DIFFERENT POLICY OBJECTIVES OF THE ROAD AUTHORITY IN THE OPTIMAL TOLL DESIGN PROBLEM

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1. INTRODUCTION

In this paper we are dealing with the optimal toll design problem for planning purposes. Uniform, (quasi)uniform and variable (time-varying) tolls during the peak are considered (see Section 2.1.), in which travellers’ responses (route and departure time choice) to these tolls are taken into account. Charging may in general lead to a lower travel demand as well, however for simplicity the total number of travellers is assumed constant in this paper. We will consider tolling schemes in which the road authority has already decided on which links to toll and which periods to toll. Then for different tolling patterns (e.g. uniform or time varying), the aim is to determine optimal toll levels given a certain policy objective, such as minimizing total travel time or maximizing total revenues.

The focus of this paper is to describe the dynamic framework of the optimal toll design problem and to mathematically formulate the problem. Furthermore, it will be illustrated that different objectives of the road authority and different tolling schemes can lead to different optimal toll levels. The main contributions of this paper are the following. First, different objective functions of the road authority are explored, namely maximizing revenues and minimizing total network travel time. Secondly, different tolling schemes and their impact on the objective of the road authority are analysed. Finally, a dynamic instead of static traffic assignment model with road pricing is proposed.

Dynamic pricing models in which network conditions and link tolls are time-varying, have been addressed in Wie and Tobin (1998), comparing the effectiveness of various pricing policies (time-varying, uniform, and step-tolls). A limitation of these models is that they are restricted to a bottleneck or a single destination network. Mahmassani and Herman (1984) and Ben-Akiva et al. (1986) developed dynamic marginal (first-best) cost pricing models for general transportation networks. As indicated by these authors, the application of their models might not be easy to implement in practice. Moreover, since tolls are based on marginal cost pricing, it is implicitly assumed that all links can be priced dynamically, which is practically infeasible.

In the work of Viti er al. (2003) a dynamic congestion-pricing model is formulated as a bi-level programming problem, in which the prices are allowed to affect the (sequentially) modelled route and departure time choice of travellers. Abou-Zeid (2003) developed some models for pricing in dynamic
traffic networks. In the work of Joksimovic et al. (2005), the time-varying pricing model including route and departure choice is solved using a simple algorithm for the road authority’s objective of optimising total travel time in the network. In this paper these models are extended to include different and more general tolling schemes and different objectives of the road authority.

The remainder of this paper is as follows. In Section 2, the problem formulation of the time-varying optimal toll design is described. The road authority objectives are mathematically formulated in Section 3. In Section 4, a solution procedure to the optimal toll design problem is proposed. Case studies for both minimizing travel time and maximizing revenue objectives are provided in Section 5. This paper finishes with a conclusion part (Section 6).

2. OPTIMAL TOLL DESIGN PROBLEM

In the optimal toll design problem described in this paper, the links and time periods to toll are given, and the aim is to determine optimal toll levels for different tolling patterns given a specific objective of the road authority (such as minimizing congestion, maximizing total toll revenues, maximizing accessibility, maximizing social welfare, etc.). The resulting road-pricing scheme describes for each link and each time period how much a traveller has to pay for entering the link at that time.

The model framework (see Figure 1) consists of two main parts, namely a road pricing part and a travel behaviour part. This framework essentially describes a bi-level problem. The upper level is the road pricing problem given travel responses, while the lower level is the travel behaviour part for given toll levels. Both parts will be explained in more detail in the following subsections.
In the road pricing model, the road authority aims to introduce the best tolling scheme depending on its goals. A tolling scheme is defined as a package in which a set of links and time intervals is chosen to toll, together with the toll levels corresponding to certain time-varying tolling patterns. Road authorities may have different goals, leading to different objective functions in the model. Depending on its goal, the road authority has to select the best tolling scheme. We assume that the links to be tolled, $A$, are given, as well as the tolled time period, $\bar{T}_a$, for each tolled link $a$. First, some possible link tolling patterns will be described. Secondly, a short description of the travel behaviour part will be given.

2.1 Link tolling patterns

Different link tolling patterns over time can be considered by the road authority. As illustrated in Figure 2, we distinguish (i) a uniform tolling scheme (toll levels are constant over the entire study time period $T$), (ii) a quasi-uniform tolling scheme (tolls levels are constant over a specified time period and zero otherwise), and (iii) a variable tolling scheme (tolls levels are time-varying).

![Figure 2](image_url)

**Figure 2** Different tolling schemes with respect to time of day

The three different tolling patterns can be formulated as follows:

Variable: 
$$\theta_a(t) = \begin{cases} \phi_a(t)\bar{\theta}_a, & \text{if } a \in \bar{A}, t \in \bar{T}_a; \\ 0, & \text{otherwise}, \end{cases}$$

where $0 \leq \phi_a(t) \leq 1$. \hfill (1)

Quasi-uniform: 
$$\theta_a(t) = \begin{cases} \bar{\theta}_a, & \text{if } a \in \bar{A}, t \in \bar{T}_a; \\ 0, & \text{otherwise}. \end{cases}$$

Uniform: 
$$\theta_a(t) = \begin{cases} \bar{\theta}_a, & \text{if } a \in \bar{A}, t \in T; \\ 0, & \text{otherwise}. \end{cases}$$

Where
\( \theta_a(t) \): variable toll value on link \( a \) at time \( t \), where \( a \in \mathcal{A}, t \in \mathcal{T}_u \)

\( \phi_a(t) \): predefined function over time for each tolled link

\( \bar{\theta}_a \): maximum toll to be paid at link \( a \in \mathcal{A} \)

\( \mathcal{T}_u \): tolled time intervals at link \( a \in \mathcal{A} \)

In all cases, there is only a single toll level \( \bar{\theta}_a \) to be determined for each tolled link \( a \in \mathcal{A} \), which indicates the maximum toll to be paid for that link. Clearly, the quasi-uniform tolling pattern is a special case of the variable tolling pattern (assuming that \( \phi_a(t) = 1, \forall t \in \mathcal{T}_u \)), while the uniform pattern is a special case of the quasi-uniform pattern (by furthermore assuming that \( \mathcal{T}_u = T \)). In case of uniform tolls, the toll levels for all time intervals are set to toll level \( \bar{\theta}_a \) for tolled link \( a \). In case of quasi-uniform tolls, only the tolls in the time intervals \( t \in \mathcal{T}_u \) will be set to the toll level \( \bar{\theta}_a \), and will be zero outside that time period. For variable tolls we assume there is a given predefined function \( \phi_a(t) \) over time for each tolled link. In other words, the proportions of the time-varying tolls are fixed (hence, the shape of the toll levels over time is given). All three toll patterns will be used in the case study in this paper (Section 5).

A tolling scheme indicates a combination of a tolling pattern and corresponding maximum toll levels, hence describing the following variables:

(a) tolled links \( \mathcal{A} \) (assumed given);

(b) tolled time intervals \( \mathcal{T}_u, \forall a \in \mathcal{A} \) (assumed given);

(c) link tolling pattern \( \phi_a(t), \forall a \in \mathcal{A}, \forall t \in \mathcal{T}_u \) (assumed given);

(d) maximum toll levels \( \bar{\theta}_a, \forall a \in \mathcal{A}, \forall t \in \mathcal{T}_u \) (to be optimised).

This means that for each tolling scheme, toll levels \( \theta_a(t) \) are known for each link \( a \) and each time interval \( t \). In this paper the tolled links, the tolled time intervals, and the link tolling patterns are assumed to be input. The road authority aims to find the set of optimal maximum toll levels \( \bar{\theta}^* \equiv \left[ \bar{\theta}_a \right] \) that optimise some given objective. The next section (Section 3) will further discuss these road authority’s policy objectives.

2.2 Modelling of travel behaviour

For a modelling travel behaviour a framework described in Joksimovic et. al. (2006) will be used. Here we will state only the general path cost function.
The general path cost function is extended to capture tolls, heterogeneous users and departure time choice.

$$c_{mp}^{rs}(k) = \alpha_m \cdot \tau_{mp}^{rs}(k) + \beta \left| k - \xi^{rs} \right| + \chi \left| k + \tau_{mp}^{rs}(k) - \xi^{rs} \right|$$

(4)

where

- $c_{mp}^{rs}(k)$: experienced generalized travel cost for traveller $m$ on path $p$ between origin $r$ and destination $s$ departing in period $k$;
- $\alpha_m$: value of time for traveller class $m$;
- $\tau_{mp}^{rs}(k)$: travel time on path $p$ at time $t$ between origin $r$ and destination $s$ if departing in period $k$;
- $\beta, \chi$: departure time parameters
- $\xi^{rs}, \xi^{rs}$: preferred departure and preferred arrival time respectively for trip $rs$ (equal for traveller classes);

For modelling of route and departure time choice a MNL model is used. In this framework we suppose that tolls are additive, as well as travel time. It should be noted that a stochastic user-equilibrium (SUE) model is used.

The dynamic link travel time function is used:

$$\tau_a(t) = \tau^0_a + b_a x_a(t)$$

where

- $\tau_a(t)$: Travel time on link $a$ for travellers entering at time $t$
- $\tau^0_a$: a free-flow travel time
- $x_a(t)$: inflow in link $a$ entering at time $t$
- $b_a$: delay parameter

Eqn. (5) relates for each link $a$ the number of vehicles $x_a(t)$ to the travel time $\tau_a(t)$ on that link as an increasing function, where each link has a free-flow travel time and a delay component (with $b_a$ a nonnegative parameter).

For more information about the modelling of the lower level of the optimal toll design problem, see Joksimovic et al. (2006).

### 3. ROAD AUTHORITY OBJECTIVES

For our experiments, two different objectives are chosen, namely (i) maximization of total toll revenues, and (ii) minimization of the total travel time.

The road authority seeks to optimise its objective by selecting the optimal maximum toll levels i.e.
\[ \tilde{\theta}^* = \arg \min_{\tilde{\theta} \in \Theta} Z(\tilde{\theta}). \]  

Where

\( \tilde{\theta}^* \): vector of optimal maximum toll levels

\( Z(\tilde{\theta}) \): the objective function to be optimised

The set of feasible maximum toll levels, which typically includes upper and lower bounds and for each tolled link, can be expressed as follows:

\[ \Theta = \{ \tilde{\theta} : \tilde{\theta}_a^\text{min} \leq \tilde{\theta}_a \leq \tilde{\theta}_a^\text{max}, \forall a \in A \}. \]  

Where

\( \Theta \): set of feasible maximum toll levels

\( \tilde{\theta}_a^\text{min} \): lower bound for each tolled link \( a \in A \)

\( \tilde{\theta}_a^\text{max} \): upper bound for each tolled link \( a \in A \)

For each evaluation of the objective function \( Z(\tilde{\theta}) \) (Eqn. (5)) a travel behaviour part (modelled as a dynamic traffic assignment (DTA) problem) has to be solved (see Figure 1).

The total revenues on the network are a product of the inflows into tolled links and the corresponding toll levels valid at the link entrance times. Given some maximum toll levels \( \tilde{\theta} \), the objective function (Eqn. (7)) describes the total toll revenues to be maximized.

\[ Z_{\text{revenue}}(\tilde{\theta}) = \sum_a \sum_t u_a(t)\tilde{\theta}_a(t), \]  

where

\( Z_{\text{revenue}}(\tilde{\theta}) \): objective function of maximising revenues

Instead of maximizing total toll revenues, the road authority may be more interested in minimizing total travel time (e.g. as a proxy for minimizing congestion or pollution). Objective function \( Z_{\text{time}}(\tilde{\theta}) \) describes the total travel time on the network,

\[ Z_{\text{time}}(\tilde{\theta}) = \sum_a \sum_t u_a(t)r_a(t), \]  

\( \sum_a \sum_t u_a(t)r_a(t) \)
4. SOLUTION ALGORITHM TO THE OPTIMAL TOLL DESIGN PROBLEM

Each component of the optimal toll design problem can be solved using various types of algorithms. The outline of the complete algorithm for the case of variable tolls is as follows. The algorithm starts with specifying the grid of considered toll levels for all links to be tolled, satisfying the constraints (lower and upper bounds). In each iteration, the algorithm solves a DTA problem (i.e. finds a dynamic stochastic multiclass user equilibrium solution) based on the current toll levels and sets new tolls that can potentially optimize the objective functions described in Eqn. (7) or Eqn. (8).

Because the algorithm is a grid-search method it stops after all feasible toll levels in the grid have been considered satisfying equilibrium level. At this stage of the research, the focus is mainly to investigate the framework of the model and the properties of the solutions for different objectives and tolling schemes, and not on the development of algorithms. More efficient algorithms will be developed in the future research.

The two-stage iterative grid-search procedure for the optimal time-varying toll problem with DTA (including joint route and departure time choice) can be outlined. Firstly, the input and output is described. Secondly, the steps of the algorithm are outlined.

Input:
Network \( G = (N, A) \), set of tolled links \( \mathcal{A} \), set of tolled time intervals link tolling patterns \( \phi_a(t) \), travel demand, logit scale par grid dimensions \( I_a \), meter, free-flow link travel times, number of DTA iterations \( J \), grid dimensions \( I_a \), road authority objective (Eqns. (8),(9))

Output:
Optimal maximum toll levels \( \bar{\theta}^* \), optimal value of objective function \( Z_{rev}^* \) or \( Z_{time}^* \).

**Grid Search algorithm**

**Outer loop: PRICING**

**Step 1: [Initialization]**

The maximum toll level grid for each link \( a \) is given by

\[
\bar{\theta}_a^{(i)} = \theta_a^{\min} + \frac{i}{I_a} \left( \theta_a^{\max} - \theta_a^{\min} \right), \quad i = 0, K \cdot I_a.
\]
All combinations of all maximum toll levels for all links determine the set of grid vectors \( \bar{\theta}^{(i)} = [\bar{\theta}^{(i)}_a] \), which contains \( I \equiv \prod_a (I_a + 1) \) elements.

Set \( i := 1 \) and set \( Z^* = +\infty \).

**Step 2: [Set toll values]**

Select grid point \( i \) for the toll levels, yielding tolls \( \theta^{(i)}_a(t) \)

**Inner loop: DTA**

**Step 3: [Dynamic traffic assignment]**

In this step the DTA model is solved. Initialisation assumes an empty network and free flow conditions. The travel costs according to Eqn. (4) are computed. The new intermediate dynamic route flow pattern is determined and updates using Method of Successive Averages (MSA) method. Finally, a dynamic network-loading (DNL) model is performed and the convergence criteria for the lower level of the optimal toll design problem is checked. For more information see Joksimovic (2006).

**Step 4: [Compute objective function]**

Compute the objective function \( Z(\theta^{(i)}) \) using Eqn. (8) or Eqn. (9).

If \( Z(\theta^{(i)}) < Z^* \), then set \( Z^* = Z(\theta^{(i)}) \) and set \( \bar{\theta}^* = \bar{\theta}^{(i)} \).

**Step 5: [Convergence of road pricing level]**

While \( i < N \), set \( i := i + 1 \) and return to Step 2.

Otherwise, the algorithm is terminated and \( \bar{\theta}^* \) is the set of optimal toll levels.

Performing this simple iterative procedure, we explore all toll level combinations and find the optimal value of the objective function. Regarding the convergence of this algorithm, the inner DTA loop using the widely used heuristic MSA procedure typically converges to an equilibrium solution, although convergence cannot be proven. In the outer road pricing loop the whole solution space is investigated with a certain grid accuracy (yielding a finite number of solutions that are evaluated).

5. CASE STUDIES

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5.1 Network description, travel demand and input parameters

The solution procedure proposed in the previous section has been applied for illustrative reasons a small network, see Figure 2.

Figure 2 Network description

The network consists of just a single OD-pair connected by two non-overlapping paths where only link 2 is tolled. Since there is only one OD pair, we will ignore the OD subindices \((r,s)\) in the variables. Two user classes with different value of time (VOT) are distinguished. The total travel demand for departure period \(K = \{1,K,20\}\) from node 1 to node 3 is assumed 86, of which 50% are high VOT travellers and 50% low VOT travellers.

The following parameter values are used on route level: preferred departure time \(\zeta = 10\) [period] preferred arrival time \(\xi = 15\) [period], value of time for class 1 \(\alpha_1 = 0.25\) [eur/periods], value of time for class 2 \(\alpha_2 = 0.75\) [eur/periods], penalty for deviating from preferred departure time \(\beta = 0.25\) [eur/periods], penalty for deviating from preferred arrival time \(\gamma = 1\), and scale parameter in MNL model is 0.8. On the link level, we assume that route 1 with a free-flow travel time of 7.0 time units is longer than route 2 (3.0 time units) by setting the free-flow link travel times in Eqn. (5) to \(\tau_1^0 = \tau_3^0 = 3.5\) and \(\tau_2^0 = 3.0\). Furthermore, it is assumed that the first route will never show congestion, hence \(b_1 = b_2 = 0\), while congestion is possible on link 2 for which we set \(b_2 = 0.005\) in Eqn. (5).

5.2 Optimal tolls schemes

In Section 2.1 three different tolling schemes have been mentioned, namely the uniform, the quasi-uniform, and the variable tolling scheme, see Eqns. (1)–(3). All three tolling regimes will be considered in the case studies below. Note that in this case study with only a single link (link 2) tolled, determining the optimal toll for each tolling regime (even for the variable tolling scheme) only requires to find a single optimum toll level, \(\theta^*_2\). For the quasi-uniform tolling scheme we assume that a toll will be levied in the peak period, i.e. the tolling period is \(\bar{T} = \{8,9,10,11,12\}\) in Eqn. (1). In the variable tolling scheme only periods 9, 10, and 11 will be tolled with fixed proportions 0.6, 1.0, and 0.6, that is
\[
\phi_2(t) = \begin{cases} 
1.0, & \text{if } t = 10; \\
0.6, & \text{if } t = 9, 11; \\
0, & \text{otherwise.} 
\end{cases}
\] (11)

6.3 Optimal tolls for maximizing total revenues

Assume that the road authority aims to maximize total revenues, as formulated in Eqn. (8), by selecting the best tolling scheme and the best toll level. The three different tolling regimes as mentioned above will be considered. For each tolling scheme and for each toll level, the dynamic traffic assignment (DTA) problem can be solved. In Figure 4 the total revenues are plotted for each tolling scheme for all \( 0 \leq \bar{\theta}_2 \leq 15 \) (although not shown here, in all cases the DTA model converged).

![Figure 4](image-url)

Figure 4  Total revenues for different tolling schemes and toll levels
In case the toll level is zero, there are clearly no revenues. For very high toll levels, all travellers will choose to travel on the untolled route, resulting in zero revenues as well. As can be observed from Figure 4, uniform tolling with $\theta_2 = 3.11$ yields the highest revenues. The variable tolling scheme is not able to provide high revenues due to the small number of tolled time periods.

5.3 Optimal tolls for minimizing total travel time

In this case study, the road authority aims at minimizing total travel time on the network (see Eqn (9)) by selecting the best tolling scheme and the best toll level. Figure 5 depicts the total travel times for different tolling schemes and toll levels.

As can be observed from Figure 5, it seems possible to decrease the total travel time on the network by imposing a toll on congested route 2. High toll levels will push all travellers during the tolled period away from route 2 to the longer route 1, yielding higher total travel times. Variable tolling with $\theta_2 = 3.24$ (yielding $\theta_2^{10} = 3.24$ and $\theta_2^{9} = \theta_2^{11} = 1.99$ in the periods 10, 9, 11 respectively, zero otherwise) according to Eqns. (1) and (11)) results in the lowest total travel time. The objective function looks somewhat irregular which...
can be explained by the rounding off of the link travel times in flow propagation

5.4 Discussion

Results of both objectives (maximizing total toll revenues and minimizing total travel time) where different tolling schemes (uniform, quasi-uniform and variable) are applied are given in

Table 1  Comparison of toll revenues and total travel time for different objectives

<table>
<thead>
<tr>
<th>Objective: maximize total toll revenue</th>
<th>Tolling scheme</th>
<th>Optimal toll</th>
<th>Total revenue</th>
<th>Total travel time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>3.11</td>
<td>64.15</td>
<td>534.48</td>
<td></td>
</tr>
<tr>
<td>Quasi-uniform</td>
<td>2.82</td>
<td>49.50</td>
<td>503.42</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>4.04</td>
<td>37.13</td>
<td>498.26</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Objective: minimize total travel time</th>
<th>Tolling scheme</th>
<th>Optimal toll</th>
<th>Total revenue</th>
<th>Total travel time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>2.41</td>
<td>60.81</td>
<td>523.56</td>
<td></td>
</tr>
<tr>
<td>Quasi-uniform</td>
<td>2.27</td>
<td>45.56</td>
<td>497.91</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>3.24</td>
<td>35.25</td>
<td>496.02</td>
<td></td>
</tr>
</tbody>
</table>

These results show that in the case of maximizing total toll revenues the best tolling scheme is uniform with toll level $\bar{\theta} = 3.11$. However, this toll will yield a high total travel time (534.48). On the other hand, in the case of minimizing total travel time, the variable tolling scheme with $\bar{\theta} = 3.24$ performs best. However, this toll will yield a low total toll revenue (35.25). In other words, maximizing total toll revenues and minimizing total travel time are opposite objectives.

This can be explained as follows. In maximizing toll revenues, the road authority would like to have as many as possible travellers on the tolled route, hence trying to push as few as possible travellers away from the tolled alternative by imposing toll. In contrast, when minimizing total travel time, the road authority would like to spread the traffic as much as possible in time and space, hence trying to influence as many travellers as possible to choose other departure times and routes. Using a uniform tolling scheme, travellers are not changing their departure times, making it suitable for maximizing revenues, while in the variable tolling scheme other departure times are good alternatives, making it suitable for minimizing travel time. In any case, depending on the objectives of the road authority, there are different optimal tolling schemes with different toll levels.
6. CONCLUSIONS

A mathematical bi-level optimisation problem has been formulated for the optimal toll network design problem. The road authority has some policy objectives, which they may optimise by imposing tolls. Second-best scenarios are considered in this paper, assuming that only a subset of links can be tolled. Different tolling schemes can be selected by the road authority, such as (quasi-)uniform and variable tolling schemes, each having a different impact on the policy objective. Due to tolls, the travellers may change their route and departure times. Heterogeneous travellers with high and low value of time are considered.

The aim of the research is to investigate the feasibility of the dynamic model framework proposed in this paper and to investigate properties of the objective function for different objectives and tolling schemes. The complex optimisation problem has been solved using a simple grid search method, but for more practical case studies more sophisticated algorithms will be developed in the future.

In the case studies the paper shows that policy objectives can indeed be optimised by imposing tolls, and that different policy objectives lead to different optimal tolling schemes and toll levels. Keeping the total travel demand fixed, introducing a uniform (fixed) toll, travellers can only avoid the toll by route changes, not by changing departure time, leading to higher toll revenues. On the other hand, having a variable toll enables travellers to avoid tolls by changing departure time, yielding lower total travel times.

References


