

## 13.012 Quiz #1

FALL 2003

PART A:

① ~~②~~ sea floor there is no velocity normal to the ground  $\vec{V} \cdot \hat{n} = 0$ ;  $\frac{\partial \phi}{\partial n} = 0$  on  $z = -H$

② ~~Streakline~~

③ pathline

④  $\frac{\partial \Gamma}{\partial t} = 0$

⑤  $\vec{\nabla} \cdot \vec{V} = 0$

⑥  $C = -4$

⑦  $\omega = 2\pi f = 2.19 \text{ rad/s} = \sqrt{gk}$ ?  $z = 0.48 \frac{1}{m}$   $kH \sim 4.8 > \pi$   
 0 is deep

• waves are deep

•  $\omega^2 = gk$

•  $\lambda = 13.1 \text{ m}$

•  $V_p = \omega/k = 4.5 \text{ m/s}$

•  $V_g = \frac{1}{2} V_p = 2.3 \text{ m/s}$

⑧  $\omega = 5.6 \text{ rad/s}$   $k = \omega^2/g$   $k = 3.14 \frac{1}{m}$   $kH = 3.51 > \pi$

still deep

$\omega^2 = gk$   $\lambda = 2\pi/k$

$V_p = \omega/k$   $V_g = \frac{1}{2} V_p$

⑨  $P = -\rho g z = 1000 \cdot 10 \cdot 100 = 10^6 \text{ N/m}^2$

⑩  $P_d = \frac{1}{2} \rho V^2$

$P_d = -\rho \frac{\partial \phi}{\partial t}$

- (10) irrotational  $\vec{\nabla} \times \vec{v} = 0$  [DOES] ✓  
 continuity  $\vec{\nabla} \cdot \vec{v} = 0 \Rightarrow$  [DOES NOT] ✓

**PART B:**

"2" waves in central Atlantic

$\omega = 1.9 \text{ rad/s}$   
 $a = 1 \text{ m}$

$\omega^2 = gk$   
 $k = 0.324 \text{ 1/m}$   
 $\lambda = 19.4 \text{ m}$

(a)  $\Rightarrow 2a/\lambda = 2/19.4 = 0.103 < 1/7$  (0.143)  
 $\therefore$  yes, linear assumption is valid but for  $a \approx 2a \approx 2 \text{ m}$

- (b)  $kH > \pi$  so @  $kH \sim \pi$  then too shallow  $\therefore$  not linear

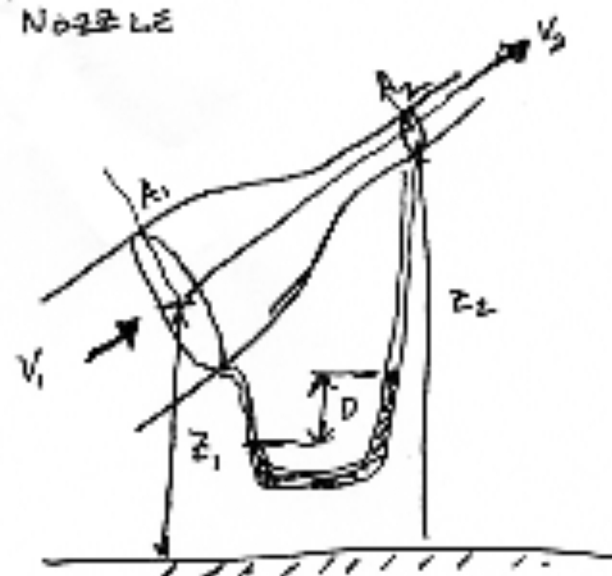
$$kH = 0.324H \sim \pi$$

$$H \sim 9.7 \text{ m}$$

(c)  $\bar{E} = \frac{1}{2} \rho g a^2 = 5000 \text{ J/m}^2$

$$\bar{kE} = \frac{1}{2} \bar{E} = \frac{1}{4} \rho g a^2 = 2500 \text{ J/m}^2$$

"1" NOZZLE



Flow Through Nozzle

Continuity:  $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$

$$V_1 A_1 = V_2 A_2$$

$$\therefore V_2 = V_1 \frac{A_1}{A_2}$$

$$\left( \frac{A_1}{A_2} = \frac{V_2}{V_1} \right)$$

Bernoulli's Eqn gives  $\Delta P$

$$P_1 + \frac{1}{2} \rho_1 V_1^2 + \rho_1 g z_1 = P_2 + \frac{1}{2} \rho_1 V_2^2 + \rho_1 g z_2$$

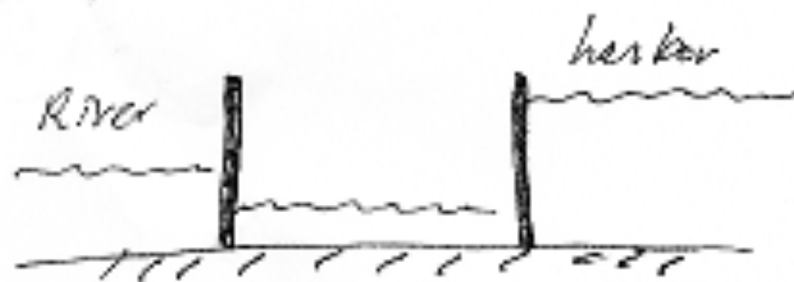
$$(P_1 - P_2) = \frac{1}{2} \rho_1 (V_2^2 - V_1^2) + \rho_1 g (z_2 - z_1)$$

From manometer

$$(P_1 - P_2) = \rho_2 g D \quad \leftarrow$$

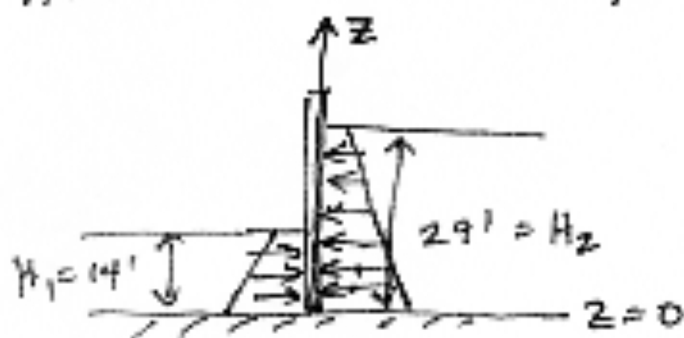
$$\rho_2 g D = \frac{1}{2} \rho_1 V_1^2 \left( \frac{A_1^2}{A_2^2} - 1 \right) + \rho_1 g (z_2 - z_1)$$

$$\frac{\rho_2 g D + \rho_1 g (z_1 - z_2)}{\frac{1}{2} \rho_1 V_1^2} + 1 = \frac{A_1}{A_2} \quad \leftarrow$$



Max force is on the harbor gate  $\rightarrow$

- ① boat goes into Lock @ Low tide  
~~is~~ from the river into the harbor
- ② tide Rises to high tide - now the water in the chamber is 14' deep harbor is 14' + 15' deep.



Pressure on wall (take bottom @  $z=0$ )

Left side

$$P(z) = +\rho g (H_1 - z)$$

$$\text{Force} = L \int_0^{H_1} \rho g (H_1 - z) dz$$

$$= \rho g L \left( H_1 z - \frac{z^2}{2} \right) \Big|_0^{H_1}$$

$$F = \frac{1}{2} \rho g L H_1^2$$

acts at  $z = \frac{H_1}{3}$

Rt side

$$P(z) = \rho g (H_2 - z)$$

$$F = L \int_0^{H_2} \rho g (H_2 - z) dz$$

$$= \rho g L \left( H_2 z - \frac{z^2}{2} \right) \Big|_0^{H_2}$$

$$= \frac{1}{2} \rho g L H_2^2$$



@  $z = \frac{H_2}{3}$

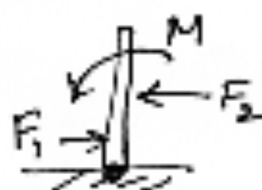
Resulting force acts to left:

$$\therefore F_R = (F_2 - F_1) = \frac{1}{2} \rho g L (H_2^2 - H_1^2) \left( \frac{\text{kg m}}{\text{s}^2} \right)$$

where  $L = 22' = 6.7 \text{ m}$   
 $H_1 = 14' = 4.27 \text{ m}$   
 $H_2 = 29' = 8.84 \text{ m}$

$$\therefore F_R = 2007 \text{ kN}$$

Note there is a moment on the gate as well!



Total force acts to the left. There is a force  $F_1$  acting to the right at  $\bar{z}_1 = \frac{14'}{3}$  and force  $F_2$  acting to left @  $\bar{z}_2 = \frac{29'}{3}$ .

These forces supply a moment to the gate as shown above.

$$F_2 \cdot \bar{z}_2 - F_1 \cdot \bar{z}_1 \Rightarrow \bar{z} = 3.4 \text{ m from bottom}$$

The forces act at the center of the gate from side to side.

⇒ Design-wise any of the 3 will work.

- Design A will require the most torque to move the gate as the force on each door in B is  $\frac{1}{2}$  of one whole gate in A. B requires less space for
- Design B requires 4 motors could be expensive though hydraulic system would not be too much different than in A
- Design C is easiest to open but hardest to seal

doors to swing through.