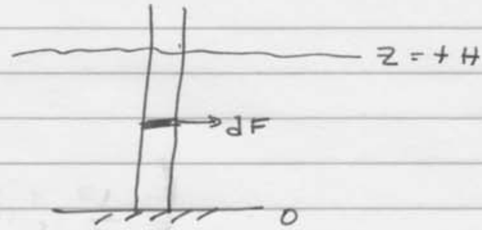
13.012
2004

HW 8 –

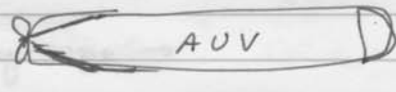


① $M = \int_0^H z\, dF$

dF from Morrison

$\lambda >> d$ no diffraction
deepwater $\omega^2 = gk$
$5 \le x/d \quad 2a/d \in [1 : 10]$ ← both drag & inertial terms must be considered

②



← $u = 3\ cm/s$

Model = $1/10^{th}$ Full Scale (proto) $\quad \ell_m = 0.75\ m$
$d_m = 0.075\ m$

Thrust to maintain position

$\frac{\ell_m}{d_m} = \frac{\ell_p}{d_p} = \frac{10}{1} \quad C_D \approx 1.0$ ← const for both model & full scale if Re # scaling used correctly

$Re_m = Re_{fs(p)} \rightarrow \frac{U_m \ell_m}{\nu} = \frac{U_p \ell_p}{\nu}$

a) $\frac{\ell_m}{\ell_p} = \frac{1}{10} \quad \boxed{U_m = 10\, u_p}$

b) Drag on Model = 10 N $\quad C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$, $A \approx \frac{\pi d^2}{4}$

$C_{D_m} = \frac{8 \cdot 10}{1000 \cdot 30 \cdot 30 \cdot \pi\, (.075)^2} = C_{Dp}$

$T_{max} \approx \frac{8 \cdot 10}{1000 \cdot 30 \cdot 30 \cdot \pi\, (0.075)^2} \cdot \frac{1}{2} \cdot 1000 (3)(3)\, \pi \frac{(0.75)^2}{4}$

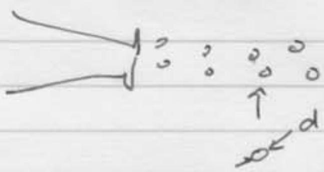
$= \frac{10}{100} \cdot 100$ ~~[illegible]~~ $\underline{10\,N}$

(c)

$$T = f(\rho, U, L, D, \mu)$$

$$c_T = \frac{T}{\frac{1}{2}\rho U^2 D^2} = f\left(\frac{\rho U L}{\mu}, \frac{L}{D}\right)$$

(3)



$$d = f(\rho, \mu, V_{jet}, D, \Upsilon)$$

$$\frac{N}{m^2} = \frac{kg\, m}{s^2 m^2}$$

n = 6

j = 3 → 3π groups →

$$\frac{d}{D} = f\left\{\frac{\rho V D}{\mu}, \text{surface tension coef}\right\}$$

Re flow in jet ; $\left(\Upsilon / \rho V^2\right)$

	d	ρ	μ	V	D	Υ
M	0	1	1	0	0	1
L	1	−3	−1	1	1	−1
T	0	0	−1	−1	0	−2

choose ρ, V, D to be 3 vars

d = key variable

$$\pi_1 = d/D$$

$$\pi_2 = \frac{\mu}{\rho V D}$$

$$\pi_3 = \Upsilon \rho^a V^b D^c$$

M ⇒ 1 + a + 0 + 0 = 0

L ⇒ −1 − 3a + b + c = 0

T ⇒ −2 + 0 − b + 0 = 0

b = −2

a = −1

c = 1

$$\boxed{\pi_3 = \frac{\Upsilon}{\rho V^2 D}} \quad \left[\frac{kg\, m^2}{s^2 m^2} \cdot \frac{m^3}{kg} \cdot \frac{s^2}{m^2\, m}\right] ✓$$

1/We ← Weber #

④ Ship → froude #

$L_s = 10m$
$B_s = L_s/10$ $\quad D = f(\rho, U, L, B, \mu, g)$
U_s

a) $$c_D = \frac{D}{\frac{1}{2}\rho U^2 LB} = f\left\{ \frac{\rho U L}{\mu}, \frac{U}{\sqrt{gL}}, \frac{L}{B} \right\}$$
(↑ or L^2)

b) $L_m = L_s/10$ $\quad V_s = 20 m/s$ $\quad F_m = 250 N$

$$C_{T_m} = C_{F_m} = \frac{F_m}{\frac{1}{2}\rho U_m^2 L_m B_m} = \frac{250}{500 \cdot U_m \cdot 1 \cdot 0.1}$$

$$\frac{U_m}{\sqrt{g l_m}} = \frac{U_s}{\sqrt{g l_s}} \qquad U_m^2 = (20)^2 \frac{l_m}{l_s} = \frac{(20)^2}{10} = 40$$

$$U_m = \sqrt{40} = 2\sqrt{10} \ m/s$$

$$C_T = \frac{1}{0.2 \cdot 2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{4}$$

Full scale → $T = C_T \cdot \frac{1}{2}\rho U_s^2 L_s B_s$
$$= \frac{\sqrt{10}}{4} \cdot \frac{1}{2} \cdot 1000 \cdot (20)^2 \cdot 10 \cdot (1.0)$$
$$T = \left(\frac{\sqrt{10}}{2} \cdot 1000 \cdot 1000\right) N$$

c) Power = F · Vel

$$P = f\{V_s, L_s, B_s, \rho, \mu, D_j, V_j, g\}$$

~~Power~~ $\quad n = 10 \; ; \; j = 3 \quad \pi \to 7$ groups

Choose ρ, V_j, D_j $\quad \left[\frac{V_s}{V_j}, \frac{L_s}{B_s}, \frac{B_s}{D_j} \right]$ F_R, Re

Power → $\frac{ML}{T^2} \cdot \frac{L^2}{T^2}$ → $\pi_1 = P \rho^a V_j^b D_j^c$

M	$1 + a = 0$	$a = -1$
L	$3 - 3a + b + c = 0$	$c = -4$
T	$-4 - 2b = 0$	$b = -2$

$3 + 3 - 2 + c = 0$
$6 - 2 \quad c = -4$

$$C_P = \frac{P}{\rho V^2 D^4}$$

π_2 = advance ratio $\Rightarrow J = \left[\frac{ND_j}{V_j}\right]$

5) $L_s = 100\,m \rightarrow \nu = 10^{-6}\ m/s^2$
$L_m = ?$

Froude = $\frac{U_m}{\sqrt{g L_m}} = \frac{U_s}{\sqrt{g L_s}}$

$\frac{U_m^2}{U_s^2} = \frac{L_m}{L_s} \rightarrow \frac{U_m^2 L_s}{L_m U_s^2} = 1$ $\qquad \frac{U_s}{U_m} = \sqrt{\frac{L_s}{L_m}}$

Re also scaled $\rightarrow$ $\frac{U_m L_m}{\nu_m} = \frac{U_s L_s}{\nu_s}$

$\frac{\nu_m}{\nu_s} = \frac{U_s L_s}{U_m L_m} = \sqrt{\frac{L_s}{L_m}} \cdot \frac{L_s}{L_m} = \sqrt{20} \cdot 20$

$$\boxed{\frac{\nu_m}{\nu_s} = 40\sqrt{5}}$$

on another planet gravity would be different!

6) d → g → Thrust. NRPM