13.012 Marine Hydrodynamics for Ocean Engineers
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Fall 2003

HW #4
Out: 4 October 2004
Due: 11 Oct 2004

Problem 1:

Given a wave with the following characteristics (frequency (ω), period (T), wavelength (λ), amplitude(a)) determine the phase and group speed of the waves:

a) $\omega = 1$ *rad/s*, $a = 0.3$*m*, in water depth $H = 1$ *m*
b) $\lambda = 2.8$ *m*, $a = 0.1$*m*, in water depth $H = 20$ *m*
c) $T = 10$*s*, $a = 1.$*m*, in water depth $H = 30$ *m*
d) $\lambda = 1$ *m*, $a = 0.1$ *m*, in water depth $H = 1$ *m*

(Hint: Determine first whether the case is shallow or deep water and whether linear wave theory will even work!)

Problem 2:

A wave maker generates a wave that is equivalently a summation of two linear mono-chromatic waves, one with frequency 1Hz and wavelength 1.0m and the other with frequency 0.8 Hz and wavelength 1.2m and both with amplitude of 0.1meter, in a *very* long and deep tank.

What will the wave surface look like (a) near to the wave maker and (b) *very* far from the wave maker?
(c) What is happening to the multi-frequency wave packet between these two points?

Use a sketch where necessary.

Problem 3:

a) Explain the significance of the dispersion relationship.
b) How does it differ in shallow versus deep water?
c) How can we determine the solution for the dispersion relationship in shallow water?

Problem 4:

Explain the conditions under which a vessel floating on the surface in water waves will heave up and down with

a) the same frequency as the waves
b) have no little or no heave motion

Problem 5:

A plane progressive wave has a wavelength of 50m in deep water (15ºC SW) and a wave *height* of 4m.

(a) What is the frequency at which wave crests pass a fixed point?

(b) What is the magnitude of the particle velocity at the surface?

(c) What is the magnitude of the particle velocity at depth of 10m below the free surface?

(d) What is the magnitude of the pressure variation at a depth of 10m?

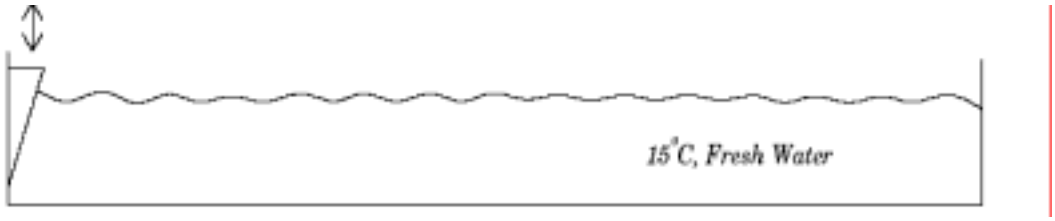
(e) What is the velocity at which the crests move?

(f) What is the average energy density?

(g) What is the rate of energy flux per meter of crest length?

Problem 6:

A wave tank 50m long, 2m wide and very deep has a wave maker at one end and a vertical wall at the other. The tank contains fresh water at 15º C. At time *t=0*, the wave maker begins to oscillate at one cycle per second.



(a) What is the wave length of the generated waves?

(b) The average power delivered to the waves is 3 Joules/s. What is the amplitude of the waves?

(c) When does the wave train reach the opposite end of the tank?

(d) If the waves are perfectly reflected, what is the magnitude of the wave force on the far wall of the tank?