13.012
Fall 2004

Ps 7

1.

a) $\dfrac{\partial \hat{\phi}_D}{\partial n} = -\dfrac{\partial \hat{\phi}_I}{\partial n} \quad \left( \dfrac{\partial \phi_D}{\partial n} = -\dfrac{\partial \phi_I}{\partial n} \right)$

b) Radiation Potential

$FS \rightarrow \dfrac{\partial^2 \phi_R}{\partial t^2} + g \dfrac{d\phi_R}{\partial z} = 0$

@ $x \rightarrow \infty$, $\phi_R \rightarrow 0$

$\left.\dfrac{\partial \phi_R}{\partial n}\right|_{seafloor} = 0$,

$\dfrac{\partial \phi_R}{\partial n} = \vec{V}_B \cdot \hat{n}$ on body

c) $F(t) = Re\left\{ [\hat{F}_I + \hat{F}_D] e^{i\omega t} + \hat{x}_3 \hat{F}_R e^{i\omega t} \right\} + F_{hydro}$

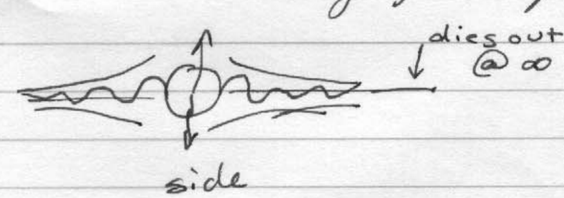
$\hat{F}_I = i\omega\rho \iint_{\bar{S}} \hat{n}\,\hat{\phi}_I \, ds$

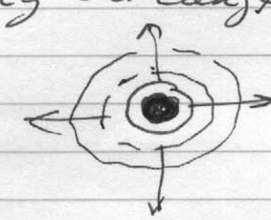
$\hat{F}_D = i\omega\rho \iint_{\bar{S}} \hat{n} \cdot \hat{\phi}_D \, ds$

$\hat{F}_R = i\omega\rho \iint_{\bar{S}} n\, \hat{\phi}_R \, ds$

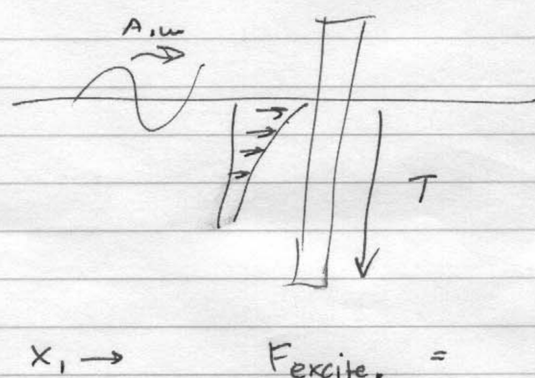
d) diffraction is negligible for long wavelengths

e)



2.

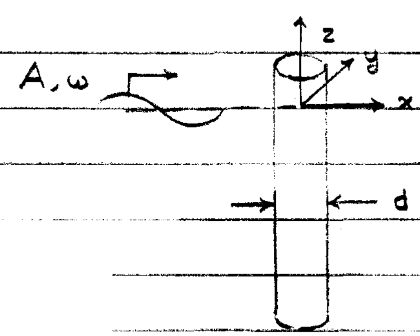



$x >> d$
$A \sim 2d$

a)
Surge $x_1 \rightarrow$ $\quad F_{excite_1} = \iint p \cdot n_x \, dS$

$p = \rho \dfrac{\partial \phi}{\partial t} + \underbrace{\rho g z}_{const}$ $\qquad p \sim \rho \dfrac{\partial \phi_I}{\partial t}$ $\qquad \phi_D \sim$ negligible

2.




- $\zeta(x, t) = A \,\mathrm{Re}\left\{ e^{-ikx + i\omega t} \right\}$
- $\phi(x, z, t) = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz - ikx + i\omega t} \right\}$

a. GIVEN $\lambda \gg d$ & $A \sim 2d$, or $h/d \sim 4 \Longrightarrow$ USE MORISON EQ.:

$$dF_x(x = 0, z, t) = \left( \rho \frac{\pi d^2}{4} + m_a \right) \frac{\partial u}{\partial t} \, dz + \frac{1}{2} \rho C_D \, d \, u|u| \, dz$$

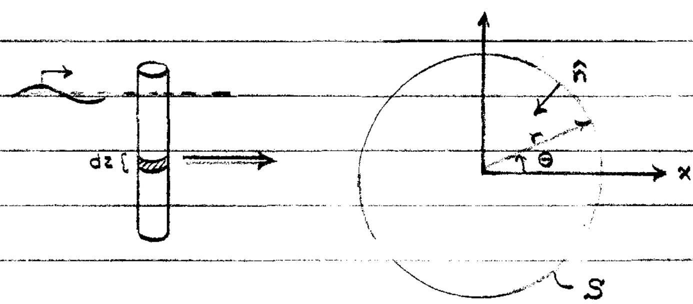
WHERE $m_a = \rho \frac{\pi d^2}{4}$ AND $u = \frac{\partial \phi}{\partial x}$.

b. THE FROUDE-KRYLOV TERM FROM a IS

$$\rho \frac{\pi d^2}{4} \frac{\partial u}{\partial t} dz \Big|_{x=0} \qquad (1)$$

IZONTAL FORCE DUE TO UNSTEADY PRESSURE INDUCED BY THE UNDISTURBED AMBIENT WAVES IS AS FOLLOWS:

- $p = -\rho \frac{\partial \phi}{\partial t} = \Re e \left\{ \rho g A e^{kz - ikx + i\omega t} \right\}$




- $dF_x = \iint_S p \hat{n} \cdot \hat{\imath} \, ds$

$$= \int_0^{2\pi} \Re e \left\{ \rho g A e^{kz} e^{-ikr\cos\theta + i\omega t} \right\} \overbrace{(-\cos\theta)}^{\hat{n}\cdot\hat{\imath}} dz \, r \, d\theta \Big|_{r = d/2}$$

$$= -\rho g A e^{kz} \frac{d}{2} dz \int_0^{2\pi} \cos\left(\omega t - k\frac{d}{2}\cos\theta\right) \cos\theta \, d\theta$$

$$= -\rho g A e^{kz} \frac{d}{2} dz \int_0^{2\pi} \Big[ \cos\omega t \cdot \cos\left(\frac{kd}{2}\cos\theta\right) +$$

$$+ \sin\omega t \sin\left(\frac{kd}{2}\cos\theta\right) \Big] \cos\theta \, d\theta$$

- IF $\lambda \gg d$, THEN $kd \ll 1$ AND $\cos\left(\frac{kd}{2}\cos\theta\right) \longrightarrow 1$

$$\sin\left(\frac{kd}{2}\cos\theta\right) \longrightarrow \frac{kd}{2}\cos\theta$$

- THEN

$$dF_x \simeq -\rho g A e^{kz} \tfrac{d}{2}\, dz \int_0^{2\pi} \left( \cos\omega t \cos\theta - \sin\omega t \frac{kd}{2} \cos^2\theta \right) d\theta$$

$$= -\rho g A e^{kz} \tfrac{d}{2}\, dz \cdot \left( \frac{kd}{2} \pi \sin\omega t \right)$$

- NOTE THAT

$$\left.\frac{\partial u}{\partial t}\right|_{x=0} = \frac{\partial^2 \phi}{\partial x \partial t} = -A\omega^2 e^{kz} \sin\omega t \overset{\text{DISP. REL.}}{=} -Agke^{kz} \sin\omega t$$

$$\therefore \; dF_x \simeq \rho \frac{\pi d^2}{4} \left.\frac{\partial u}{\partial t}\right|_{x=0} dz \quad \checkmark$$

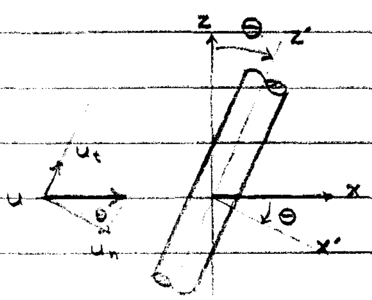
c. THE ADDED MASS TERM REPRESENTS THE DIFFRACTION FORCE.

d. THE PRESENCE OF A STRUCTURE WITH DIAMETER $\sim O(\lambda)$ WILL SIGNIFICANTLY DISTURB THE AMBIENT WAVEFIELD.

e. DEEP WATER: $\phi = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz - ikx + i\omega t} \right\}$

$$u = \frac{\partial \phi}{\partial x} = \mathrm{Re}\left\{ \omega A e^{kz - ikx + i\omega t} \right\}$$

SEPARATION PRINCIPLE (NEGLECTING VERTICAL VELOCITIES)




- $\theta = 15°$

$u_n = u \cos\theta$

$u_t = u \sin\theta$

- $dF_n = (\rho\nabla + m_a)\frac{\partial u_n}{\partial t}\,dz' + \frac{1}{2}\rho C_D d\,|u_n|\,u_n\,dz'$

- $dF_t = \frac{1}{2}\rho C_F(\pi d)\,u_t\,|u_t|\,dz'$

$$dz' = dz\cos\theta$$

LIFT: $dF_L = -dF_n\sin\theta + dF_t\cos\theta$

DRAG: $dF_D = dF_n\cos\theta + dF_t\sin\theta$

b. FOR A VERTICAL CYLINDER, THE DRAG FORCE WILL BE ENTIRELY DUE TO THE NORMAL COMPONENT.

— i.e. $dF_D = dF_n = (\rho\nabla + m_a)\frac{\partial u}{\partial t}\,dz + \frac{1}{2}\rho C_D d\,|u|\,u\,dz$

4a. $|F_x| \sim |F_I| = 5.6\times10^4$ N.

b. $F_x(t) = \underset{\text{INERTIA,}}{F_I(t)} + \underset{\text{VISCOUS DRAG TERM}}{F_v(t)}$

- $|F_I| \gg |F_v|$

- SINCE $F_I \sim \frac{\partial u}{\partial t}$, WHICH IS 90° OUT-OF-PHASE WITH $\eta(t)$, THE MAXIMA OCCUR WHEN $\eta = 0$.

d. $M_{\text{SEAFLOUR}} = \int_{-H}^{0}(H+z)\,dF_x$ WHERE

$$dF_x = (\rho\nabla + m_a)\frac{\partial u}{\partial t}\,dz + \frac{1}{2}\rho C_D d\,|u|\,u\,dz$$

e. $$z_{cp} = \frac{\int_{-H}^{0} z \, dF_x}{\int_{-H}^{0} dF_x}$$

RELATIVE TO THE SEAFLOOR, $H + z_{cp}$.