1. INTRODUCTION

Road pricing presents one of the market-based policy instruments having influence on travel behaviour of users of a transportation network. Road pricing is a type of responsive pricing that can change travel patterns by influencing user’s travel choices at various levels (e.g. departure time choice, route choice). Many researchers have been working on road pricing problems (see e.g. Vehoev, 2000, 2002; May and Milne, 2000), though almost all of these modelling studies consider static traffic models only. Dynamic models, however, describe the problem more accurate and are required for studies that look at time-varying road pricing. However, formulating and solving dynamic models with time-varying pricing is much more complex compared to static models.

Dynamic optimal toll design problem with homogenous users was the focus in the work of Joksimovic et al. (2005). In this paper we extend the previous work dealing with the dynamic optimal toll design problem with focus on travel behaviour of heterogeneous users. From the travellers’ point of view, route choice and departure time choice are taken into account. Thus, the focus of this paper is to study the potential responses of heterogeneous users due to road pricing.

The problem of road pricing has been studied in the literature from different modelling perspectives and under various assumptions. The (economic) theory of road pricing dates back to Pigou (1920) and first-best congestion tolls are derived in static deterministic models (Beckman et al. 1956; Dafermos, 1973; Yang and Huang, 1996) and static stochastic models (Yang, 1999).

This paper is organized as follows. In Section 2 the problem of optimal tolls with heterogeneous users is defined and described. The modelling of travel behaviour in the optimal toll design problem is given in Section 3 where the dynamic traffic assignment model is derived. In Section 4 the solution procedure is outlined. Different case studies to show the travel behaviour of heterogeneous users in the optimal toll design problem are performed in Section 5. This paper finishes with a conclusion part (Section 6).
2. PROBLEM DEFINITION

The road authority set the tolls on the network. When the road authority has determined the toll levels, the travellers are faced with these tolls while traversing the network and may change their travel behaviour in order to optimise their own objective following utility maximization theory. It is assumed that each traveller individually chooses his or her subjective most preferable route and travel time. We assume that each traveller has the choice to select the route and departure time for his or her trip, which have been argued in the literature to be the most important behavioural responses to road pricing. All other choices, such as trip choice, mode choice, and destination choice, are not considered in this paper.

When travellers make route and departure time changes for their trips after the road authority has introduced tolls on the network, the network conditions (traffic flows and travel times) may change. This may be reason for the road authority to reconsider their tolling scheme in order to optimise its objective. Hence, there is an interaction between the road authority and the travellers, as depicted in Figure 1.

On the left hand side in the road pricing model, the road authority sets levels for the link tolls based on a given tolling pattern (spatial and temporal pattern of tolls, stating when and where to toll) and the objective considered. On the right hand side in the dynamic traffic assignment model, the travellers are simulated on a transportation network aiming towards a (stochastic) dynamic
equilibrium state taking the link tolls into account. In the next section the framework will be formulised and described in more detail.

3. MODELLING OF TRAVEL BEHAVIOUR IN OPTIMAL TOLL DESIGN PROBLEM

Let $G = (N, A)$ denote a given transport network $G$ with nodes $N$ and directed links $A$. Furthermore, let $D_{m}^{rs}$ describe the given travel demand (total number of travellers) for each origin-destination (OD) pair $(r, s)$ and for each user class $m \in M$. Between each OD pair there exist routes $p \in P^{rs}$, where $P^{rs}$ denotes the set of all feasible routes. All user classes are assumed to use the same infrastructure. In this paper the problem is considered in discrete time, hence defined in terms of time intervals instead of time instant. The total time horizon considered is denoted by set $T$, and each time interval is denoted by $t \in T$. The travel demand period is defined by subset $K \subset T$, where each $k \in K$ is a feasible departure time interval. Not all links and all time intervals need to be tolled. The set of tolled links is denoted by $\mathcal{A} \subseteq A$, while the tolled time intervals (in terms of link entrance) for each link $a \in \mathcal{A}$ is defined by $\mathcal{T}_{a} \subseteq T$.

The aim is to determine the optimal tolls which maximize the given road authority objective function. Because we focus on the travel behaviour of heterogeneous users, only uniform tolls will be considered. Uniform tolls are applied in all time periods. The objective function used in this work is revenue maximization. For other tolling regimes (e.g. quasi(uniform) or time-varying tolls) and different policy objectives we refer to Joksimovic et al. (2006).

3.1 Dynamic Traffic Assignment (DTA) Model

The dynamic traffic assignment (DTA) model consists of two components: (i) a simultaneous route choice and departure time choice component, and (ii) a dynamic network-loading component. In the route and departure time choice component, travellers are modelled as utility maximisers in which they choose the route and departure time that minimizes a certain generalized costs, yielding dynamic route flows. It should be noted that heterogeneous travellers are considered in which travellers may differ in their values of time only. The dynamic loading component then dynamically propagates these route flows over the network, yielding (new) experienced travel times and toll costs. The interaction between the two components is depicted in Figure 1. Both components will be explained in more detail below.
3.1.1 Simultaneous route choice and departure time choice component

Let traveller class \( m \) experience generalized travel cost for each route \( p \) from origin \( r \) to destination \( s \) and departure time interval \( k \) (denoted by \( c_{mp}^a(k) \)) be given by a linear combination consisting of the route travel time \( \tau_p^a(k) \), penalties for scheduling delays, and route toll costs \( \theta_p^a(k) \):

\[
c_{mp}^a(k) = \alpha_m \tau_p^a(k) + \beta |k - \xi^a| + \gamma |k + \tau_p^a(k) - \xi^a| + \theta_p^a(k),
\]

where \( |k - \xi^a| \) denotes the deviation of the actual departure time \( k \) from the preferred departure time \( \xi^a \), and where \( |k + \tau_p^a(k) - \xi^a| \) denotes the deviation of the actual arrival time \( k + \tau_p^a(k) \) from the preferred arrival time \( \xi^a \). The parameters \( \alpha_m \), \( \beta \), and \( \gamma \) convert time to a monetary value. Parameter \( \alpha_m \) denotes the value-of-time (VOT) of class \( m \) travellers. Note that the VOT is the only class-specific parameter in this modelling framework. The travel times (speeds) and tolls are assumed to be the same for all traveller types. Hence, we are not assuming different vehicle types, but focus on travellers with e.g. different purposes like business trips (high VOT) versus leisure trips (low VOT).

The route travel times and the route toll costs are determined from the corresponding link travel times and toll costs along the route. Let \( \delta_{ap}(k,t) \) be a dynamic route-link incidence indicator, which is one if link \( a \) is reached on route \( p \) from \( r \) to \( s \) at time \( t \) when departing at time \( k \), and zero otherwise. This indicator can be computed when the link travel times \( \tau_a(t) \) are known. Then the route travel time can be computed from consecutive link travel times,

\[
\tau_p^a(k) = \sum_{a \in p} \tau_a(t) \delta_{ap}(k,t).
\]

Similarly, the route toll costs can be computed from consecutive link toll costs \( \theta_a(t) \),

\[
\theta_p^a(k) = \sum_{a \in p} \theta_a(t) \delta_{ap}(k,t).
\]

Based on the experienced generalized travel costs \( c_{mp}^a(k) \), each traveller is assumed to simultaneously choose the route and departure time that he or she perceives to have the least travel costs, yielding a stochastic user-equilibrium assignment. Assuming that the random components on the generalizes travel costs are independently (which may not hold if routes overlap) and identically extreme value type I distributed, then according to
McFadden (1974) the joint probability of choosing route $p$ and departure time $k$ is given by the following multinomial logit (MNL) model:

$$
\psi^r_{mp}(k) = \frac{\exp(-\mu_c^{r_0}(k))}{\sum_{p' \in P^r} \sum_{k'} \exp(-\mu_c^{r_0}(k'))}, \quad \forall (r,s), p \in P^r, m.
$$

(4)

Given the class-specific travel demand $D_m^r$, the dynamic class-specific route flows can be determined by

$$
f_m^r(k) = \psi_m^r(k)D_m^r, \quad \forall (r,s), p \in P^r, m.
$$

(5)

Solving the DTA model is basically a fixed-point problem since generalized route travel costs yield route flows, while route flow affect the travel times and therefore the generalized route travel costs. The relationship between the route flows and the travel times is given by the dynamic network-loading component.

3.1.2 Dynamic Network Loading

The dynamic network-loading (DNL) component ‘simulates’ the route flows on the network, yielding link flows, link volumes, and link travel times. The DNL model used in this paper is a simple system of equations adapted from Chabini (2000) and Bliemer and Bovy (2003) in which the flow propagation equation is simplified by assuming that there are no subintervals within one time interval and that the link travel time is stationary. In this case, the equations are similar to the ones proposed by Ran and Boyce (1996).

The following set of equations describe the dynamic network loading model:

$$
u_{ap}^r(t + \delta(t)) = u_{ap}^r(t),
$$

(6)

$$
u_{ap}^r(t) = \begin{cases} 
\sum_m f_{mp}^r(t), & \text{if } a \text{ is the first link on route } p, \\
\nu_{a'p}^r(t), & \text{if } a' \text{ is the previous link on route } p.
\end{cases}
$$

(7)

$$
u_a(t) = \sum_{(r,s) \in P^p_P} u_{ap}^r(t).
$$

(8)

$$
u_s(t) = \sum_{(r,s) \in P^p_P} \nu_{ap}^r(t).
$$

(9)

$$
x_a(t) = \sum_{w \in \mathcal{W}} u_a(w) - v_a(w).
$$

(10)

$$
t_a(t) = t_a^0 + b_a x_a(t).
$$

(11)
The flow propagation equations in Eqn. (6), which determine the propagation of the inflows through the link and therefore determine the outflows, relate the inflows $u^{m}_{ap}(t)$ and outflows $v^{m}_{ap}(t)$ of link $a$ at time interval $t$ of vehicles travelling on route $p$ from $r$ to $s$, respectively. This equation simply states that traffic that enters link $a$ at time $t$ will exit the link when the link travel time $\tau_{a}(t)$ elapses. Note that since we are dealing with a discrete-time problem, the link exit time $t+\tau_{a}(t)$ needs to be an integer value. Therefore, $\rho_{a}(t)$ is used, which simply rounds off the travel time (expressed in time intervals) to the nearest integer.

Eqn. (7) describes the flow conservation equations. If link $a$ is the first link on a route, the inflow rate is equal to the corresponding route flows determined by the simultaneous route and departure time choice model. Since we have assumed that all vehicles travel at the same speed, all travellers can be combined in the DNL model by summing them up. If link $a$ is not the first link on a route, then the link inflow rate is equal to the link outflow rate of the previous link.

Eqns. (8)–(10) are definitions. The first two simply stating that the total link inflows $u_{a}(t)$ (or outflows $v_{a}(t)$) are determined by adding all link inflows (or outflows) for all routes that flow into (out of) link $a$ at that time interval. Eqn (10) defines the number of vehicles on link $a$ at the beginning of time interval $t$, $x_{a}(t)$, which is by definition equal to the total number of vehicles that have entered the link until time interval $t$, $\sum_{w=1}^{t} u_{w}(w)$, minus the total number of vehicles that have exited the link, $\sum_{w=1}^{t} v_{w}(w)$.

Finally, Eqn. (11) relates for each link $a$ the number of vehicles to the travel time on that link as an increasing function, where each link has a free-flow travel time $\tau_{a}^{0}$, and a delay component $b_{a}x_{a}(t)$, (with $b_{a}$ a nonnegative parameter).

4 SOLUTION METHOD

In order to solve the optimal toll design problem, a simple grid search is used. Namely, the set of different tolls is chosen for which is the DTA model solved. The resulting values of objective functions are compared and the best toll (which optimise the given objective function) is chosen. For each toll iteration from the road-pricing level, the equilibrium DTA model (described in Section 3) is solved. More sophisticated solution method can be applied to solve this problem but for the purpose of analysing the travel behaviour of heterogeneous users, simple grid search was satisfying.

Input: Network $G = (N, A)$, set of tolled links $\overline{A}$, set of tolled time intervals $\overline{t}$, link tolling patterns $\phi_{a}(t)$, travel demand $D^{m}$, logit scale parameter $\mu$, free-
flow link travel times $\tau^0_a$, link delay parameters $b_a$, number of DTA iterations $J$, grid dimensions $I_a$, road authority objective.

Output: Optimal maximum toll levels $\bar{\theta}^*$, optimal value of objective function $Z^*_\text{time}$.

**Step 1: Initialization**

[Upper level: Road pricing level]

**Step 2: Set the toll value**

The maximum toll level grid for each link $a$ is given by

$$\bar{\theta}^{(i)}_a = \bar{\theta}^{\min}_a + \frac{1}{\tau_a} \left( \bar{\theta}^{\max}_a - \bar{\theta}^{\min}_a \right), \quad i = 0, K, I_a.$$

All combinations of all maximum toll levels for all links determine the set of grid vectors $\bar{\theta}^{(i)} = [\bar{\theta}^{(i)}_a]$, which contains $I = \prod_a (I_a + 1)$ elements. Set $i := 1$ and set $Z^* = +\infty$.

[Lower level: Perform DTA and DNL (according to the model proposed in Section 3)]

**Step 3a: [Initialization]**

Set $j = 1$. Assume an empty network and free-flow network conditions, i.e. $\tau_a^{(j)}(t) = \tau^0_a$.

**Step 3b: [Compute dynamic route costs]**

Compute travel costs $c^{\sigma_a(j)}_{mp}(k)$ using Eqns. (1)–(3).

**Step 3c: [Compute new intermediate route flows]**

Determine the new intermediate dynamic route flow pattern $f^{\sigma_a(j)}_{mp}(k)$ using Eqns. (4)–(5).

**Step 3d: [Flow averaging]**

Use the Method of Successive Averages (MSA) to update the route flows:

$$f^{\sigma_a(j)}_{mp}(k) = f^{\sigma_a(j)}_{mp}(k) + \frac{1}{J} \left( f^{\sigma_a(j)}_{mp}(k) - f^{\sigma_a(j)}_{mp}(k) \right).$$

**Step 3e: [Perform dynamic network loading]**

Dynamically load $f^{\sigma_a(j)}_{mp}(k)$ onto the network using Eqns. (7)–(11), yielding new link travel times $\tau_a^{(j+1)}(t)$.

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Step 3f: [Convergence of DTA level]

If the dynamic duality gap is sufficiently small, go to Step 4; otherwise set $j := j + 1$ and return to Step 3b.

Step 4: Compute the objective function

Maximization revenues

$$Z_{\text{revenue}}(\bar{\theta}) = -\sum_a \sum_t u_a(t) \theta_a(t),$$

If $Z(\theta^{(i)}) < Z^*$, then set $Z^* = Z(\theta^{(i)})$ and set $\bar{\theta}^* = \bar{\theta}^{(i)}$.

Step 5: [Convergence of road pricing level]

While $i < N$, set $i := i + 1$ and return to Step 2.

Otherwise, the algorithm is terminated and $\bar{\theta}^*$ is the set of optimal toll levels.

Performing this simple iterative procedure, we explore all possibilities for all toll level combinations and find the optimal value of the objective function. Regarding the convergence of this algorithm, the inner DTA loop using the widely used heuristic MSA procedure typically converges to an equilibrium solution, although convergence cannot be proven. In the outer road pricing loop the whole solution space is investigated with a certain grid accuracy (yielding a finite number of solutions that are evaluated).

5. CASE-STUDIES

5.1 Network Description, Travel Demand and Input Parameters

The model formulation (Section 3) and solution procedure proposed in the previous section, has been applied to a small network, see Figure 2.

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The network consists of just a single OD-pair connected by two non-overlapping paths where only link 2 is tolled. Since there is only one OD pair, we will ignore the OD subindices \( (r,s) \) in the variables. Two user classes with different value of time (VOT) are distinguished. The total travel demand for departure period \( K = \{1, K, 20\} \) from node 1 to node 3 is \( D = 86 \), of which 50% high VOT travellers and 50% low VOT travellers. The following parameter values are used on route level: preferred departure time \( \zeta = 10 \), preferred arrival time \( \xi = 15 \), value of time for class 1 \( \alpha_1 = 0.25 \), value of time for class 2 \( \alpha_2 = 0.75 \), penalty for deviating from preferred departure time \( \beta = 0.25 \), penalty for deviating from preferred arrival time \( \gamma = 1 \), and scale parameter in MNL model \( \mu = 0.8 \). On the link level, we assume that route 1 with a free-flow travel time of 7.0 time intervals is longer than route 2 (3.0 time intervals) by setting the free-flow link travel times in Eqn. (11) to \( \tau_1^0 = \tau_3^0 = 3.5 \) and \( \tau_2^0 = 3.0 \). Furthermore, it is assumed that the first route never have congestion, hence \( b_1 = b_3 = 0 \), while congestion is possible on link 2 for which we set \( b_2 = 0.005 \) in Eqn. (11).

5.2 Zero Toll Case

For the case in which the tolls are zero on link 2, the route flows and costs are depicted in Figure 3.
It can be seen from Figure 3, that the flows are almost evenly spread between the two routes. The departure time profiles indicate that travellers that use the longer route 1 will depart earlier in order to arrive as close as possible to their preferred arrival time. Since high VOT users attach a higher weight to the travel time in their cost, more high VOT users will be using route 2 as this route will typically have a lower travel time (even with some congestion), whereas route 1 is primarily used by low VOT users who take mainly the penalty for arriving late or early at the destination into account.

Only uniform toll regime will be considered in the case studies below. Note that in this case study with only a single link (link 2) is tolled.
5.3 Optimal Tolls for Maximizing Revenues

Assume that the road authority aims to maximize total revenues, as formulated in Eqn. (12), by selecting the best tolling scheme and the best toll level. For uniform tolling scheme (for each toll level), the dynamic traffic assignment (DTA) problem can be solved. In Figure 4 the total revenues are plotted for each tolling scheme for all $0 \leq \bar{\theta} \leq 15$ (although not shown here, in all cases the DTA model converged). In case the toll level is zero, there are clearly no revenues. For very high toll levels, all travellers will choose to travel on the untolled route, resulting in zero revenues as well. As can be observed from Figure 4, uniform tolling with $\bar{\theta} = 3.11$ yields the highest revenues. The variable tolling scheme is not able to provide high revenues due to the small number of tolled time periods.

![Figure 4: Total travel time for time-varying scheme](image)

**Figure 4** Total travel time for time-varying scheme

The route costs and flows are depicted in Figure 5, together with the optimal toll levels for the objective of maximizing revenues.

The route flows and costs are also depicted in Figure 5, together with the optimal toll levels for the objective of maximizing revenues. Compared with the case of zero tolls in Figure 3, it can be seen that travellers shift towards (non-congested and untolled) route 1 and also shift their departure time (mostly later). Furthermore, it can be observed that there are many more
travellers with a high VOT tolled route 2 than travellers with a low VOT. This is to be expected, as travellers with a high VOT care less about toll costs and more about a short trip time.

Figure 5  Path flows, costs and toll levels when maximizing revenues
6 CONCLUSIONS

In this paper the optimal toll design problem with departure time choice and heterogeneous users is considered. Route and departure time choice of travellers are simultaneously modelled. The DTA and DNL are modelled and mathematically explained. The model is solved using simple grid search and applied on a small hypothetical network.

The contributions of this paper are the following. First, the whole modelling framework is dynamic. Secondly, the departure time choice is included in the framework. Thirdly, the heterogeneity of travellers is analysed showing the differences between travellers.

Compared with the case of zero tolls, it can be seen that travellers shift towards (non-congested and untolled) route and also shift their departure time (mostly later). Furthermore, it can be observed that there are many more travellers with a high VOT tolled route than travellers with a low VOT. This is to be expected, as travellers with a high VOT care less about toll costs and more about a short trip time.

In future research the following lacks can be relaxed. Modelling of trip choice, value of schedule delay of travellers. More appropriate solution algorithm can be applied. A bigger, more realistic network (e.g. with more OD pairs) can be applied in this framework.

REFERENCES


Joksimovic D., M.C.J. Bliemer and P.H.L. Bovy (2006). Different policy objectives of the Road authority in the Optimal Toll Design Problem’
presented at 11\textsuperscript{th} International Conference on Travel Behaviour Research, Kyoto, Japan.


