1. INTRODUCTION

Transport systems planning and designing are based on the application of systems of transport simulation models, whose forecast reliability and goodness-of-fit strongly influence the results and the quality of the planned/designed interventions. The implementation of a reliable and effective system of models should be based on the disaggregate estimation of each model component (supply, demand, assignment). The resulting estimated system of models should be then validated as a whole by comparing its output with corresponding observed measures (normally link flows). For a number of reasons, this aggregated validation generally fails. Therefore, in order to assure an overall consistency to the system of models, the observed measures are used to correct part of the model and improve its reliability: this usually involves the correction of the o-d matrix, e.g. through a GLS estimator.

This procedure is so widely applied and trusted in practice that researchers and practitioners often adopt sub-models already estimated in different contexts, therefore leading to a further approximation in the model system. As a consequence, the corrected model system should be further validated by either an hold-out or a before-after approach, as described by Cascetta et al. (2005). For a number of reasons - mainly the lack of data - this important validation is normally not carried out. As a consequence, the model reliability is in practice almost entirely addressed by means of the model correction through traffic counts. In spite of that, few researchers have focused their attention on a systematic analysis of to what extent this procedure is able to correct the whole system of models and consistently guarantee its forecast reliability.

This paper reports some preliminary results of a research project whose main objective is a thorough investigation of the reliability of the methods for static o-d matrix correction. The adopted approach, following some previous literature contributions, is analysing the issue through a set of laboratory experiments wherein a demand matrix, a supply and an assignment model as well as the corresponding link flows (i.e. resulting from demand assignment to the network) are all assumed to be “true”. This allows carrying out a series of experiments wherein both the true o-d matrix and the whole set of (observed) unbiased link flows are available for the o-d matrix correction. In more detail, the following issues have been explored in this phase: (a) given a random perturbation of the true o-d demand, checking the capability of different
subsets of link flows to reproduce the starting true demand through the GLS estimator; (b) introducing a random perturbation of link flows in the preceding experiment so to mimic assignment and/or sampling errors.

The paper is structured as follows. Section 2 reports a brief literature review and introduces the notation adopted throughout the paper. Section 3 proposes a number of different experiments, according to the aforementioned approaches, in order to explore the capability of the correction procedure to correct the o-d matrix in presence of different possible biases involving each demand choice dimension (generation and distribution), and with reference to different levels of precision in the input data. Finally, section 4 presents some conclusions and research developments.

2. LITERATURE REVIEW

The estimation/correction of the o-d matrix from traffic counts is one of the most classical problems in transport engineering. Most of the studies proposed in the literature can be classified, according to their theoretical approach, either in the “classical” framework, i.e. the Maximum Likelihood (ML) estimator proposed by Maher (1983) and Bell (1983) and the Generalised Least Squares (GLS) estimator proposed by Cascetta (1984), or in the “Bayesian” framework proposed by Maher (1983).

Following Cascetta and Nguyen (1988) and Cascetta (2001), classical estimators are based on the hypothesis that link counts and o-d data by sample are available. In order to combine these two experimental sources, a Maximum Likelihood estimate \( \hat{d}_{ML} \) can be obtained by maximizing the probability (likelihood) of observing both the o-d sampling survey data and the link counts (under the usually acceptable assumption that these two probabilities are independent), yielding:

\[
\hat{d}_{ML} = \max_{\mathbf{x} \in \mathbb{D}} \left[ \ln L(\hat{d} / \mathbf{x}) + \ln L(\hat{f} / \mathbf{x}) \right]
\]

wherein \( \mathbf{x} \) is the variable demand, \( \hat{d} \) is the demand by sample and \( \hat{f} \) the vector of link counts. The log-likelihood functions in equation (1) can be specified on the basis of hypotheses on the probability laws of demand counts \( \hat{d} \) and of traffic counts \( \hat{f} \), conditional on the demand vector \( \mathbf{x} \). Normally, traffic counts can be assumed as independently distributed as Poisson random variables, or following a Multivariate Normal random variable, while the log-likelihood function of O-D demand counts depends on the type of sampling adopted. Generalized Least Squares (GLS) demand estimation \( d_{GLS} \) provides the estimate of the O-D demand flow vector, starting from a system of linear stochastic equations, leading to the following optimization problem:
\[
d_{\text{GLS}} = \min_{x, \hat{d}} \left\{ \frac{1}{2} (x - \hat{d})^{\top} Z^{-1} (x - \hat{d}) + \frac{1}{2} (\hat{f} - \text{AP}x)^{\top} W^{-1} (\hat{f} - \text{AP}x) \right\} 
\]

where \( \text{AP} \) is the assignment map and \( Z \) and \( W \) the covariance matrices related to the sampling error underlying the demand estimation and the measurement/assignment errors respectively.

Bayesian methods estimate unknown parameters by combining experimental information (traffic counts in this case) with non-experimental information (a priori or "subjective" expectations on o-d demand, e.g. coming from an out-date estimation or from a model system). In detail, a family of Bayesian estimators \( \hat{d} \) can be obtained by maximizing the logarithm of the a posteriori probability:

\[
\hat{d} = \max_{x \in S} \left[ \ln g(x/d^*) + \ln L(\hat{f}/x) \right] 
\]

wherein \( g(x/d^*) \) expresses the distribution of subjective probability attributed to the unknown vector given the a priori estimate \( d^* \) and \( L(\hat{f}/x) \) expresses the probability of observing the vector of traffic counts \( \hat{f} \) conditional on the unknown demand vector \( x \). Again, the detailed specification of a Bayesian estimator depends on the assumptions made about the probability functions \( g(x/d^*) \) and \( L(\hat{f}/x) \). Normally, the unknown demand vector can be assumed to follow a multinomial random variable (in this case \( \ln g(x/d^*) \) becomes the entropy function of the unknown vector \( x \)), a Poisson random variable (in this case \( \ln g(x/d^*) \) becomes the information function of the unknown vector \( x \)), or a Multivariate Normal random variable.

Some authors compared the preceding methods on small-size test network, through a procedure analogous to that adopted in this paper: it is worth mentioning the works by Di Gangi (1988) and Lam et al. (1990 and 1991). A detailed comparison of these methods will be also explored in the second part of the research project.

Moreover, within this framework, a number of theoretical and operational studies have been carried out. For instance, Bell (1991) explored further theoretical properties of the GLS method. Yang et al. (1992) dealt with the hypothesis of congested network, incorporating o-d estimation and traffic assignment; this problem has been eventually studied as a bilevel optimization problem, among others, by Florian and Chen (1995), Yang (1995) and Cascetta and Postorino (2001). Lo et al. (1996) introduced an explicit representation of the stochastic nature of observed flows, eventually generalized by Vardi (1996); Lo et al. (1999) describe an optimization method for the application of this approach to large-scale networks. A further generalization is proposed in Lo et al. (2003). Hazelton (2000) proposes a method which can also use only information on link counts, but it requires explicit path enumeration and therefore is practically strong time-requiring for large-size networks. Finally, as pointed out by Lo et al (2003) and Hazelton
(2003), a promising research development deals with considering time-series link counts (e.g. referred to several days) as a key aspect for improving the reliability of o-d matrix estimation.

Notably, the most common software packages incorporate some of those methods, and often in practice the formulation (2) in the congested network case is applied. Therefore, before moving to a thorough analysis of all the above mentioned approaches (this activity will carried out in the second part of the research project), attention has been primarily focused on the reliability of the estimator (2), through a laboratory experiment described in the next section.

3. THE LABORATORY EXPERIMENTS

The laboratory experiment has been carried out on the network of Fuorigrotta, a district in the city centre of Napoli (Italy), made up by 225 links and 1369 o-d pairs. An o-d matrix available for the study area, coming from model estimates, has been assumed to be the “true” origin-destination demand matrix \( \mathbf{d}_{true} \). This matrix has been eventually assigned to the network through a SUE-Probit assignment, determining a vector of “true” link flows \( \mathbf{f}_{true} \).

In a first step, link counts are assumed to be equal to the “true” link flows, without any perturbation (i.e. no measurement errors or stochastic fluctuations). Obviously, the hypothesis of unbiased link counts is explicitly introduced in equation (2) through small variances within the matrix \( \mathbf{W} \).

Alternatively, the approach described in Cascetta et al. (2005) can be applied, that is introducing the consistency with link counts as constraints in the optimization problem (which can be therefore solved through a convex simplex algorithm). These link flows are used to correct some o-d matrices obtained through perturbations of the matrix \( \mathbf{d}_{true} \), in order to mimic different modelling errors. In more detail, the following perturbations are considered:

- random: perturbations were defined through the relation \( \tilde{d}_{od} = d_{od} + z(\alpha d_{od})^\beta \) where \( z \) is a draw taken from a normal standard and \( \alpha \) and \( \beta \) are parameters, in order to mimic errors in demand distribution;
- amplification, i.e. multiplying all the values of the true o-d matrix for the same factor (equal to 1,50) in order to reproduce errors in traffic demand generation:
  \[ d_{od}^{pert} = \alpha d_{od}^{true}, \]
- spreading: given the overall true demand \( d_{tot}=\sum_{o}\sum_{d}d_{od} \) and the number of centroids \( n \), each value of the o-d matrix is set to \( d_{od}^{true}/n^2 \).

The distance between the true o-d matrix \( \mathbf{d}_{true} \) and the corrected o-d matrix \( \mathbf{d}_{corr} \) is measured through the well-known indicators:

\[
MSE = \frac{\sum_{od} (d_{od}^{true} - d_{od}^{corr})^2}{n_{od}}
\]
The same indicators have been also applied in order to measure the distance between link counts and simulated link flows. Moreover, in order to provide for a measurement reference, those indicators have been also computed for the initial perturbations imposed on demand and/or link counts.

A first simulation, which does not correspond to a real situation but can provide information about the reliability of estimator (2), is based on using all link counts for the o-d matrix correction. Table 1 reports the results, referred both to demand and flows, for each type of o-d matrix perturbation.

![Table 1 – Perturbed o-d matrix correction using all true link flows.](image)

Interestingly, link counts are always well-reproduced, as a consequence of the assumption on the covariance matrix $W$, while the demand correction is always very poor, apart from the amplification perturbation – as results from the comparison between perturbation and correction RMSE% in the demand side of Table 1. Moreover, in some cases the corrected o-d matrix is farther than the perturbed o-d matrix from the true o-d matrix.

Such result can be explained considering that a link flow is the sum of percentages of o-d flows: therefore, excluding the amplification perturbation which "propagates" the error also in term of link flows, the effect of a general perturbation of the o-d matrix exhibit a sort of compensation with respect to link flows. A confirmation comes from the comparison of the RMSE% values for perturbed demand and the corresponding RMSE% for perturbed flows.

In virtue of these results, the further experiments and analyses pointed out in the introduction (which are expected to reduce the correction capability of the GLS estimator) will be carried out in the following only with reference to the amplification perturbation.

A first experiment dealt with the correction by using different subsets of link counts. For this aim, the maximum flow selection method - for details see for instance Yang et al. (1998) – has been applied, choosing a set of link counts
made up by 35, 50, 75, 100 and 120 links respectively. Results of the correction procedure are reported in Table 2.

<table>
<thead>
<tr>
<th>Max Flow (# sections)</th>
<th>Demand correction</th>
<th>Flows correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>RMSE%</td>
</tr>
<tr>
<td>35</td>
<td>13.73</td>
<td>15.62%</td>
</tr>
<tr>
<td>50</td>
<td>9.77</td>
<td>12.60%</td>
</tr>
<tr>
<td>75</td>
<td>7.13</td>
<td>10.51%</td>
</tr>
<tr>
<td>100</td>
<td>3.56</td>
<td>7.98%</td>
</tr>
<tr>
<td>120</td>
<td>2.72</td>
<td>7.06%</td>
</tr>
<tr>
<td>All</td>
<td>2.33</td>
<td>7.00%</td>
</tr>
</tbody>
</table>

Table 2 – Correction of the amplified o-d demand (factor 1.50) using a subset of link counts (selection method: maximum flow)

RMSE% values show that a correction using 100 link counts provides results close to those obtained using all 225 link counts. This also suggests to analyze (carrying out experiments on several networks) whether the approximation in o-d matrix correction depends on the absolute number, rather than the percentage, of used link counts.

In a second test, the count section selection method based on drawing screen-lines has been adopted. In more detail, two sets of screen-lines have been selected, named respectively SLA and SLB. The first is made up by five screen-lines (4 horizontally and 1 vertically oriented), leading to 67 count sections; the second is made up by three horizontal screen-lines, leading to 82 count sections. Different subset of count sections within the two screen-line sets have been also considered. The correction results are reported in Table 3, which also reports the results of a correction carried out using the same number of count sections determined through the maximum flow approach.

<table>
<thead>
<tr>
<th># sections</th>
<th>Demand (amplification)</th>
<th>Flows</th>
<th>Max flow</th>
<th>Demand (amplification)</th>
<th>Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>RMSE%</td>
<td>MSE</td>
<td>RMSE%</td>
<td>MSE</td>
</tr>
<tr>
<td>35</td>
<td>23.46</td>
<td>20.62%</td>
<td>4885.52</td>
<td>8.85%</td>
<td>15.79</td>
</tr>
<tr>
<td>48</td>
<td>36.93</td>
<td>26.48%</td>
<td>6545.93</td>
<td>9.69%</td>
<td>9.91</td>
</tr>
<tr>
<td>57</td>
<td>19.33</td>
<td>17.62%</td>
<td>2878.4</td>
<td>6.68%</td>
<td>9.5</td>
</tr>
<tr>
<td>67</td>
<td>9.29</td>
<td>10.69%</td>
<td>624.31</td>
<td>3.04%</td>
<td>6.72</td>
</tr>
<tr>
<td>82</td>
<td>10.79</td>
<td>11.17%</td>
<td>627.04</td>
<td>3.08%</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Table 3 – Correction of the amplified o-d demand using a subset of link counts (selection method: maximum flow vs. screen-lines)

Apart from the subjectivity inherent the choice of appropriate screen-lines, the maximum flow selection method dominates the screen-line selection method. Moreover, the two methods tend to provide similar results when the number of used link counts increases, i.e. if the number of count sections is adequate the importance of the selection method is not entirely significant.

Another relevant experiment can be carried out by considering perturbations also in link counts so as to mimic measurement/assignment errors. Therefore, the vector of true link flows has been perturbed with the random method already described for demand perturbation. The magnitude of the perturbations is reported in the following Table 4, while the corresponding o-d
matrix results are reported in Table 5. Note also that Table 5 reports the distances between corrected flows and true flows (i.e. not perturbed)
The main result is that small perturbations (in terms of RMSE%) on link flows do not influence significantly the quality of o-d matrix correction, while over a certain threshold (e.g. 15%) the correction results tend to become very poor.

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>True flows perturbation</th>
<th>MSE</th>
<th>RMSE%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha=0.2\ \beta=1$</td>
<td>18096.58</td>
<td>14.96%</td>
<td></td>
</tr>
<tr>
<td>$\alpha=0.2\ \beta=0.5$</td>
<td>90.59</td>
<td>1.20%</td>
<td></td>
</tr>
<tr>
<td>$\alpha=0.5\ \beta=0.5$</td>
<td>226.46</td>
<td>1.90%</td>
<td></td>
</tr>
<tr>
<td>$\alpha=0.5\ \beta=1$</td>
<td>112675.38</td>
<td>37.35%</td>
<td></td>
</tr>
<tr>
<td>$\alpha=1\ \beta=0.5$</td>
<td>452.91</td>
<td>2.69%</td>
<td></td>
</tr>
<tr>
<td>$\alpha=2\ \beta=0.5$</td>
<td>905.8</td>
<td>3.79%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 – True link flows perturbations

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>Demand (amplification)</th>
<th>Flows</th>
<th>MSE</th>
<th>RMSE%</th>
<th>MSE</th>
<th>RMSE%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha=0.2\ \beta=1$</td>
<td>52.16</td>
<td>28.50%</td>
<td>12887.61</td>
<td>11.89%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha=0.2\ \beta=0.5$</td>
<td>7</td>
<td>10.13%</td>
<td>347.61</td>
<td>2.19%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha=0.5\ \beta=0.5$</td>
<td>3.58</td>
<td>7.77%</td>
<td>437.07</td>
<td>2.53%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha=0.5\ \beta=1$</td>
<td>383.53</td>
<td>77.88%</td>
<td>83251.33</td>
<td>29.18%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha=1\ \beta=0.5$</td>
<td>4.35</td>
<td>8.50%</td>
<td>572.99</td>
<td>2.91%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha=2\ \beta=0.5$</td>
<td>2.89</td>
<td>7.22%</td>
<td>908.09</td>
<td>3.66%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>True link flows</td>
<td>2.33</td>
<td>7.00%</td>
<td>242.2</td>
<td>1.78%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5 – Correction of the amplified o-d demand using all perturbed link flows

4. CONCLUSIONS

In this paper some preliminary results of a research project whose main objective is a thorough investigation of the reliability of the methods for static o-d matrix correction have been proposed. For this aim, a set of laboratory experiments wherein a demand matrix, a supply and an assignment model as well as the corresponding link flows (i.e. resulting from demand assignment to the network) are all assumed to be “true”, have been presented.
The main result up to now determined is that the GLS estimator can provide a reliable correction if the initial demand estimate is affected only by generation errors, while distribution errors can hardly be reduced through the observation of link counts, which tend in most cases to average demand distribution biases.
A thorough analysis of the different correction procedures described in section 2, as well as a generalization of the results obtained so far through a set of tests on different network of different sizes, will be carried out in the second part of the research project.

Bibliography


