PART B -

(2) Initial Acceleration

$$\Sigma F = M\ddot{x}_3\Big|_{0=t} = \vec{F}_g - \vec{F}_B$$ only forces at work!

$M = m + m_a \qquad m = \rho\forall = 2\rho_w\forall$

$\forall = \forall_{sphere} + 2\forall_{cylinders}$

$= \frac{4}{3}\pi(2a)^3 + 2\cdot\pi\left(\frac{a}{2}\right)^2(4a)$

$= \frac{32}{3}\pi a^3 + 2\pi a^3 = \boxed{\frac{38}{3}\pi a^3 = \forall}$

(i) vertical

$m_a \cong M_{a\,sphere\ only} + \phi$ ← negligible compared to sphere

$= \frac{1}{2}\rho_w\forall_s = \underline{\underline{\frac{16}{3}\rho_w\pi a^3}}$

(↓+)

$(m+m_a)\ddot{x}_3 = mg - \rho_w g\forall$

$= 2\rho_w g\forall - \rho_w g\forall$

$\ddot{x}_3 = \frac{\rho_w g\forall}{m+m_a}$

take as convention

$= \frac{\cancel{\rho_w}\, g\, \frac{38}{3}\cancel{\pi a^3}}{\frac{76}{3}\cancel{\rho_w\pi a^3} + \frac{16}{3}\cancel{\rho_w\pi a^3}}$

(accel is + down)

$\ddot{x}_3 = \frac{38g}{92} = \frac{19}{46}g\ m/s^2$

lower added mass so slightly larger initial accelration

$\ddot{x}_3 = 4.13\ m/s^2$ (↓+)

(ii) horizontal

$m_a\big|_h = \frac{1}{2}\rho\forall_s + \underbrace{2\pi\left(\frac{a}{2}\right)^2(4a)\rho_w}_{cylinders}$

$M_a\big|_h = \underline{\underline{\frac{22}{3}\pi\rho_w a^3}}$

$(m+m_a)\ddot{x}_3 = mg - \rho_w g\forall = 1\rho_w g\forall$

$\ddot{x}_3 = \frac{\cancel{\rho_w}\, g\left(\frac{38}{3}\cancel{\pi a^3}\right)}{\left(\frac{76}{3} + \frac{22}{3}\right)\cancel{\rho_w\pi a^3}}$

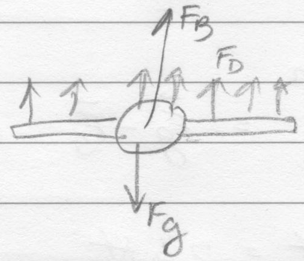
$= \frac{38}{98}g$

$\ddot{x}_3 = \frac{19}{49}g\ \frac{m}{s^2}$ (down +)

slightly lower due to higher added mass

$\ddot{x}_3 = 3.88\ m/s^2$ (↓+)

(b)



Terminal Vel $\quad \Sigma F = 0 = m\ddot{x}_3 \Rightarrow \ddot{x}_3 = 0$ no added mass per se.

$$mg - \rho g \forall - \tfrac{1}{2}\rho U^2 C_D A = 0$$

$$\rho g \forall = \tfrac{1}{2}\rho U^2 (C_D A) \qquad u^2 = \frac{\rho g \forall}{\tfrac{1}{2}\rho C_D A} = \frac{2g\forall}{(C_D A)}$$

$$(C_D A) = A_{sphere} C_{Dsphere} + 2A_{cyl} C_{Dcyl}$$

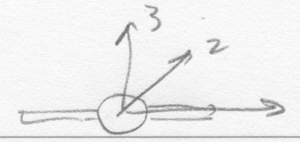
$$= 4\pi a^2 \cdot C_{D_s} + 2a(4a) C_{D_c}$$

$$U = \sqrt{\frac{\cancel{2} g \cdot \frac{38}{3}\pi a^{\cancel{3}}}{2\cancel{4} a^{\cancel{2}}(\pi C_{D_s} + 2C_{D_c})}} = \sqrt{\frac{19 g \pi a}{3(\pi C_{D_s} + 2C_{D_c})}}$$

if a small $(a \to 0)$ $\quad U \to 0$

a large then $U \doteq \sqrt{\frac{19 g \pi a}{3(\pi C_{D_s} + 2C_{D_c})}}$

($C_{D_?}$ chosen based on Re#)



(c) $M_1 = ? \; -\varepsilon_{jkl} u_i u_k m_{li}$

$M_2 = ?$

$\boxed{M_3 = 0}$ by symmetry

$u = (0, -u_2, u_3, 0, 0, 0)$

no accelerations on angular velocities!

Munk moments only –

$M_1 = -\overset{(+1)}{\varepsilon_{123}} u_2 u_3 m_{33} - \overset{(-1)}{\varepsilon_{132}} u_3 u_2 m_{22}$

$j = 1$

$(k = 2, 3)$ 2 → $i = 3$ only

$i = 2, 3$ 3

$l = 1 - 3$

$m_{li} =$ 12, 13; 22, 23; 32, 33

$1 \to 2$, $+$, 3

$m_{11}, m_{22}, m_{33} \neq 0$

$m_{55}, m_{66} \neq 0$

Rest are zero!

$\boxed{m_1 = -(u_2 u_3)[m_{22} - m_{33}]} = 0$

$j = 2 \Rightarrow$ $\boxed{m_2 = 0}$

$-\varepsilon_{2kl}$

$j = 2$

$k = 3$ only

$l = 1$ then

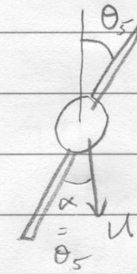
$i = 1$ only

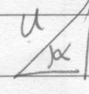
but $u_1 = 0$ ∴

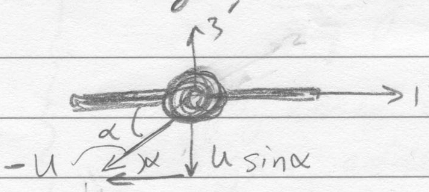
b/c $m_{22} = m_{33}$ by symmetry!

for $j = 3$ then $k = 2$ only, $l = 1$ only so & $i = 1$ only $u_1 = 0$ ∴ $m_3 = 0$

(d) Munk moment only







$U = (-U\cos\alpha, 0, U\sin\alpha, 0, 0, 0)$

$m_{33} = M_{aH}$ from part a

$m_{11} = M_{aV}$ from pt a.

Want $M_2 = -\varepsilon_{213} u_1 u_3 m_{33} - \varepsilon_{231} u_3 u_1 m_{11} = u_1 u_3 [m_{33} - m_{11}] \neq 0$

$j = 2$ $k = 1, 3$ $i = 1, 3$ $l = 1, 3$ for nonzero m_{il}