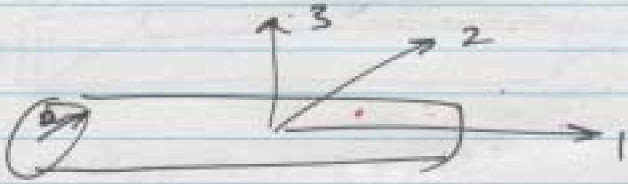
13.012 PS 5 FALL 2004 (A. Techet)



1.

$$\begin{bmatrix} X & 0 & 0 & 0 & 0 & 0 \\ & X & 0 & 0 & 0 & 0 \\ & & X & 0 & 0 & 0 \\ & \text{Symmetric} & & 0 & 0 & 0 \\ & & & & X & 0 \\ & & & & & X \end{bmatrix}$$

- m_{11} cannot be determined by strip theory

Approximate As the equivalent added mass for a sphere

$$m_{11} = \frac{1}{2}\rho\frac{V}{5} = \frac{4}{3.2}\pi \cdot a^2 \rho$$

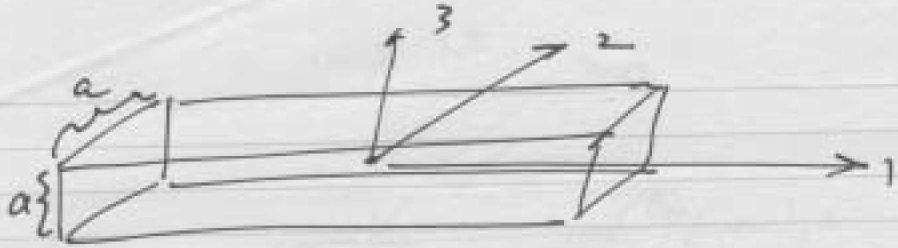
- m_{22} by strip theory $\int_{-L/2}^{+L/2} a_{HH}\, dx$

$$a_{HH} = \rho\pi a^2 \quad a \neq f(x) = \text{const}$$

$$\therefore m_{22} = \rho\pi a^2 L$$

- m_{33} by strip theory $m_{33} = \rho\pi a^2 L = m_{22}$ by symmetry
- $m_{55} = \int_{-L/2}^{L/2} x^2 \cdot \rho\pi a^2 dx = \rho\pi a^2 \frac{x^3}{3}\Big|_{-L/2}^{L/2} = \frac{1}{12}\rho\pi a^2 L^3$
- $m_{66} = m_{55}$ by symmetry

(2)



$a_{vv} = a_{HH} = 1.51\pi\rho a^2$

$a_{\theta\theta} = 0.234\pi\rho a^4$

$$\begin{bmatrix} X & O & O & O & O & O \\ & X & O & O & O & O \\ & & X & O & O & O \\ & & & X & O & O \\ & & & & X & O \\ & & & & & X \end{bmatrix}$$

Symmetric

$m_{11} \Rightarrow$ not exactly a sphere but again it's a decent estimation.

$m_{22} = m_{33}$ by symmetry

$$m_{22} = \int_{-L/2}^{L/2} a_{vv}\, dx$$

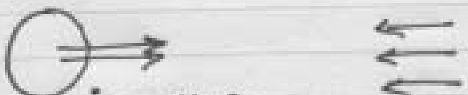
$$m_{22} = 1.51\rho\pi a^2 L = m_{33}$$

$$m_{55} = \int_{-L/2}^{L/2} 1.51\pi\rho a^2 x^2\, dx = 1.51\pi\rho a^2 \frac{L^3}{12}$$

$m_{66} = m_{55}$ by symmetry

$$m_{44} = \int_{-L/2}^{L/2} m_{\theta\theta}\, dx = 0.234\rho\pi a^4 \cdot L$$

(3)



$\dot{v} = -2\,m/s$ $\quad \forall = 2m^3$

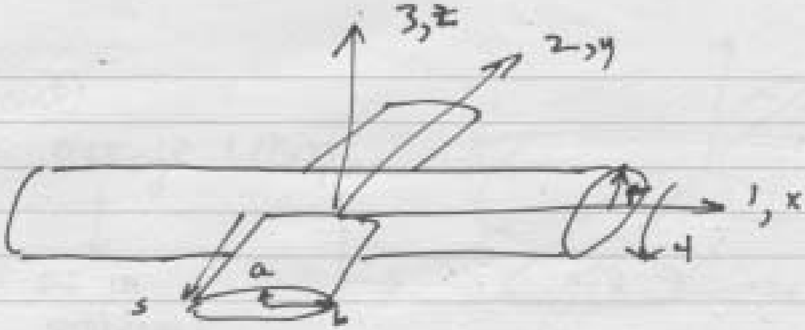
$\dot{u} = 5\,m/s^2$

$$F_x = \rho\forall\dot{v} + m_a(\dot{v} - \dot{u}) \Rightarrow m_a = \tfrac{1}{2}\rho\forall \quad (\forall = \tfrac{4}{3}\rho\pi a^3)$$

$$\rho\forall\left[\dot{v} + \tfrac{1}{2}\dot{v} - \tfrac{1}{2}\dot{u}\right] = \rho\forall\left[\tfrac{3}{2}\dot{v} - \tfrac{1}{2}\dot{u}\right] = -\tfrac{11}{2}\rho\forall = -11\rho\,(N)$$

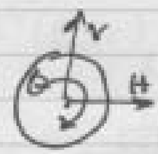
$$F_x = 0 \Rightarrow \tfrac{3}{2}\dot{v} = \tfrac{1}{2}\dot{u} \Rightarrow \tfrac{3}{\cancel{2}}(-2) = \tfrac{1}{\cancel{2}}\dot{u} \Rightarrow \boxed{\dot{u} = -6\,m/s}$$

④



$$\begin{bmatrix} x & 0 & 0 & 0 & 0 & 0 \\ & x & 0 & 0 & 0 & 0 \\ & & x & 0 & 0 & 0 \\ & & & x & 0 & 0 \\ & & & & x & 0 \\ & & & & & x \end{bmatrix}$$

need m_{ij} $(i=j)$



$m_{HH} = m_{VV} = \rho\pi a^2$
$m_{\theta\theta} = 0$

$m_{VV} = \rho\pi b^2$
$m_{HH} = \rho\pi a^2$
$m_{\theta\theta} = \rho(a^2 - b^2)^2$

- $m_{11} \rightarrow$ cylinder $\cong \rho \forall_{sphere}$
  - $\rightarrow$ 1 ellipse $\Rightarrow m_{11} = \int_r^s \rho\pi a^2 \, dy$

$$m_{11\,total} = m_{11_{cyl}} + 2m_{11_{ellipse}} = \rho\frac{4}{3}\pi r^3 + 2\rho\pi a^2(s-r)$$

- $m_{22} \Rightarrow$ predominate contribution is the cylinder. the lifting surfaces add negligibly

$$m_{22} = \int_{-L/2}^{L/2} \rho\pi r^2 \, dx = \rho\pi r^2 L$$

- $m_{33} \Rightarrow m_{33cyl} + 2m_{33,ellipse} = \rho\pi r^2 L + 2\rho\pi b^2(s-r)$

- $m_{44} \Rightarrow m_{44}/_{cyl} = 0$ $\quad m_{44}$ due to fins $\Rightarrow$

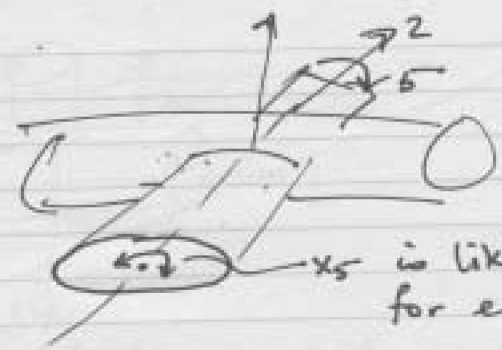
$$m_{44} = 2\int_r^s y^2 \rho\pi b^2 \, dy = 2\rho\pi b^2 \left.\frac{y^3}{3}\right|_r^s$$

both ellipses $\quad m_{44_{TOT}} = \frac{2}{3}\rho\pi b^2\left[s^3 - r^3\right]$

(Problem 4 con't)

- $M_{55} = m_{55\,cyl} + m_{55\,fins}$

$m_{55\,cyl}$: as in problem 1

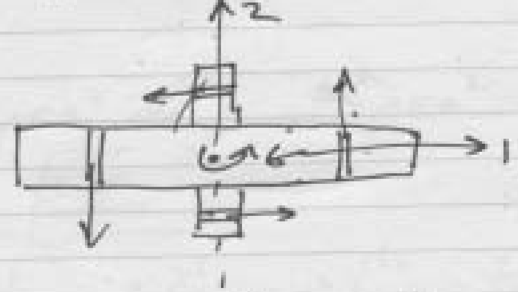


x_5 is like θ for ellipse part

$$= \int_{-L/2}^{L/2} x^2 \rho \pi r^2 \, dx + 2\int_r^s m_{\theta\theta}\big|_{ellipse} \, dy$$

$$m_{55} = \rho \pi r^2 \frac{L^3}{12} + 2 \cdot \rho (a^2 - b^2)^2 (s - r)$$

- $m_{66} \Rightarrow$ Top view



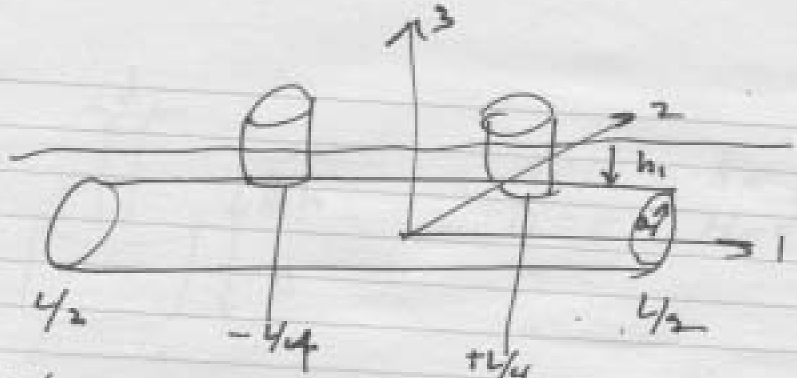
$$m_{66} = m_{66}\big|_{cyl} + 2\, m_{66\,ellipses}$$

$$m_{66\,cyl} = \int_{-L/2}^{L/2} x^2 \rho \pi r^2 \, dx = \frac{1}{12} \rho \pi r^2 L^3$$

$$m_{66\,ellipse} = \int_r^s \rho \pi a^2 y^2 \, dy = \frac{2}{3} \rho \pi a^2 (s^3 - r^3)$$

$$m_{66\,Tot} = \frac{1}{2} \rho \pi r^2 L^3 + \frac{2}{3} \rho \pi a^2 (s^3 - r^3)$$

(5)



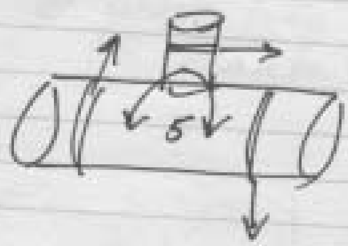
pontoon → ignore added mass in heave due to vertical struts since they pierce f. surf.

Heave $m_{33} \Rightarrow \rho\pi a^2 L$

Pitch $m_{55} = \int_{-L/2}^{L/2} x^2 \rho\pi a^2 \, dx = \frac{L^3}{12}\rho\pi a^2$

due to pontoons

$$2 \cdot m_{55} = 2 \cdot \int_r^{h_1} z^2 \rho\pi r^2 \, dz = 2 \cdot \frac{z^3}{3} \rho\pi r^2 \Big|_r^{h_1} = \frac{2}{3}\rho\pi r^2 \left[h_1^3 - r^3\right]$$



b) Restoring $C_{33} = \rho g A_{wp} = 2\rho\pi r^2 g$

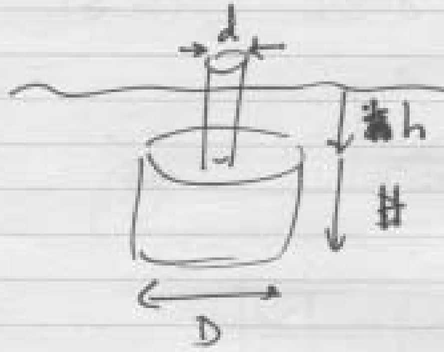
$F_z = C_{33} x_3(t) \qquad x_3(t) = \hat{x}_3 \cos \omega t$

$= 2\pi\rho g r^2 (\hat{x}_3 \cos \omega t)$

c) $\omega_n \Rightarrow \sqrt{\dfrac{k}{m + m_{33}}}$

$$\begin{cases} m = \text{equilibrium mass} \approx \rho \forall_{displaced} \\ M_{33} = \rho\pi a^2 L \\ k = C_{33} \end{cases}$$

⑥

6) 

$d=1 \quad h=5$
$D=5 \quad H=4$

a) $m_{33} \approx \frac{1}{2}\rho\,\forall_s = \frac{1}{2}\rho\left(\frac{4}{3}\pi\frac{D^3}{8}\right) = \frac{1}{12}\rho\pi D^3$

$F_{restore} = \rho g A_{wp}\,\Delta x \Rightarrow \rho g \frac{\pi d^2}{4}\Delta x$

c) $k = \rho g A_{wp} = \rho g \frac{\pi d^2}{4} \qquad \omega_n = \sqrt{k/m+m_a} \qquad M = \rho\forall$ displaced @ equilibrium position!

d) @ wave freq ~ ω_n amplitude is high. (Resonance)

b) F