1. INTRODUCTION

The microeconomic models that include time assignment as an argument in utility have been in the literature for more than four decades now. They have been developed from various theoretical angles: labour supply, home production and transport (Becker, 1965; Johnson, 1966; Oort, 1969; De Serpa, 1971; Evans, 1972; Gronau, 1986; Winston, 1987; Juster, 1990). The more complete framework is the one that includes all activities and consumption as sources of individual utility, plus three types of constraints: money income, time available and technical relations between activities and consumption.

On the other hand, discrete choice models rest on the estimation of a conditional indirect utility function (CIUF) that represents the maximum satisfaction an individual can reach for each of the available alternatives. In the case of transport, the first theoretical framework built on these grounds was proposed by Train and McFadden (1978), who derived the CIUF from a consumer behaviour model that included goods and leisure as the only sources of utility, implicitly based on the approach by Becker (1965). Recently, Jara-Díaz and Guevara (2003) developed a discrete choice model that is originated in a version of the more general consumer approach by De Serpa (1971), which includes work time in the utility function as well. In this case, the derivation of the CIUF includes as an intermediate step an explicit equation for the time assigned to work (a labour supply model) that is estimated to obtain the values of leisure and work. Combining this results with those from a mode choice model (value of travel time savings) they were able to calculate the value of time assigned to travel as well.

In this paper we generalise the approach just described to consider explicitly the restrictions on all activities and goods. The result is a complete set of equations formed by time assigned to work (labour supply), time assigned to all discretionary activities, consumption of unrestricted goods and as many CIUF as discrete choices are considered. In the following section we present the complete model including the calculation of the values of leisure and work. The third section the model is expanded to include discrete choices. The fourth section contains a synthesis of the empirical experience and the options of the analyst are shown, namely to estimate a subset of equations depending on the available information. The last section contains a synthesis and conclusions.
2. THE GENERAL MODEL OF CONSUMPTION AND TIME USE

The generic model we postulate encompasses a utility function that depends on time assigned to each activity, $T_i$, and on goods consumption, $X_j$, in a period $\tau$. Besides the constraints on time and money available, technical relations representing minimum consumption and time uses are included as well.

$$\max_{U(T,X)} \text{subject to } U(T,X)$$

Income constraint
Total time constraint
Technical constraints

The analytical solution of this model yields, in general, equations for time use and goods consumption, $T^*(\ldots)$ and $X^*(\ldots)$, which in turn will determine a maximum level of utility (indirect utility) given by $U[T^*(\ldots), X^*(\ldots)] = \mathcal{V}(\ldots)$. This indirect utility will be the origin of the discrete models that the analyst would like to specify, which, as evident already, have to be compatible with the equations for $T_i$ and $X_j$.

In order to get a specific form for the generic functions, in expressions (1) to (5) we use a Cobb-Douglas function for utility, $T_w$ is time assigned to work, $w$ is the wage rate, $P_j$ are the goods’ prices, $I_f$ is non-work income, and there are minimum values for both time assigned to activities and goods consumed. The rest are parameters in the utility function and the multipliers for each constraint.

$$U = \Omega T^\theta_w \prod_i T_i^\theta_i \prod_j X_j^\theta_j$$  \hspace{1cm} (1)

$$I_j + wT_w - \sum_j P_j X_j \geq 0 \leftarrow \lambda$$  \hspace{1cm} (2)

$$\tau - T_w - \sum_i T_i = 0 \leftarrow \mu$$  \hspace{1cm} (3)

$$T_i - T_i^{Min.} \geq 0 \leftarrow \kappa_i \forall i$$  \hspace{1cm} (4)

$$X_j - X_j^{Min.} \geq 0 \leftarrow \varphi_j \forall j$$  \hspace{1cm} (5)

The multiplier $\mu$ is the marginal utility of time as a resource and $\kappa_i$ is the increase in utility after an exogenous relaxation of the minimum time necessary to perform activity $i$. Dividing these by $\lambda$ (marginal utility of income), the marginal utilities can be expressed in money units per unit time (values of time). There are two important relations that are valid for any utility function. They come from the first order conditions (FOC) on work time (equation 6) and each activity (equation 7).

$$\frac{\mu}{\lambda} = w + \frac{\partial U/\partial T_w}{\lambda}$$  \hspace{1cm} (6)
\[
\frac{\kappa_i}{\lambda} = \frac{\mu}{\lambda} \frac{\partial U/\partial T_i}{\lambda} - \partial U/\partial T_w - \frac{\partial U/\partial T_i}{\lambda}
\]

(7)

The first part of equation (7) establishes that the money value of relaxing a restricted activity (constraint 4 active) equals the value of time as a resource minus the value of the marginal utility of that activity. The multiplier \(\kappa_i\) is zero if the individual assigns to the corresponding activity more time than the necessary minimum, i.e. if the activity classifies as leisure according to De Serpa’s definition. In these cases, the value of the marginal utility is equal to \(\mu/\lambda\), which is why the value of time as a resource is also called the value of leisure. On the other hand, equation (6) establishes that the value of leisure equals the wage rate (the consumption equivalent reward of working more) plus the value of the marginal utility of work (the utility reward of working more). For short, this equation states that the individual will assign time such that the total values of leisure and work are equal.

At the optimum in problem (1)-(5) there will be activities and goods that end up restricted to the minimum feasible, whose duration or consumption could not be reduced if wanted. Let us call \(R\) and \(J\) the set of restricted activities and goods respectively, and define

\[
G_f = \left( \sum_{j \in J} P_j X_j^{\text{Min}} - I_f \right) \quad T_f = \sum_{r \in R} T_r^{\text{Min}}.
\]

(8)

The FOC for those activities that end up not being restricted, contained in a set \(I\), are

\[
\frac{\partial U}{\partial T_i} = \frac{\mu}{T_i} U = \mu \quad \Rightarrow T_i = \frac{\mu}{\theta_i} U \quad \forall i \in I.
\]

(9)

Combining this with the time restriction (3) and the definition of committed time in (8), and calling \(A\) the sum of the exponents of activities in \(I\), we get

\[
\frac{\mu}{U} = \frac{A}{(\tau - T_w - T_f)}.
\]

(10)

A similar procedure with unrestricted consumption, contained in a set \(K\), yields

\[
\frac{\partial U}{\partial X_k} = \frac{\eta_k}{X_k} U = \lambda P_k \quad \Rightarrow P_k X_k = \frac{\eta_k}{\lambda} U \quad \forall k \in K.
\]

(11)

\[
\frac{\lambda}{U} = \frac{B}{(wT_w - G_j)}
\]

(12)

where \(B\) is the sum over the exponents of goods in \(K\). Note that obtaining equations (10) and (12) have conduced very naturally to the aggregation of
leisure activities and goods consumption through $A$ and $B$ respectively. From the FOC applied to the work time we get

$$
\frac{\partial U}{\partial T_w} = \frac{\theta_w}{T_w} U = \mu - \lambda w \Rightarrow \frac{\theta_w}{T_w} = \frac{\mu}{U} - \frac{\lambda}{U}.
$$

(13)

Replacing (10) and (12) in (13) we get a second order equation for $T_w$

$$
\frac{\theta_w}{T_w} + w \frac{B}{(wT_w - G_f)} - \frac{A}{(\tau - T_w - T_f)} = 0.
$$

(14)

Let us normalize the exponents of utility defining

$$
\alpha = \frac{(A + \theta_w)}{2(A + B + \theta_w)} \quad \beta = \frac{(B + \theta_w)}{2(A + B + \theta_w)}
$$

$$
\gamma_j = \frac{\theta_j}{(A + B + \theta_w)} \forall j \in K \quad \vartheta_i = \frac{\eta_i}{(A + B + \theta_w)} \forall i \in I.
$$

(15)

(16)

The solution of equation (14) involves a square root where only the positive sign is valid, as the negative leads to a nil value for work time if $\theta_w$ was zero. All this yields three models:

$$
T_w^* = \beta(\tau - T_f) + \alpha \frac{G_f}{w} + \sqrt{\left(\beta(\tau - T_f) + \alpha \frac{G_f}{w}\right) - (2\alpha + 2\beta - 1)(\tau - T_f) \frac{G_f}{w}}.
$$

(17)

$$
T_i^* = \frac{\vartheta_i}{(1-2\beta)} \left(\tau - T_w^* \left(\frac{G_f}{w}, T_f\right) - T_f\right) \forall i \in I.
$$

(18)

$$
X_k^* = \frac{\gamma_k}{(1-2\alpha)} \frac{w}{P_k} \left(\tau - T_w^* \left(\frac{G_f}{w}, T_f\right) - G_f \frac{w}{w}\right) \forall k \in K.
$$

(19)

Equation (17) gives individual work time (labour supply) as a function of committed time, committed expenses and the wage rate. Equations (18) and (19) correspond to time use demand and goods consumption respectively. Note that equation (19) can be trivially expressed as expense in the $k$-th good by simply moving the price to the right-hand-side. In the stochastic version of this model system, the parameters to estimate are $\alpha$, $\beta$, $\vartheta_i$ and $\gamma_k$, the observed dependent variables are $T_w$, $T_i$ and $X_k$, and the independent variables are $G_f$, $T_f$ and $w$.

The labour supply equation can be estimated as a single model to obtain values for $\alpha$ and $\beta$. If equation (17) is replaced in equations (18) and (19), one can estimate $\alpha$, $\beta$ and $\vartheta_i$ or $\alpha$, $\beta$ and $\gamma_k$, respectively; if not, the observed value of $T_w$ could be used in both equations to estimate $\vartheta_i/(1-2\beta)$ or $\gamma_k/(1-$
2\alpha), respectively. In the case of equations (18) and (19), one can estimate up to N-1 simultaneously, where N is the number of activities or goods consumed freely; otherwise linear dependency would be introduced as they have to sum all time or income available.

The set of equations (17) – (19) looks as a powerful system for the modelling of work time, leisure time and consumption, depending on the available information. As a most interesting result, the estimated parameters permit the calculation of estimates for the values of leisure and work, as shown now. From equations (10) and (12) plus definitions (15), an estimate of the value of leisure is

\[
\frac{\mu}{\lambda} = \frac{(1-2\beta)(wT_w^* - G_f)}{(1-2\alpha)(\tau - T_w^* - T_f)}
\]  

(20)

Combining this with equation (6) the value of time assigned to work can be calculated as

\[
\frac{\partial U}{\partial T_w} = \frac{(2\alpha + 2\beta - 1)(wT_w^* - G_f)}{(1-2\alpha)T_w^*}.
\]  

(21)

3. THE DISCRETE CHOICE MODEL

Let us see now how to build the discrete choice models from the system just obtained. To do this, we have to obtain the indirect utility function by replacing the work equation (17), optimal amounts of time assigned to activities (18) and goods consumption (19) in the direct utility function (1). Collecting all multiplicative constants in \( \hat{\Omega} \), this yield

\[
V = \hat{\Omega} w^{1-2\alpha} \left( T_w^* - \frac{G_f}{w} \right)^{1-2\alpha} \left( \tau - T_w^* - T_f \right)^{-2\beta} T_w^{2\alpha + 2\beta - 1} \prod_{r \in R} T_r^{\text{Min} \gamma} \prod_{j \in J} X_j^{\text{Min} \gamma}
\]

(22)

This expression for \( V \) represents the maximum utility with a wage rate \( w \), fixed expenses \( G_f \) and total time assigned to constrained activities \( T_f \). Let us assume that one of these later has to be done by choosing one among many alternatives, each one characterized by its required time \( t_i \) and cost \( c_i \). In this case, expression (22) can be transformed trivially into a conditional indirect utility function (CIUF) \( V_i \) by simply considering \( t_i \) and \( c_i \) explicitly as part of \( T_f \) and \( G_f \) respectively, i.e.

\[
G_f = G_i' + c_i \quad y \quad T_f = T_i' + t_i
\]

(23)
The resulting function \( V_i(t_i, c_i, w, G_{r_i}, T_{r_i}) \) is, by definition, the maximum possible utility conditional on the choice of the i-th alternative, which can be estimated with known econometric tools. The independent variables are the total time assigned to restricted activities but the one modelled, expenses in restricted goods except \( c_i \), the wage rate, cost and time of each discrete alternative. Note that if simplified versions of \( V_i \) are estimated, for example linear in \( t_i \) and \( c_i \) within population segments, variables \( w, G_{r_i} \) and \( T_{r_i} \) could be used to segment appropriately.

From the discrete choice model the value of diminishing the time assigned to those activities modelled in this fashion can be calculated as the ratio between \( \partial V_i^e / \partial t_e \) and \( \partial V_i^e / \partial c_e \) in the chosen alternative \( e \). By construction, this is an estimator of \( \kappa / \lambda \) in the original problem (see, for instance, Jara-Díaz and Guevara, 2003), such that equation (7) can be used to calculate the value of time assigned to the modelled activity by making the difference with the value of leisure estimated through equation (20), i.e.

\[
\frac{\partial U_i}{\partial T_i} = \frac{\mu}{\lambda} \frac{\kappa}{\lambda} = \frac{(1-2\beta)(wT_{w_r}^* - G_{r_i})}{(1-2\alpha)(t - T_{w_r}^* - T_{r_i})} \frac{\partial V_i}{\partial t_e} - \frac{\partial V_i}{\partial c_e}
\]

This approach to model discrete choices can be applied to as many restricted activities as desired. The important thing is to recognize that the CIUF that commands the discrete choice model should be compatible with the equations that represent individual labour supply, time use and consumption.

In this manner, the complete system formed by equations (17), (18), (19) and (24) form a powerful tool to understand work, activities, consumption and travel, and to estimate all the relevant values of time.

4. EMPIRICAL MODELLING: RESULTS, OPTIONS AND LESSONS

The theoretical framework described in the preceding sections has been applied to various data bases that involve activities, trips and consumption. Jara-Díaz and Guevara (2003) used a sample of workers extracted from the 2001 Origin-Destination survey in Santiago, Chile, to estimate the labour supply model and mode choice model for the trip to work. Besides being the first paper using this approach, it prompted the identification of some theoretical and econometric limitations, which have been solved to date. For example, the authors estimated only one of the relevant continuous equations, work, and formulated the model with only one constrained activity, travel to work, which translated into travel cost and time as the only explanatory

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variables besides the wage rate. As this is not reasonable, it provoked the revision of the theoretical problem, which ended up with the inclusion of all restricted variables (activities and consumption) as done in this paper. This reformulation generated variables $T_r$ and $G_r$, which are definitely much more reasonable as explanatory for the assignment of time and money to discretionary activities and consumption. Thanks to this expansion, now we have a theoretical justification to modify the referential time $\tau$ in both discrete and continuous models, reducing it by the committed mandatory time. A second limitation, of an econometric nature, was superseded by Munizaga et al (2006), where correlation between the continuous model and all travel alternatives was included, as it should be.

Later on, the econometric calibration procedure has been further improved by expanding the simultaneous estimation of the equations of the model system, using maximum likelihood techniques to estimate up to three continuous equations (work and two leisure activities) and one discrete model (travel), including error correlation among all of them. This econometric procedure has been applied to the theoretical framework presented here using three samples collected with different methods in different cities (countries). In all three bases the information was detailed regarding time use, but mandatory expenses had to be generated exogenously as this information could not be collected (neither consumption).

The most detailed of the three samples in terms of activities was collected in Santiago, Chile, including information regarding time use, trips, income and work conditions of 290 workers living along a specific corridor. This TASTI sample, described in Jara-Díaz et al (2004), has been used to model activities only and activities-travel jointly. The results obtained are very interesting (see below) and illustrate quite well the properties of the model. Nevertheless, the activities system of equations (work, entertainment and personal care) presented multiple solutions and the sign of the square root argument in the labour supply equation became negative for a few individuals at the chosen solution point.

The model has been applied also to the German sample MOBIDRIVE (Axhausen et al, 2002), both to estimate the activities system (work and entertainment) as well as the activities-travel joint model. The sample includes 26 individuals only but observed during six weeks in the city of Karlsruhe. The activities model system converged to one solution and no problems with the sign of the square root appeared. The same happened with the model system estimated with information collected by the same team in the Thurgau Canton, Swiss (activities only, work and entertainment). There, 75 individuals were observed during 5 weeks.

Although the time use model system was estimated with three (Santiago) and two (Karlsruhe and Thurgau) equations, in all three samples the values of leisure and work could be calculated according to equations (20) and (21). We obtained very reasonable models and results in the three cases, such that comparisons within and between samples make sense. All the values of leisure are positive, statistically significant and larger, in absolute value, than
the corresponding values of time assigned to work. These latter happened to be negative in both the Chilean and Swiss cases and positive for the German one (not significant in Thurgau and Karlsruhe). To have a quantitative idea, the values of leisure in TASTI, MOBIDRIVE and Thurgau are approximately 3, 13 and 27 US$/hora respectively, following the same relative order as the satisfaction index (subjective welfare) reproduced in Frey and Stutzer (2002) for the same countries. As the corresponding average wage rates are 4.4, 10.6 and 30.4 US$/hour respectively, it is worth controlling for this. A simple manipulation of equation (6) permits a controlled comparison among simples, by simply dividing both sides by \( w \), which yields

\[
\frac{\mu}{\lambda} + \left( \frac{\partial U}{\partial T_f} \right) \frac{\lambda}{w} = 1
\]  

(26)

For the Chilean sample, the two components of equation (26) are 0.66 for leisure and 0.34 for work. For the Swiss sample these are 0.88 and 0.12 and for the German sample they are 1.2 and -0.2. The (marginal) dislike for work relative to income is remarkably superior for the Chileans, which is very likely influenced by longer work schedules, larger amount of time spent in transport and an unattractive working environment (Jara-Diaz et al, 2005).

The empirical experience has helped us identify various elements that, together with the theoretical derivation of the model system, suggest some possible variations in future applications. One is the identification of activities as restricted or frees (leisure), which so far has been decided a priori in our empirical models, when in fact can be subject to experimentation. Other element is the aggregation of activities into a few generic ones intuitively searching for internal homogeneity according to something relatively unclear. Thirdly, considering the wage rate as an exogenous variable has implied assuming that the individuals are in a long run equilibrium regarding work, which means that they have adjusted their working conditions (time and salary) through changes and negotiations although in the short run they have fixed schedules and incomes. The extension to a fixed income framework should be explored.

Lastly, it is relevant and useful to report that the models estimated simultaneously and including error correlation always gave better results statistically than the independent estimation of isolated equations. This is an important point as the values of leisure and work can be calculated from and estimated labour equation only.

5. SYNTHESIS AND CONCLUSIONS

On the basis of a consumer’s behaviour model that includes time use and goods consumption as decision variables, we have deduced a system of equations that includes labour supply and the assignment of time to activities besides the demand for goods. The explanatory variables are the wage rate \( w \), the time committed in constrained activities \( T_f \) and the expenses committed

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in constrained consumption $G_t$. The system permits the calculation of the values of leisure and work.

From this general model we have been able to derive discrete choice models for any desired restricted activity that presents alternatives. It is sufficient to acknowledge explicitly the time and cost of each discrete alternative in the expressions for committed time and expenses, $T_t$ and $G_t$, replacing the resulting equations for work, time use and goods consumption in the direct utility function to obtain the CIUF. From another viewpoint, obtaining the CIUF necessarily requires the derivation of (conditional) demand functions for time assigned to activities and for goods consumption. Furthermore, these demands and the CIUF have to be compatible in terms of the common parameters for both the theoretical derivation and the stochastic version.

The system can include as many discrete choices as restricted activities, or not include any at all. In this manner, the analyst has the option of modelling a subset of activities and goods consumption equations, as well as to include discrete choices or not. We have presented the empirical experience so far, including systems of activity equations only and joint systems of travel and activities.

**ACKNOWLEDGEMENTS**

This research was partially funded by Fondecyt, Chile, Grant 1050643, and the Millennium Nucleus in Complex Engineering Systems.

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