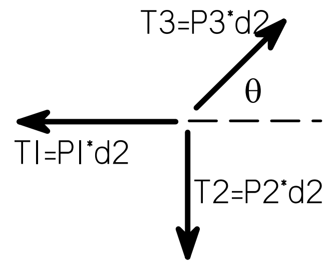
Gere 3.3-8 page 253

N.B. subscripts used [$ORIGIN := 1$

$P_1 := 100N$ $\quad d_1 := 10mm$ $\quad r_1 := \dfrac{d_1}{2}$ $\quad d_2 := 75mm$ $\quad \theta := 45deg$

using right hand rule vector representation for the moments applied to the system and using equilibrium for torque:



$P_1 \cdot d_2 - P_3 \cdot d_2 \cdot \cos(\theta) = 0$ $\quad$ => ...

$P_3 := \dfrac{P_1}{\cos(\theta)}$ $\quad P_3 = 141.421\,N$

$i := 1..3$ $\quad P_2 \cdot d_2 - P_3 \cdot d_2 \cdot \sin(\theta) = 0$ $\quad$ => ... $\quad P_2 := P_3 \cdot \sin(\theta)$ $\quad P_{(2)} = 100\,N$

$T_i := P_i \cdot d_2$ $\quad T = \begin{pmatrix} 7.5 \\ 7.5 \\ 10.607 \end{pmatrix} J$

maximum is P3 => .. $\quad$ from torque equation (Gere 3-11) $\quad \tau_{max} = \dfrac{T \cdot r}{I_P}$

by symmetry all rods have same IP $\quad I_P := \pi \cdot \dfrac{(r_1)^4}{2}$ $\quad I_P = 9.817 \times 10^{-10} m^4$

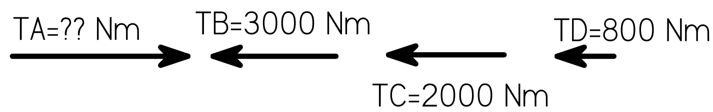
$\tau_{max} := \dfrac{P_3 \cdot d_2 \cdot r_1}{I_P}$ $\quad \tau_{max} = 54.019 \dfrac{N \cdot 10^6}{m^2}$

ORIGIN := 1

Gere problem 3.4-2 page 256

$T_B := 3000N\cdot m \qquad T_C := 2000N\cdot m \qquad T_D := 800\cdot N\cdot m \qquad G := 80\cdot 10^{9}Pa$

using right hand rule vector representation for the moments applied to the system and using equilibrium for torque:



$T_1 = T_{AB} \qquad T_1 := T_B + T_C + T_D \qquad T_1 = 5.8\times 10^{3} N\cdot m \qquad d_1 := 80mm \qquad r_1 := \frac{d_1}{2} \qquad L_1 := 0.5m$

$T_2 = T_{BC} \qquad T_2 := T_C + T_D \qquad T_2 = 2.8\times 10^{3} N\cdot m \qquad d_2 := 60mm \qquad r_2 := \frac{d_2}{2} \qquad L_2 := 0.5m$

$T_3 = T_{CD} \qquad T_3 := T_D \qquad T_3 = 800 N\cdot m \qquad d_3 := 40mm \qquad r_3 := \frac{d_3}{2} \qquad L_3 := 0.5m$

from torque equation (Gere 3-11) $\qquad \tau_{max} = \frac{T\cdot r}{I_P} \qquad i := 1..3$

$I_{P_i} := \pi\cdot\frac{(r_i)^4}{2} \qquad I_P = \begin{pmatrix} 4.021\times 10^{-6} \\ 1.272\times 10^{-6} \\ 2.513\times 10^{-7} \end{pmatrix} m^4$

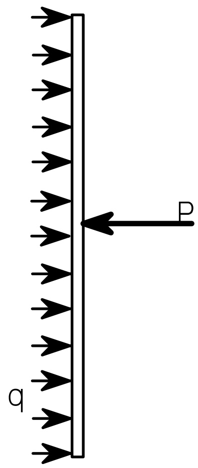
$\tau_{max_i} := \frac{T_i\cdot r_i}{I_{P_i}} \qquad \tau_{max} = \begin{pmatrix} 57.694 \\ 66.02 \\ 63.662 \end{pmatrix} \frac{N\cdot 10^6}{m^2} \qquad \tau_{max\_all} := \max(\tau_{max}) \qquad \tau_{max\_all} = 66.02\,\frac{N\cdot 10^6}{m^2}$

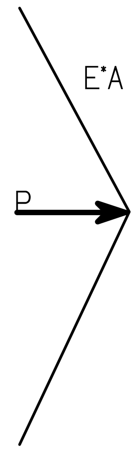
for twist ... Gere 3-14 $\qquad \theta = \frac{T\cdot L}{G\cdot I_p}$ $\qquad$ and for non-uniform torque .. Gere 3-20. $\qquad \theta = \sum_i \phi_i = \sum_i \left(\frac{T\cdot L_i}{G\cdot I_p}\right)_i$

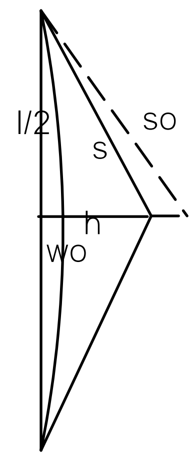
$\theta := \sum_{i=1}^{3} \frac{T_i\cdot L_i}{G\cdot I_{P_i}} \qquad \theta = 2.444\,deg$

13.014J/1.052J extra problem from PhD qualifiers January 2002

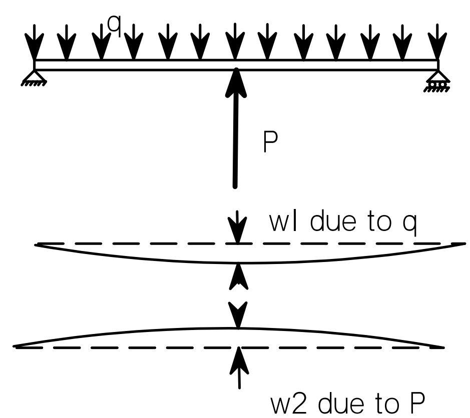
consider mast (beam) and stay (cable) separately:







mast as a beam: superposing deflections from constant load/length and concentrated force @ L/2



$$w_1 := \frac{5}{384}\cdot q\cdot l^4 \qquad w_2 := \frac{1}{48}\cdot P\cdot l^3$$

$$EIw_o := \frac{5}{384}\cdot q\cdot l^4 - \frac{1}{48}\cdot P\cdot l^3$$

stay as a cable: first obtain strain:

$$\varepsilon := \frac{s - s_o}{s_o} \qquad s_o := \sqrt{\left(\frac{l}{2}\right)^2 + h^2} \qquad s := \sqrt{\left(\frac{l}{2}\right)^2 + \left(h + w_o\right)^2}$$

neglect terms in wo^2

$$s_o := \frac{1}{2}\sqrt{l^2 + (2\cdot h)^2} \qquad s := \sqrt{\left(\frac{l}{2}\right)^2 + h^2 + 2\cdot h\cdot w_o + w_o^{\,2}} \qquad s := \frac{1}{2}\sqrt{l^2 + (2\cdot h)^2 + 8\cdot h\cdot w_o}$$

$$\varepsilon := \frac{s - s_o}{s_o} \qquad \varepsilon \rightarrow 2 \cdot \frac{\left[\frac{1}{2}\cdot\left(l^2 + 4\cdot h^2 + 8\cdot h\cdot w_o\right)^{\frac{1}{2}} - \frac{1}{2}\cdot\left(l^2 + 4\cdot h^2\right)^{\frac{1}{2}}\right]}{\left(l^2 + 4\cdot h^2\right)^{\frac{1}{2}}}$$

factor out $l^2 + 4\cdot h^2$

$$\varepsilon := \frac{\left[\left(l^2 + 4\cdot h^2\right)\cdot\left(1 + \frac{8\cdot w_o\cdot h}{l^2 + 4\cdot h^2}\right)\right]^{\frac{1}{2}} - \left(l^2 + 4\cdot h^2\right)^{\frac{1}{2}}}{\left(l^2 + 4\cdot h^2\right)^{\frac{1}{2}}}$$

$\left(l^2 + 4\cdot h^2\right)^{\frac{1}{2}}$ cancels out

$$\varepsilon := \left(1 + \frac{8\cdot w_o\cdot h}{l^2 + 4\cdot h^2}\right)^{\frac{1}{2}} - 1$$

$\frac{8\cdot w_o\cdot h}{l^2 + 4\cdot h^2}$ << 1 and as $(1 + x)^{\frac{1}{2}} = 1 + \frac{x}{2} + h\_o\_t$ $\quad \varepsilon := 1 + \frac{1}{2}\left(\frac{8\cdot w_o\cdot h}{l^2 + 4\cdot h^2}\right) - 1 \quad \varepsilon := \frac{4\cdot w_o\cdot h}{l^2 + 4\cdot h^2}$

to solve cable relationship, we could use geometry to relate force in cable to force in stay (see below at end) or as suggested use the energy approach:
In that, the force is P and the displacement of P is wo. there is strain energy in bending in the mast, and in the stretch of the cable. It is expected that the bending contribution will be << cable so ...

cable solution energy method $\quad \Pi = U - W$

Π total potential energy
U elastic energy
W work done

$$\Pi = \frac{1}{2}\cdot E\cdot\varepsilon^2 - P\cdot w_o = \frac{1}{2}\cdot E\cdot A\cdot l_{cable}\cdot\left(\frac{4\cdot w_o\cdot h}{l^2 + 4\cdot h^2}\right)^2 - P\cdot w_o \qquad l_{cable} := s_o \qquad l_{cable} \rightarrow \frac{1}{2}\cdot\left(l^2 + 4\cdot h^2\right)^{\frac{1}{2}}$$

$\frac{d}{dw_o}\Pi = 0$ for equilibrium

$$\frac{d}{dw_o}\left[\frac{1}{2}\cdot E\cdot A\cdot s_o\cdot\left(\frac{4\cdot w_o\cdot h}{l^2 + 4\cdot h^2}\right)^2 - P\cdot w_o\right] \rightarrow 8\cdot E\cdot\frac{A}{\left(l^2 + 4\cdot h^2\right)^{\frac{3}{2}}}\cdot w_o\cdot h^2 - P$$

=> $$P := \frac{8\cdot E\cdot A\cdot\left(h^2\cdot w_o\right)}{\left(l^2 + 4\cdot h^2\right)^{\frac{3}{2}}}$$

$K := \frac{q \cdot l}{w_o}$ lbs per inch e.g. from above: $EIw_o := \frac{5}{384} \cdot q \cdot l^4 - \frac{1}{48} \cdot P \cdot l^3$

$$E \cdot I \cdot w_1 = \frac{5 \cdot q \cdot l^4}{384} = E \cdot I \cdot w_o + \frac{P \cdot l^3}{48}$$

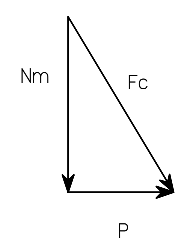
$$E \cdot I \cdot w_o + \frac{P \cdot l^3}{48} \rightarrow E \cdot I \cdot w_o + \frac{1}{6} \cdot E \cdot \frac{A}{\left(l^2 + 4 \cdot h^2\right)^{\frac{3}{2}}} \cdot w_o \cdot h^2 \cdot l^3$$

$$\frac{5 \cdot q \cdot l^4}{384} = w_o \cdot \left[E \cdot I + \frac{1}{6} \cdot \frac{E \cdot A \cdot h^2 \cdot l^3}{\left(l^2 + 4 \cdot h^2\right)^{\frac{3}{2}}}\right]$$

$$\frac{ql}{w_o} = \frac{384 \cdot E}{5 \cdot l^3} \cdot \left[I + \frac{1}{6} \cdot \frac{A \cdot h^2 \cdot l^3}{\left(l^2 + 4 \cdot h^2\right)^{\frac{3}{2}}}\right]$$

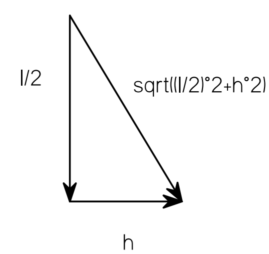
$$\text{tension\_cable} := \sigma_Y \cdot A$$

$$N_y = \text{tension\_cable} \cdot \frac{\frac{1}{2}}{\sqrt{\left(\frac{1}{2}\right)^2 + h^2}} = \sigma_Y \cdot A \cdot \frac{\frac{1}{2}}{\sqrt{\left(\frac{1}{2}\right)^2 + h^2}}$$



$$P_y = \text{tension\_cable} \cdot \frac{h}{\sqrt{\left(\frac{1}{2}\right)^2 + h^2}} = \sigma_Y \cdot A \cdot \frac{h}{\sqrt{\left(\frac{1}{2}\right)^2 + h^2}}$$

from P(w.o) above



$$P_y = \frac{8 \cdot E \cdot A \cdot \left(h^2 \cdot w_{o\_yield}\right)}{\left(l^2 + 4 \cdot h^2\right)^{\frac{3}{2}}} = \sigma_Y \cdot A \cdot \frac{h}{\sqrt{\left(\frac{1}{2}\right)^2 + h^2}}$$

$$w_{o\_yield} := \frac{\sigma_Y \cdot A}{8 \cdot E \cdot A \cdot h^2} \cdot \frac{h}{\sqrt{\left(\frac{1}{2}\right)^2 + h^2}} \cdot \left(l^2 + 4 \cdot h^2\right)^{\frac{3}{2}}$$

or ...

$$w_{o\_yield} \text{ simplify } \rightarrow \frac{1}{4} \cdot \frac{\sigma_Y}{E \cdot h} \cdot \left(l^2 + 4 \cdot h^2\right)$$

stress in the beam at cable yield

compression bending

$$\sigma_{max} := \frac{N_y}{A_{mast}} + \frac{M_{max} \cdot R}{I} \qquad M_{max} := \frac{q \cdot l - P_y}{2} \cdot \frac{l}{2} - \frac{q \cdot l^2}{8}$$

$R_1 = R_2$ by symmetry $\qquad 2 \cdot R_1 = q \cdot l - P_y \qquad R_1 = \dfrac{q \cdot l - P_y}{2}$

$$M_{max} = R_1 \cdot \frac{l}{2} - \frac{q \cdot l}{2} \cdot \frac{l}{4} = \frac{q \cdot l - P_y}{2} \cdot \frac{l}{2} - \frac{q \cdot l^2}{8} \qquad P_y = \sigma_Y \cdot A \cdot \frac{h}{\sqrt{\left(\frac{l}{2}\right)^2 + h^2}}$$

$$M_{max} = \frac{q \cdot l^2}{4} - \frac{q \cdot l^2}{8} - \frac{l}{4} \cdot \left[\sigma_Y \cdot A \cdot \frac{h}{\sqrt{\left(\frac{l}{2}\right)^2 + h^2}}\right] \qquad N_y = \sigma_Y \cdot A \cdot \frac{\frac{l}{2}}{\sqrt{\left(\frac{l}{2}\right)^2 + h^2}}$$

everything determined

using the geometry above to relate P (the interface force) to cable strain ...

$$F_{cable} := P \cdot \frac{\sqrt{\left(\frac{l}{2}\right)^2 + h^2}}{h} \qquad \varepsilon := \frac{F_{cable}}{A \cdot E} \qquad \varepsilon \rightarrow \frac{4}{\left(l^2 + 4 \cdot h^2\right)} \cdot w_o \cdot h$$

$$\varepsilon = \frac{F_{cable}}{A \cdot E} = \frac{P}{A \cdot E} \cdot \frac{\sqrt{\left(\frac{l}{2}\right)^2 + h^2}}{h} = \frac{4 \cdot w_o \cdot h}{l^2 + 4 \cdot h^2} \qquad => \ldots$$

$$P = \frac{A \cdot E \cdot w_o \cdot h^2}{\left[\left(\frac{l}{2}\right)^2 + h^2\right]^{\frac{3}{2}}} = \frac{A \cdot E \cdot w_o \cdot h^2}{\left[\left(\frac{1}{2}\right)^2 \cdot \left(l^2 + 4 \cdot h^2\right)\right]^{\frac{3}{2}}} = \frac{8A \cdot E \cdot w_o \cdot h^2}{\left(l^2 + 4 \cdot h^2\right)^{\frac{3}{2}}}$$

which should (and fortunately does) match the result arrived at from energy considerations above

$$P := \frac{8 \cdot E \cdot A \cdot \left(h^2 \cdot w_o\right)}{\left(l^2 + 4 \cdot h^2\right)^{\frac{3}{2}}}$$