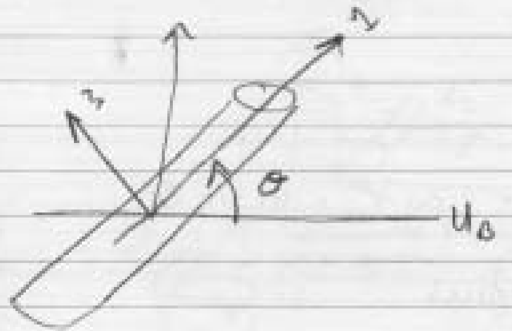
SOLUTIONS

13.012 PRACTICE PROBLEMS

DUE 10/28/03



Inclined cylinder $d = 0.5\,m$; $L = 10\,m$; $\theta = 10^\circ$

Munk Moment

$$M_j = -\varepsilon_{jkl} U_k U_i m_{li}$$

$j = 2$

$$M_2 = -\varepsilon_{2kl} U_k U_i m_{li}$$

Assuming
$\Omega_k = 0$
$\dot{U}_i = 0$

$$U_1 = U_B \cos 10^\circ$$

$$U_3 = -U_B \sin 10^\circ$$

$$U_2 = 0$$

$k = 1, 3$ $\quad m_{11} \sim 0$ $\quad m_{33} = \rho\pi\left(\frac{d}{2}\right)^2 L$

$i = 1, 3$ $\quad m_{31} = m_{13} \neq 0$

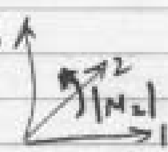
$$M_2 = -\varepsilon_{213}\left\{ U_1 U_3 m_{33} \right\}$$

$$\varepsilon_{213} = -1$$

$$\therefore M_2 = -U_B^2\left(\cos 10^\circ \sin 10^\circ\right)\frac{\rho\pi d^2 L}{4}$$

$$M_2 = -U_B^2\ (335.8)$$

let $U_B = 1\ m/s$ $\Rightarrow$ $\boxed{M_2 = -335.8\ Nm}$



b) Critical Velocity

Restoring Moment $= \rho g \forall \cdot H \sin\theta$

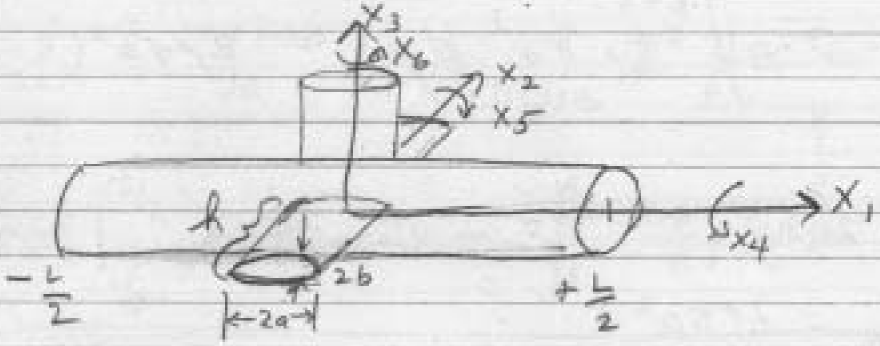
$H = 1 \cdot \text{Radius}$

$$\rho g \frac{\pi}{4} d^2 \cdot \frac{d}{2} \cdot \sin 10^\circ + - U_0^2 [335.8] = 0$$

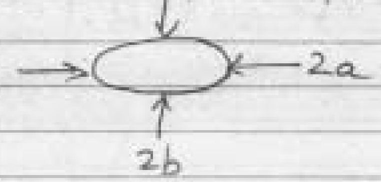
CRITICAL VELOCITY is $\boxed{U_{0_{CR}} \simeq [illegible].5 \ m/s}$

<u>PROBLEM 2:</u>

HYDRO 13.012 EXAMPLE



- 2 WINGS & SAIL $\Rightarrow$ elliptical cross section



- Length of wings & sail $= h$
- Cylindrial body Radius $= R$, Length L
- Center of gravity @ mid line; $x = 0$
- Unbounded fluid

$$M_{33} = M_{33}\Big|_{-L/2,\ hull}^{L/2} + M_{33}\Big|_{wings} + \cancel{M_{33}\Big|_{sail}}$$

~ 0 b/c we ignore longitudinal added masses

$$M_{33}\Big|_{hull} = \int_{-L/2}^{L/2} m_{vv}\Big|_{circle} dx = \int_{-L/2}^{L/2} \rho \pi R^2 dx$$

$$M_{33}\Big|_{hull} = \rho \pi R^2 L$$

$$M_{33}\Big|_{wings} = 2\cdot\int_R^{R+h} \rho\pi a^2\,dy = 2\rho\pi a^2(R+h-R)$$

2wings

$$M_{33}\big|_{wings} = 2\rho\pi a^2 h$$

$$\boxed{\therefore M_{33}\Big|_{total} = \underbrace{2\pi\rho a^2 h}_{wings} + \underbrace{\pi\rho R^2 L}_{hull}}$$

$$\Rightarrow \underline{\underline{M_{35}}} = \int x\, M_{vv}\,dx = \sum_{sim} m_{35}$$

$$M_{35}\big|_{cyl} = \int_{-\frac{L}{2}}^{L/2} \rho\pi R^2 x\,dx = \frac{1}{2}\rho\pi R^2\, x^2\Big|_{-L/2}^{L/2} = \frac{\rho\pi R^2}{2}\cancel{\left[\frac{L^2}{4}-\frac{L^2}{4}\right]} \to 0$$

etc → M_{53} is definitely zero by intuition

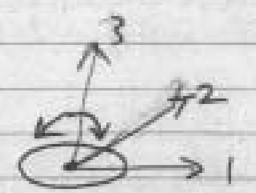
thus since added mass matrix is

symmetric $\boxed{M_{53} = M_{35} = 0.}$

$$\Rightarrow \underline{\underline{M_{55}}} = (m_{55})_{body} + (M_{55})_{wings} + (M_{55})_{sail}$$

$$\underset{hull}{M_{55}} = \int_{-L/2}^{L/2} x^2 (m_{22})_{circle}\,dx$$

$$\underset{wings}{M_{55}} = 2\int_R^{R+H} (m_{66})_{ellipse}\,dy$$



$$M_{55_{sail}} = \int_R^{R+H} (m_{11})_{ellipse}\, z^2\, dz$$

$$M_{55} = \int_{-L/2}^{L/2} \rho\pi R^2 x^2\, dx + 2\int_R^{R+H} \frac{\pi}{8}\rho(a^2-b^2)^2\, dy + \int_R^{R+H} \rho\pi b^2 z^2\, dz$$

$$\boxed{\therefore\ M_{55} = \rho\frac{\pi}{12}\left[R^2L^3 + 3h(a^2-b^2)^2 + 4b^2\left[(R+H)^3-R^3\right]\right]}$$

$$\Rightarrow \underline{M_{44}} = \cancel{M_{44}\big|_{cyl}} + M_{44}\big|_{wings} + M_{44}\big|_{sail}$$

0 in Roll

$$M_{44}\big|_{\substack{wing \\ (1\,wing)}} = M_{44}\big|_{sail}$$

$$M_{44} = 3\cdot M_{44}\big|_{sail} = 3\int_R^{R+H} (m_{22})_{ellipse}\, z^2\, dz$$

$$= 3\int_R^{R+H} \rho\pi a^2 z^2\, dz = \cancel{3}\rho\pi a^2 \frac{z^3}{\cancel{3}}\Bigg|_R^{R+H}$$

$$\boxed{\therefore\ M_{44} = \rho\pi a^2\left[(R+H)^3 - R^3\right]}$$

$\Rightarrow M_{24}$ = added mass in sway due to roll

$M_{24} = M_{42}$ = added mass in roll due to sway

$$M_{42} = \underbrace{M_{42}\Big|_{cyl}}_{\emptyset} + \underbrace{M_{42}\Big|_{wings}}_{\emptyset} + M_{42}\Big|_{sail}$$

by symmetry

$$M_{24} = M_{42} = M_{24}\Big|_{sail} = M_{42}\Big|_{sail}$$

$$M_{42} = \int_R^{R+H} z \cdot \rho\pi a^2 \, dz$$

$$= \rho\pi a^2 \cdot z^2 \Big|_R^{R+H}$$

$$\boxed{\therefore M_{24} = \frac{\rho\pi a^2}{2}\left[2RH + H^2\right]}$$

X	O	O	O	X	O
	X	O	X	O	O
		X	O	O	O
			X	O	O
				X	O
					X

$u_i = [1, 0, 1, 1, 2, 1]$

$\dot{u}_i = [3, 1, 2, 1, 2, 0]$

$$F_j = -\dot{u}_i m_{ji} - \varepsilon_{jkl} m_{li} \Omega_k u_i$$

$$F_1 = -\dot{u}_1 m_{11} - \dot{u}_5 m_{15} - \varepsilon_{123} m_{33} u_5 u_3 - \varepsilon_{132} m_{24} u_4 \cancel{\Omega_3}^{u_6}$$

$j = 1$
$k, l = 2, 3$
$i = 1, 5$

$i = \cancel{1}, 3, \cancel{4}, \cancel{5}, \cancel{2}$ $\quad k = 2 \quad l = 3$

$i = \cancel{1}, \cancel{3}, 4, \cancel{5}, \cancel{2}$ $\quad k = 3 \quad l = 2$

$$\boxed{\therefore F_1 = -\dot{u}_1 m_{11} - \dot{u}_5 m_{15} - m_{33} u_5 u_3 + m_{24} u_4 u_6}$$

$F_2, F_3, \cdots, F_6$ are found similarly

$$M_j = -\dot{u}_i m_{j+3\,i} - \varepsilon_{jkl} \Omega_k u_i m_{l+3\,i} - \varepsilon_{jkl} u_k u_i m_{li}$$

$$M_1 = -\dot{u}_2 m_{42} - \dot{u}_4 m_{44} - \varepsilon_{123} \overset{\Omega_2 = u_5}{u_5} u_6 m_{66} -$$

$j = 1$
$i = 2, 4$
$j + 3 = 4$

$j = 1$
$k = 2 \quad l = 3$
$i = 6$

$$- \varepsilon_{132} \overset{\Omega_3 = u_6}{u_6} u_1 m_{51} - \varepsilon_{132} u_6 u_5 m_{55}$$

$j = 1$
$k = 3 \quad l = 2$
$i = 1, 5$

$$- \varepsilon_{123} u_2 u_3 m_{33} - \varepsilon_{132} u_3 m_{22} u_2 - \varepsilon_{132} u_3 u_4 m_{24}$$

$j = 1 \quad k = 2$
$l = 3 \quad i = 3$

$l = 2 \quad i = 2, 4$

$$\therefore M_1 = -\dot{u}_2 m_{42} - \dot{u}_4 m_{44} - u_5 u_6 m_{66} + u_1 u_6 m_{51} + u_5 u_6 m_{55}$$
$$- \cancel{u_2 u_3} m_{33} + \cancel{u_2 u_3} m_{22} + u_3 u_4 m_{24}$$

$0 \qquad 0$