13.012 PS #6 Solutions 2004 (A. Techet)

(1) 

a) $\omega_n = \sqrt{\frac{k}{M+M_a}}$ $\quad M_a = \rho \pi \frac{d^2}{4}$ $\quad M, k$ givens.

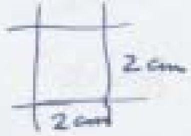
b) $U = 10\ m/s$, $d = 0.1\ m$ $\quad f_v = f_s = \frac{St \cdot U}{d} = \frac{0.2 \cdot 10}{0.1} = 20\,Hz$

c) Lockin → $f_v = f_n = f_{lift} = \frac{1}{2} f_{drag}$

$2\pi f_n = \sqrt{\frac{k}{m+m_a}}$ $\quad f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m+m_a}}$

$U_{flow} = \frac{f_v \cdot d}{St} = \frac{d}{2\pi} \cdot \frac{1}{St} \cdot \sqrt{\frac{k}{m+m_a}}$ for lock-in.

$St = 0.2$

(2)  U_{tow} $\quad \phi = 1\,mm$  300 ft^2 @ 3 knots

Area = 4 cm^2 = 0.0004 m^2; A = 300 ft = 27.9 m^2

$\#squares = \frac{\text{Area Net}}{\text{Area Square}} = 69{,}750$ 

Assuming net is square there are approx 264 squares per side

 # lines in horiz = 264 + 1 = 265

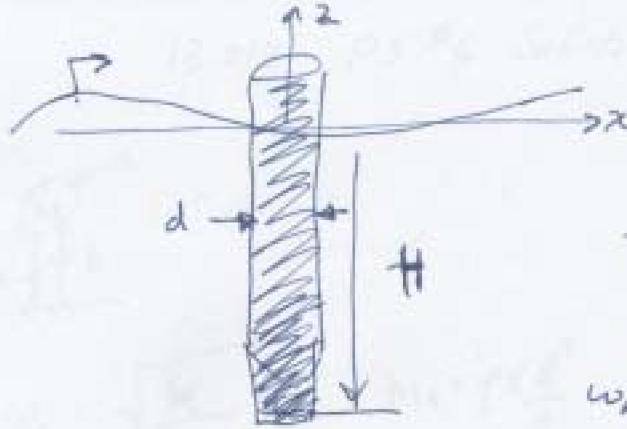
length of horiz is 264 · 2 cm = 5.28 m

same in vertical $\quad$ Total Drag = $\frac{1}{2}\rho U^2 C_D \cdot d \cdot 5.28 \cdot (530)$ ← # of lines total

Drag on one row = $\frac{1}{2}\rho U^2 C_D \cdot d \cdot 5.28\,m$ $\quad D = 4081.6\,N$

$P = D \cdot V = 6285.6\,W$ ~ 8.4 Hp

(3)



T = heave period

$T \sim A_{wp}, m, \rho$

$\omega_n = \sqrt{\frac{k}{m + m_a}}$

$T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{m + m_a}{k}}$

$k = \rho g A_{wp} = \rho g \frac{\pi d^2}{4}$

$m = \rho g \forall_{ol}$

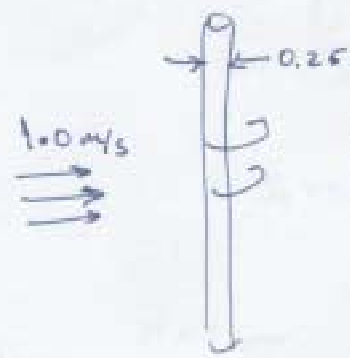
$m_a \simeq \frac{1}{2} \rho \left(\frac{4}{3} \pi r^3\right) \sim 0$

$m \uparrow$ by 2, then $T_n \uparrow$ by $\sqrt{2}$

$K \sim A_{wp}$ so $2 \times A_{wp} \Rightarrow 2 \times k \Rightarrow T_n \downarrow$ by $\sqrt{2}$

optimal period small waterplane area (skinny!)
large mass (so long is good)

(4)



0.25 m

$f_v = \frac{St \cdot U}{d} = \frac{0.2 \cdot 1}{0.25} = 0.8\,Hz$

$F_{lift} = f_v = 0.8\,Hz$

$F_{drag} = 2 F_{lift} = 1.6\,Hz$

(5) $f = 100\,Hz \quad U = 8\,m/s \quad St \sim 0.2$

$d = \frac{St \cdot U}{f} = \frac{0.2 \cdot 8}{100} = 0.016\,m = 1.6\,cm$