Ocean
ENGINEERING

13.012 Marine Hydrodynamics for Ocean Engineers
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Fall 2004

Recitation Example Problems

1 October 2004

Problem 1:

Linear Waves:

Determine if the following waves are linear. If so determine the phase and group speed of the waves:

a) $\lambda = 1.2\ m$, $a = 0.3m$, in water depth $H = 1\ m$

b) $\lambda = 1.8\ m$, $a = 0.1m$, in water depth $H = 2\ m$

c) $\lambda = 29.0\ m$, $a = 1.0m$, in water depth $H = 100\ m$

d) $\lambda = 10.0\ m$, $a = 0.5m$, in water depth $H = 4.8\ m$

Problem 2:

A scuba diver is diving in shallow water under incoming waves with frequency, f = 1Hz. and wavelength, $\lambda = 5$ meters, and amplitude 0.5 meters (it is shallow but the waves are not breaking waves). The water has high turbidity (cloudiness due to particles in the water), what does the diver observe these particles doing?

As the diver swims out into deeper water, the wave frequency and amplitude have not changed. How do the particle motions look now?

The diver is beginning to get seasick, how deep does the diver need to go to avoid the wave action and get relief from his motion sickness?

10/1/04

Problem 1:

a) $\lambda = 1.2_m$ $a = 0.3$ m $H = 1$ m
b) $\lambda = 1.8$ m $a = 0.1$ m $H = 2$ m
c) $\lambda = 29$ m $a = 1.0$ m $H = 100$ m
d) $\lambda = 10.0$ m $a = 0.5$ m $H = 4.8$ m

Equations $\omega^2 = gk \tanh(kH)$

$k = \frac{2\pi}{\lambda}$ $kH \rightarrow$ tells us if we have deep water
$kH > \pi \Rightarrow$ deep $(H/\lambda > 1/2)$

$2a/\lambda < 1/7 \Rightarrow$ linear waves

if deep $V_p = \omega/k$ & $V_g = 1/2 V_p$

otherwise $V_p = \omega/k$ & $V_g = \frac{\partial \omega}{\partial k}$

a) $k = \frac{2\pi}{\lambda} = 5.2\ 1/m$ $kH = 5.2 > 3.14$ so deep ✓

? linear: $0.3 \cdot 2/1.2 = 1/2 > 1/7$ non linear!

b) $k = \frac{2\pi}{\lambda} = 3.5\ 1/m$ $kH = 7 > \pi$ ✓ deep

? linear $2a/\lambda = 0.2/1.8 = 0.1111 < 1/7$ so is linear

$\omega^2 = gk$ so $\omega = \sqrt{gk} = \sqrt{10 \cdot 3.5} = 5.9$ rad/s

$V_p = \frac{\omega}{k} = 1.7$ m/s
$V_g = 0.84$ m/s

c) $k = 2\pi/\lambda = 2\pi/29 = 0.216\ 1/m$ $kH = 21.6 > \pi$ ✓ deep

? linear $2a/\lambda = 2/29 < 1/7$ ✓ linear

$\omega^2 = gk \Rightarrow \omega \simeq 1.5$ rad/s

$V_p = 6.8$ m/s $V_g = 1/2 V_p = 3.4$ m/s

d) $\lambda = 10m$ $k = 0.628\ 1/m$ $KH = 3.01 \Rightarrow$ Not deep!

$H/\lambda = 0.48$

$2\pi/\lambda = 1/10 < 1/7$ ✓ linear $\quad 1/20 < H/\lambda < 1/2$ intermediate

use dispersion Rel in full

$$\omega^2 = gk \tanh kH \rightarrow \omega^2 = 6.25$$

$$\omega = 2.5\ rad/s$$

$V_p = \omega/k = 3.98\ m/s \quad V_g = ?$

whole eqn $\left(V_g = \frac{d\omega}{dk}\right)$

$$V_g = \frac{1}{2} V_p \left\{ 1 + \frac{kH}{\sinh kH \cosh kH} \right\} \ m/s$$

e) bonus → $H = 1m \quad \lambda = 21m \quad H/\lambda < 1/20$

Shallow: $k = 2\pi/21 = 0.3\ 1/m$

$$\omega = \sqrt{gH}\ k \Rightarrow 0.95\ rad/s$$

$$V_p = \frac{0.95}{0.3} = 3.15\ m/s = V_g$$

② Diver needs to go below $\lambda/2$ in deep water