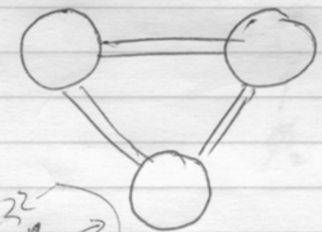
13.012 Hw #2. Fall 2004

PROBLEM 1



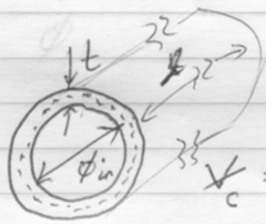
$\gamma = 2.7$ aluminum $= \dfrac{\rho_{alum}}{\rho_{H_2O}}$

$t = 0.5\,cm$

$l = 1.25\,m$

$\phi_{in} = 15.25\,cm$

EACH CANISTER:



$\forall_c = 2\pi \bar{r} \cdot t \cdot L$ canister volume (to find weight) of Aluminum

$\forall_c \cong 0.00309\,m^3 = 3090\,cm^3$

Displaced Volume $\forall_d = \pi r^2 \cdot L = 0.0259\,m^3 = 25{,}900\,cm^3$

$\left[\pi \left(\frac{16.25}{2}\right)^2 \cdot 125\ cm^3\right]$

Neutrally Buoyant: Payload + canister weight = Displacement

Displacement for 3 canisters

$\Delta = \rho_{sw} \cdot 3\forall_d = 1025 \cdot 3 \cdot 0.0259\ m^3 = 79.6\,kg$

Weight 3 canisters $W_c = 3 \cdot \forall_c \cdot 2.7 \cdot 1000 = 25.05\,kg$

$\Delta = W_c + W_p \Rightarrow 79.6 = W_p + 25.05$

$$\boxed{\therefore W_p \simeq 54.5\,kg}$$

② a) Eulerian & Lagrangian

b) continuum

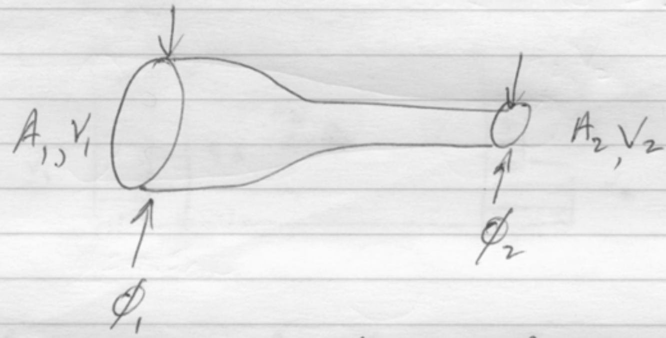
c) pathline & streamlines are same for steady flow

Pathline follows path of a particle in time

Streamline is line tangent everywhere to flow velocity

d)

(3)



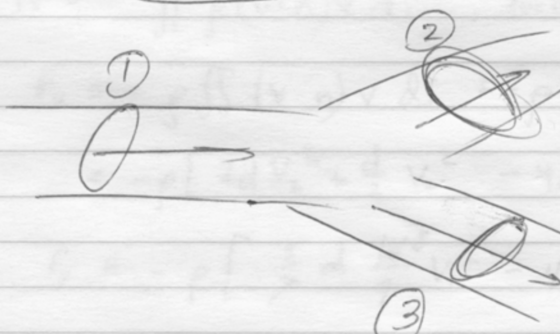
$A_1 V_1 = A_2 V_2$ conservation of mass

$$A_2 = \frac{\pi \phi_2^2}{4} = \frac{A_1 V_1}{V_2} = \frac{\pi \phi_1^2}{4} \frac{V_1}{V_2}$$

$$\phi_2^2 = \phi_1^2 \frac{V_1}{V_2} = (2.5\,cm)^2 \left(\frac{1\ m/s}{1.5\ m/s}\right) = 4.167\ cm^2$$

$$\boxed{\therefore \phi_2 = 2.04\ cm}$$

(4)



$\gamma_{Blood} = 1.05$

$A_1 V_1 = A_2 V_2 + A_3 V_3$ conservation of mass.

$$A_1 V_1 = \frac{\cancel{\pi} d_1^2}{\cancel{4}} V_1 = \frac{\cancel{\pi} d_2^2}{\cancel{4}} V_2 + \frac{\cancel{\pi} d_3^2}{\cancel{4}} V_3$$

$$\boxed{(20\,mm)^2 (1.5\ m/s) = (15\,mm)^2 V_2 + (12\,mm)^2 V_3}$$

Mass flow Rates @ 2 + 3 are equal

2 eqns, 2 unknowns

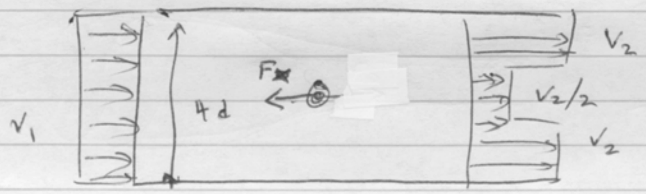
$\dot{m} = \rho A V \Rightarrow \rho_2 A_2 V_2 = \rho_3 A_3 V_3$

units $\left[\frac{kg}{s}\right]$ ✓ $\left[\frac{kg}{\cancel{m^3}} \cancel{m^2} \frac{m}{s}\right]$

$d_2^2 V_2 = d_3^2 V_3$ $\boxed{V_2 = \left(\frac{d_3}{d_2}\right)^2 V_3}$

$$\boxed{\therefore V_3 = 2.08\ m/s \; ; \quad V_2 = 1.33\ m/s}$$ ✓

(5)



find V_2 as fcn of V_1

Conserv mass $\quad A_1 V_1 \Rightarrow \quad 4d\,V_1 = dV_2 + 2d \cdot V_2/2 + dV_2$

$$4\cancel{d}V_1 = 3\cancel{d}V_2$$

$$\boxed{\frac{4}{3}V_1 = V_2}$$

$$-F_x = \iint \rho(V\cdot n)V\,ds \quad \text{force on cylinder}$$

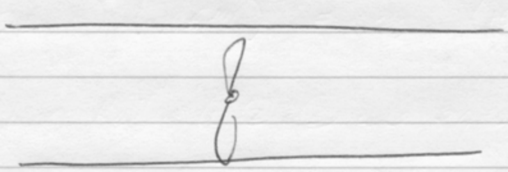
$$F_x = -\rho\iint (V\cdot n)V\,ds = -\rho\left[-V_1^2\,4d + dV_2^2 + 2d\left(\frac{V_2}{2}\right)^2 + d(V_2)^2\right]$$

$$= -\rho\left[2dV_2^2 + \frac{d}{2}V_2^2 - 4dV_1^2\right] = -\rho\left[\frac{5}{2}dV_2^2 - 4dV_1^2\right]$$

$$F_x = -\rho\left[\frac{5}{\cancel{2}}d\,\frac{\cancel{16}\,8}{9}V_1^2 - 4dV_1^2\right] = -\rho\,\frac{4}{9}dV_1^2$$

So Force on cylinder is acting to Right with magnitude $\left|\frac{4}{9}\rho d V_1^2\right|$.

(6)



(a) $p_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 = C$

(b) along a streamline
incompressible flow
no work added...

(c) if the fan is moving the flow will be the same at the inlet & at the outlet assuming it has been running for some time.

(d) No b/c work is being added by the fan.