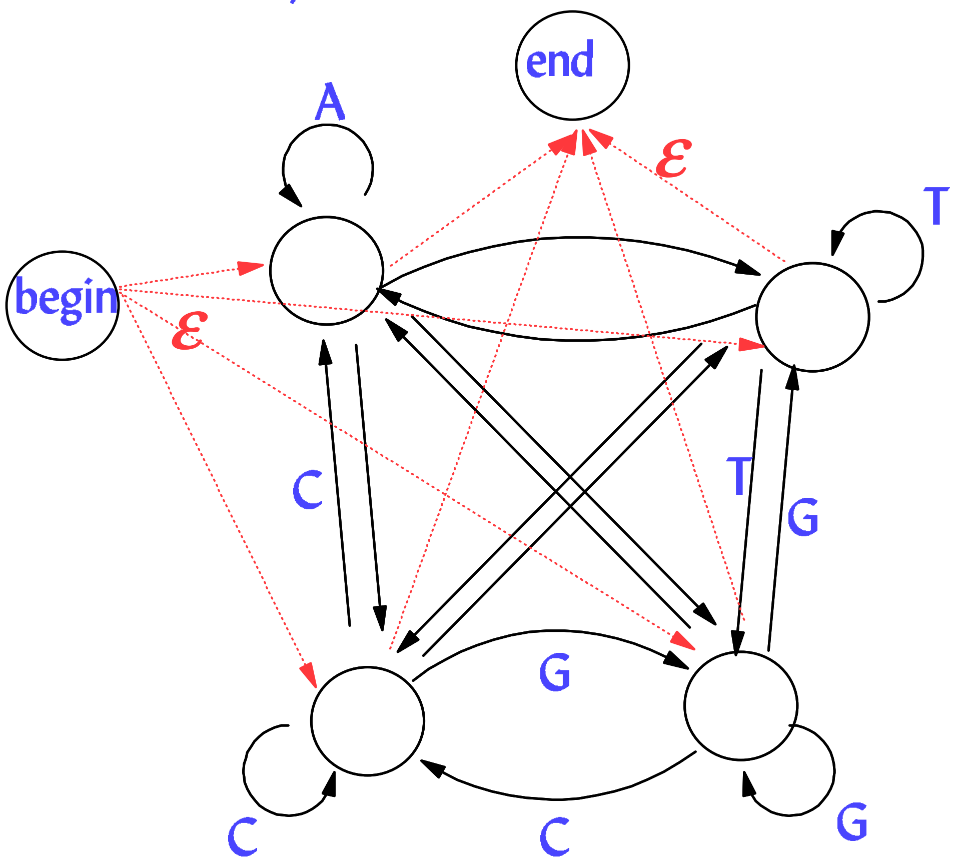
- Moore and Mealy machines are equivalent!

Trivial Extensions



More Extensions

- a f.a. that "accepts" all DNA inputs
  (not all transitions are labelled)



- what is the limitation of this state diagram?

My Not-So-Fair Casino / Part 2

- what if my croupier could switch coins on-the-fly while at his/her table?
- can you win?
- can you lose?
- can you tell?
- how is this different from the previous situation?
- Markov Chain vs. Hidden Markov Model