PS 1 — 13.012 Fall 2004

① $\omega^2 = gk \tanh(kH)$

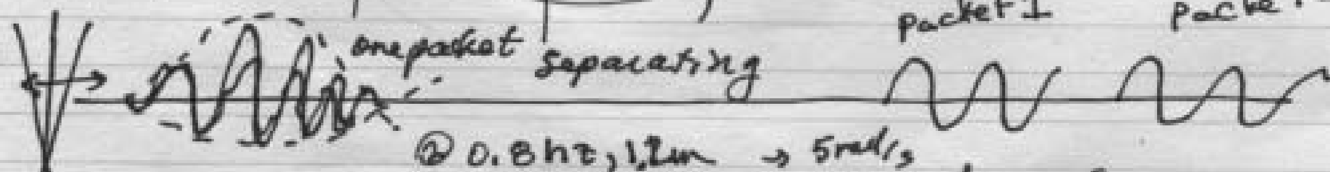
| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| (rad/s) $\omega$ | 1.0 | 9.74 | 0.628 | 7.93 |
| (m) $a$ | 0.3 | 0.1 | 0.1 | 0.1 |
| (m) $\lambda$ | 19.537 | 2.8 | 139.37 | 1.0 |
| (m) $H$ | 1.0 | 20.0 | 30.0 | 1.0 |
| $2a/\lambda$ | 0.015 ×2 | 0.036 ×2 | 0.[illegible] ×2 | 0.2 |
| $H/\lambda$ | 0.05 | 7.14 | 0.2 | 1 |
| linear? | Yes | Yes | Yes | ~~Yes~~ NO |
| deep? | No | Yes | No | Yes |
| dispersion rel | $\omega^2 = gk\tanh(kH)$ | $\omega^2 = gk$ | $\omega^2 = gk\tanh(kH)$ | $\omega^2 = gk$ |
| $V_p$ | 3.1 m/s | 2.1 m/s | 13.93 m/s | [illegible] |
| $V_g$ | 3.01 m/s | 1.0 m/s | 9.5 m/s | [illegible] |
| | shallow | deep | intermediate | deep (not linear) |

$a/\lambda < 1/7 = 0.14 \Rightarrow$ linear

$H > \lambda/2$ $\quad H/\lambda > 1/2 \Rightarrow$ deep

$$V_g = \frac{d\omega}{dk} = \frac{1}{2} V_p \left[ 1 + \frac{kH}{\sinh kH \cdot \cosh kH} \right]$$

②




Wave maker

near ② 0.8 hz, 1.2 m → 5 rad/s

① 1.0 hz, 1.0 m → 6.28 rad/s v. far

$\omega^2 = gk$ $\quad k = \omega^2/g$ — separate packets

packet 1: $V_g = \frac{1}{2}\omega/k = \frac{1}{2} g/\omega =$ [illegible] 0.796 m/s

2: $V_g = 1.0$ m/s

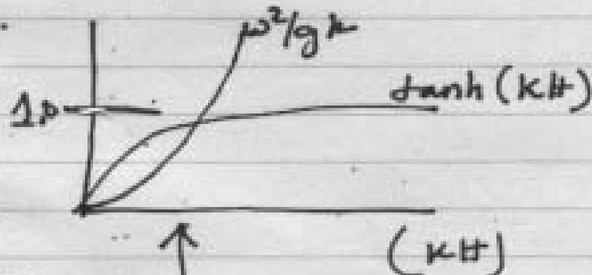
wave freqs travel @ different group speeds thus each freq. component separates out eventually

③ dispersion rel relates the frequency of waves to wavelength and depth

It dictates the relationship between phase speed & group speed

$\omega^2 = gk \tanh(kH)$

↑ general form




as $H \to \infty$ $\quad \omega^2 = gk$

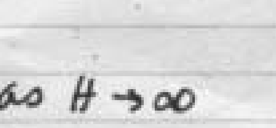
iteration or plotting used to find soln to disp rel for shallow water

④ if the object is small & the system is linear

deeper draft vessels have less heave

also ship length ~ wave length heave ↑

⑤



$\lambda = 50m$ $\quad 2a = h = 4m$

$\rho = 1025 \, kg/m^3$

deep water

a) $\omega^2 = gk$ $\quad k = 2\pi/\lambda = 0.1256$ $\quad \omega = 1.1 \, rad/s$

b) $|V| = a\omega = 2 \cdot 1.1 = 2.2 \, m/s$

c) $|V(z)| = a\omega e^{kz} = 2.2\left(e^{-k(10)}\right) = 0.639 \, m/s$

d) $p = \rho g \eta e^{kz} \Rightarrow |\Delta p| = \rho g h e^{kz}$ (h ← 2a) $= 1025 \cdot 10 \cdot 4 \cdot e^{-1.24}$

$=$ ~~14.6~~ $11.8 \, kPa$

e) $V_p = 1.1/0.1256 = 8.9 \, m/s$

5f) $E = \frac{1}{2}\rho g a^2 = 2.05\times10^4$ ~~[scribbled out]~~ $\left[\frac{kg}{m^3}\frac{m}{s^2} m^2\right] = \left[\frac{kg}{s^2}\right] = \left[\frac{Energy}{Area}\right]$

g) $\mathcal{F} = E \cdot V_g =$ ~~[scribbled out]~~ $= 2 \cdot \frac{1}{2} \cdot 8.9 \times 10^4 = 8.9\times10^4 \frac{kg\,m}{s^3}$

6) a) $\lambda \Rightarrow \omega^2 = gk = g\frac{2\pi}{\lambda}$ $\quad \lambda = \frac{2g\pi}{\omega^2} = 1.59\text{ m}$

$\omega = 2\pi(1\text{ Hz})$

c) $\Delta t = 50m / V_g = 50 \cdot 2 / \frac{\omega}{k} = \frac{100\, 2\pi}{\lambda\omega} = 62.9\text{ sec}$

d) $F \Rightarrow P = F \cdot Vel \rightarrow a = \text{constant}$

$F = \frac{\text{Power}}{\text{groupspeed}}$