13.012 Homework #3

Prof. A. Techet; Fall 2004
Issued: September 28, 2004
Due: October 5, 2004

Problem 1: Answer and discuss briefly the following:

a. Give the boundary conditions for linear waves at the free surface and bottom. Explain their physical significance.
b. Discuss the motion of fluid particles beneath a linear wave propagating from left to right. How does the particle's acceleration and speed change over one wave period. How does water depth affect this motion? (use sketches where appropriate).
c. Give the relationship between group speed and phase speed for deep and shallow water.

Problem 2:

Assuming deep water wave of amplitude a = 2.5ft, wavelength λ = 130ft and frequency, ω, dictated by the dispersion relation. Use the expressions for a linear wave, derived in class, to plot:

a. Plot the dynamic pressure, the horizontal velocity and acceleration, the vertical velocity and acceleration as a function of the variable x, for a fixed value of the time t (all quantities evaluated at $z = z_n$ (see the note below for the value of z_n). Align all plots so that the relative phase can be deduced, and plot the wave elevation, also as a function of x, for comparison.

b. Plot the same quantities as in the first question as a function of time t (at a fixed value of x and for $z = z_n$).

c. The inviscid force is proportional to the fluid acceleration. When is the horizontal inviscid force maximum, under the wave crest, the wave trough, or the wave nodal point? Answer the same question for the vertical inviscid force.

d. Calculate the TOTAL pressure under a plane progressive wave at depth H_1 that you should find under the wave crest, the wave trough, and the wave nodal point. Explain from where you measure the vertical distance z used in the hydrostatic pressure.

NOTE To find the value of zn, first find m, an integer corresponding to the reverse-alphabetical (Z-A) order of the first letter in your last name (for example SMITH: letter S is the 8th to last letter in the alphabet, hence m = 8). Then $z_n = m\lambda/120$, where λ is the wavelength.

HINTS: Non-dimensionalize position, x, and time, t, by λ = wavelength and T = wave period respectively before plotting. Note the appropriate units and labels for the Y-axis.