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Reynolds # $Re = \rho \frac{UL}{\mu} = \frac{UL}{\nu}$

using $\nu_{air} = 1.5 \times 10^{-5}\ m^2/s$ @ 20°C
$\nu_{water} = 10^{-6}\ m^2/s$

a) baseball in air $\begin{cases} \emptyset = 10\ cm = 0.1\ m = L \text{ (diameter)} \\ U = 90\ mph = 40\ m/s \end{cases}$

$$Re = \frac{(0.1)\cdot(40)}{1.5\times10^{-5}} = 267{,}000$$

b) Hammer head shark $L \cong 15\ ft$ (Great Hammerhead)
$L = 4.5\ m$ $U = 3\ mph \cong 1.34\ m/s$ in water ← cruising speed

$$Re = \frac{(4.5)(1.34)}{10^{-6}} = 603{,}000$$

c) Tadpole: $L = 0.2\ cm = 0.002\ m$ $U \cong 2$ body length/sec $= 0.4\ cm/s = 0.004\ m/s$

$$Re = \frac{(0.002)(0.004)}{10^{-6}} = 8$$

d) Aircraft Carrier Ronald Regan (Built 1998)
LWL ≈ LOA = 330 m $V \simeq 30^{+}$ knots $\cong 15\ m/s$

$$Re = \frac{330\cdot 15}{10^{-6}} = 4.95\times10^{7}$$

e) Tech Dinghy $L = 12' = 3.66\ m$ $U = 5\ kts \cong 2.57\ m/s$

$$Re = \frac{3.66\cdot 2.57}{10^{-6}} = 940{,}620$$

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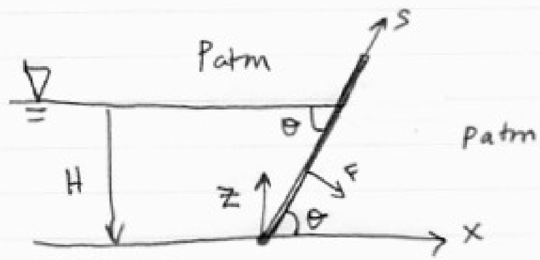
$$\rho = \rho_0 + \tfrac{1}{2} m z^2 \qquad \frac{\partial p}{\partial z} = -\rho g = -g\left(\rho_0 + \tfrac{1}{2} m z^2\right)$$

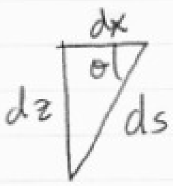
$$\int dp = -g \int \left(\rho_0 + \tfrac{1}{2} m z^2\right) dz$$

$$\therefore \boxed{p(z) = p_{atm} - \rho_0 g z - \tfrac{1}{6} g M z^3}$$

3



(NB: take $z = 0$ @ bottom)



small elemental section of wall

- $F = \sqrt{F_x^2 + F_z^2}$
- Patm acts everywhere so can use gage pressure, p_g.

$p_g = \rho g (h - z)$ acts Normal to wall.

$dF = p_g\, dA \qquad dA = w\, ds$ ← length along wall; w ← wall width

$dF_x = dF \sin\theta = p_g (w\, ds) \sin\theta \qquad ds \sin\theta = dz$

$dF_z = dF \cos\theta = p_g (w\, ds) \cos\theta \qquad ds \cos\theta = dx$

$F_x = \int_0^h \rho g (h - z) w\, dz = \tfrac{1}{2} \rho g w h^2$ SAME AS ON HORIZONTAL WALL

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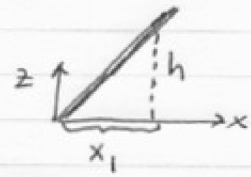
$$dF_z = \rho g (h-z)\, w\, (ds \cos\theta)$$



$$F_z = \int_0^{x_1} \rho g (h-z)\, w\, dx$$

$$x_1 = h/\tan\theta \;\;\&\;\; z = x\tan\theta$$

$$F_z = \int_0^{h/\tan\theta} \rho g (h - x\tan\theta)\, w\, dx$$

$$= \rho g w \left(hx - \frac{x^2}{2}\tan\theta \right) \Big|_0^{h/\tan\theta}$$

$$\therefore F_z = \rho g w h^2 \left[\frac{1}{\tan\theta} - \frac{1}{2\tan\theta} \right] = \boxed{\frac{\rho g w h^2}{2\tan\theta} = F_z}$$

Could alternately consider weight of water on wall:

$$F_z = \rho g\, A \cdot w$$ ← area of wedge of water

$$A = \frac{1}{2}\frac{h^2}{\tan\theta}$$

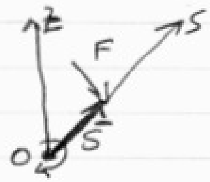
$$\boxed{\therefore F_z = \frac{\rho g h^2 w}{2\tan\theta}}$$ ✓✓ same as above

$$F_t = \sqrt{F_x^2 + F_z^2} = \sqrt{\left(\frac{\rho g w h^2}{2}\right)^2 + \left(\frac{\rho g w h^2}{2}\right)^2 \frac{1}{\tan^2\theta}} = \frac{\rho g w h^2}{2} \cdot \sqrt{1 + \frac{1}{\tan^2\theta}}$$

From Trig: $1 + \tan^2\theta = \dfrac{1}{\cos^2\theta} \longrightarrow 1 + 1/\tan^2\theta = \dfrac{\tan^2\theta + 1}{\tan^2\theta} = \dfrac{\cancel{\cos^2\theta}}{\sin^2\theta} \cdot \dfrac{1}{\cancel{\cos^2\theta}}$

$$\boxed{\therefore F_t = \frac{\rho g w h^2}{2\sin\theta}}$$

Moment Balance to find $\bar{s}$:



$M_o = \bar{s} \times \vec{F_T}$

$dM_o = s \times d\vec{F} = \rho g (h-z) s \; w \, ds$

$M_o = \int_0^s \rho g \, (h-z) s \, w \, ds$

$\bar{s} = \frac{M_o}{F_T} = \underbrace{\frac{1}{\left(\frac{\rho g w h^2}{2 \sin\theta}\right)}}_{1/F_T} \cdot \int \rho g (\underbrace{h-z}) s \, \cancel{w} \, ds \quad z = s \cdot \sin\theta$

$= \frac{2\sin\theta}{\cancel{\rho g w} h^2} \int_0^{h/\sin\theta} \cancel{\rho g w} \left[hs - s^2 \sin\theta \right] ds$

$= \frac{2\sin\theta}{h^2} \left[\frac{hs^2}{2} - \frac{s^3}{3} \sin\theta \right]_0^{h/\sin\theta}$

$= \frac{2 \cancel{\sin\theta}}{\cancel{h^2}} \left[\frac{h^{\cancel{3}}}{2\sin^{\cancel{2}}\theta} - \frac{h^{\cancel{3}}}{3\sin^{\cancel{2}}\theta} \right]$

$\bar{s} = \frac{h}{\sin\theta} - \frac{2h}{3\sin\theta} = \frac{h}{3\sin\theta}$

Recall for Vertical wall:

$\bar{z} = h/3 \Rightarrow z = s \cdot \sin\theta \quad s = z/\sin\theta$

$\therefore \bar{s} = \frac{\bar{z}}{\sin\theta} = \frac{h}{3\sin\theta}$ ✓ makes sense!

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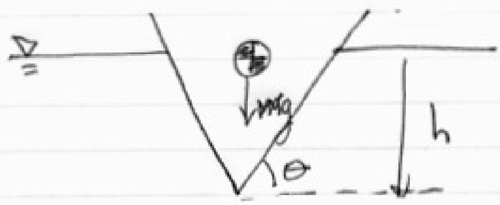


Static EQUILIBRIUM

Left/Rt symmetry

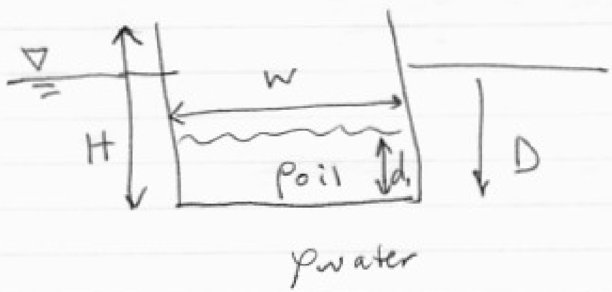
$\Sigma F_x = 0 \qquad F_{x_{left}} = F_{x_{Rt}}$

$F_z = Mg \qquad M = \rho V = 2\rho w\left(\tfrac{1}{2} b h\right) \qquad b = h/\tan\theta$

$$\boxed{\therefore F_z = 2\left(\frac{\rho g w h^2}{2\tan\theta}\right)}$$

2x the force from problem 3! thus we see that force due to pressure equals the weight of the displaced water thus allowing the vessel to float!

5



$F_z = \rho_w g w D L \rightarrow$ No oil

Oil Mass $\Rightarrow \rho_{oil}\, d_1\, W\, L$

Vessel about to sink : $D' = H$

$$\underbrace{\rho_w g\, H \cdot W \cdot L}_{\text{new displaced volume}} = \underbrace{\rho_w g D W L}_{\text{weight of barge w/o oil}} + \underbrace{\rho_o g w d_1 L}_{\text{weight of oil}}$$

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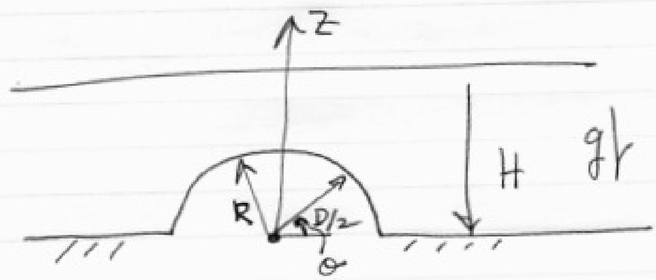
$\rho_w H = \rho_w D + \rho_{oil} d_1$

$\rho_w (H - D) = \rho_{oil} d_1$

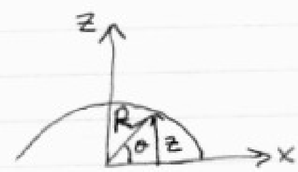
$$\boxed{\therefore \quad d_1 = \frac{\rho_w (H-D)}{\rho_{oil}}}$$

(6)



a) hydrostatic $p(z) = \rho g (H - z)$

on sphere $z = R\sin\theta$; $R = D/2$



$p(\theta) = \rho g (H - R\sin\theta) = \rho g (H - \frac{D}{2}\sin\theta)$

assume pressure is axissymmetric!

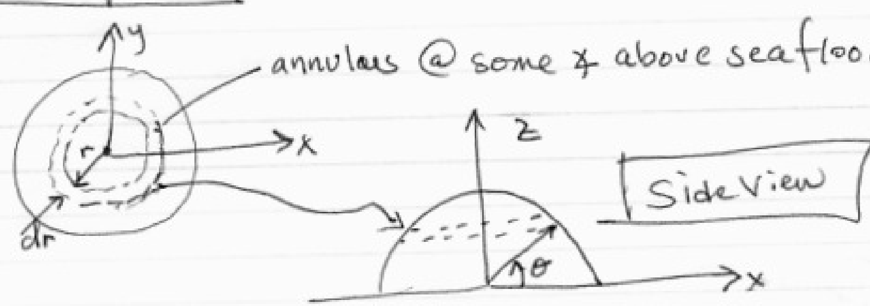
b) $F_x = 0$ b/c No external forces (by symmetry $F_{x_{Rt}} = F_{x_{left}}$)

$F_z \neq 0$ due to weight of water

Top View of Sphere

annulus @ some z above seafloor

Side View



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differential Force on annulus $\quad r = \frac{D}{2}\cos\theta \, ; \, dr = r\sin\theta \, d\theta$

$$dF_z = \underbrace{\underbrace{(2\pi r)}_{\text{circumfrence of annulus}} \cdot \underbrace{(r\sin\theta \, d\theta)}_{\text{differential width of annulus}}}_{\text{area}} \cdot \underbrace{p(\theta)}_{\text{pressure as fcn of } \theta}$$

$$dF_z = 2\pi\left(\frac{D}{2}\right)^2 \cos^2\theta \sin\theta \; \rho g\left[H - \frac{D}{2}\sin\theta\right] d\theta$$

$$F_z = 2\pi\rho g\left(\frac{D}{2}\right)^2 \int_0^{\pi} \cos^2\theta \sin\theta \left[H - \frac{D}{2}\sin\theta\right] d\theta$$

$$\boxed{\therefore F_z = \rho g H \pi R^2 - \rho g \frac{2}{3}\pi R^3} \text{ where } R = D/2$$

$$\boxed{\text{Moment} = \emptyset \; !}$$