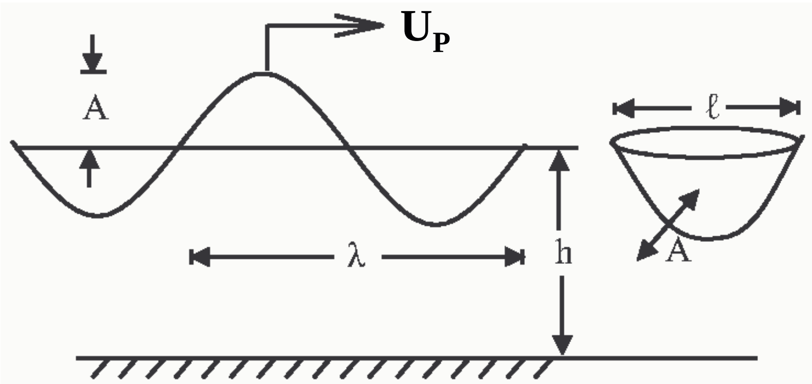
13.021 - Marine Hydrodynamics
Lecture 21

Wave Forces on a Body



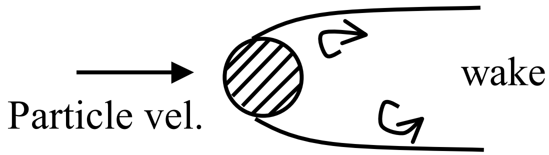
$$U = \omega A$$
$$R = \frac{Ul}{\nu} = \frac{\omega A l}{\nu}$$
$$K_c = \frac{UT}{l} = \frac{A\omega T}{l} = 2\pi \frac{A}{l}$$

$$C_F = \frac{F}{\rho g A l^2} = f\left(\underbrace{\frac{A}{\lambda}}_{\text{Wave steepness}}, \underbrace{\frac{l}{\lambda}}_{\text{Diffraction parameter}}, R, \frac{h}{\lambda}, \text{roughness}, \ldots \right)$$

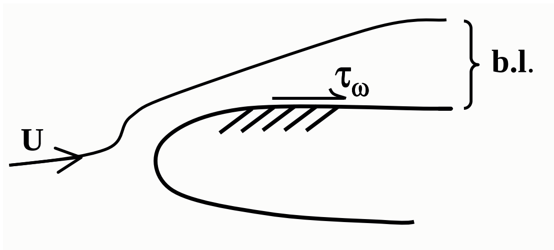
Type of Forces

1. Viscous forces (form drag, viscous drag) : $f(R, K_c, \text{roughness}, \ldots)$.

   (a) Form drag (C_D). Associated primarily with flow separation - normal stresses.



   (b) Friction drag (C_F). $\vec{F} \sim \iint_{\text{body}} \tau_\omega dS$



2. Inertial forces (Froude-Krylov force, diffraction force, radiation forc): forces arising from potential flow wave theory.

$$\vec{F} = \iint_{\substack{\text{body} \\ \text{(wetted surface)}}} p\hat{n}dS \text{ where } p = -\rho\left(\frac{\partial\phi}{\partial t} + gy + \underbrace{\frac{1}{2}|\nabla\phi|^2}_{\substack{=0 \text{ if linear theory,} \\ \text{small amplitude waves}}}\right)$$

In general:

$$\phi = \underbrace{\phi_I}_{\text{(a) Incident wave potential}} + \underbrace{\phi_D}_{\text{(b.1) Diffracted wave potential}} + \underbrace{\phi_R}_{\text{(b.2) Radiated wave potential}}$$

$$p = -\rho\left(\underset{(a)}{\frac{\partial \phi_I}{\partial t}} + \underset{(b.1)}{\frac{\partial \phi_D}{\partial t}} + \underset{(b.2)}{\frac{\partial \phi_R}{\partial t}} + gy + \dots\right)$$

(a) Incident wave potential: Froude-Krylov Force approximation, when $l << \lambda$, the incident wave field is not significantly modified by the presence of the body, therefore ignore ϕ_D and ϕ_R:

$$\phi \approx \phi_I$$

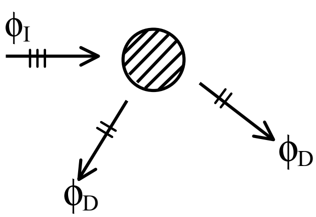
$$\vec{F}_{FK} = \underset{\text{body surface}}{\iint} \underbrace{-\rho\left(\frac{\partial \phi_I}{\partial t} + gy\right)}_{p_I} \hat{n} dS \Leftarrow \text{can calculate knowing (incident) wave kinematics (and body geometry)}$$

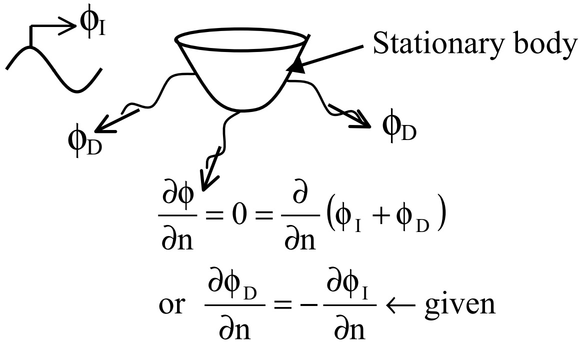
Further mathematical approximationis valid if the body is really small. After applying the divergence theorem, the above integral can be replaced by:

$$\vec{F}_{FK} = -\underset{\text{body volume}}{\iiint} \nabla p_I d\forall \approx \underset{\text{at body center}}{-\nabla p_I|} \underbrace{\forall}_{\text{body volume}}$$

(b) Diffraction and Radiation Forces - hydrodynamic coefficients: added mass, wave damping and wave excitation ...

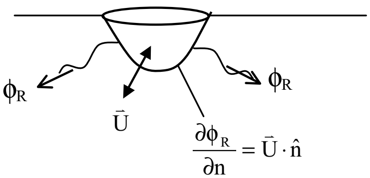
$(b.1)$ Diffraction or scattering force: when l not $<< \lambda$, wave field near body will be affected even if body is stationary, so that no-flux B.C. is satisfied.





$$\vec{F}_D = \int\!\!\int_{\text{body}} -\rho \left(\frac{\partial \phi_D}{\partial t} \right) \hat{n} dS$$

(b.2) Radiation Force - added mass and damping coefficient: even in the absence of an incident wave, body in motion creates waves and hence wave forces, and experiences also inertial forces.



$$\vec{F}_R = \int\!\!\int_{\text{body}} -\rho \left(\frac{\partial \phi_R}{\partial t} \right) \hat{n} dS = -\underbrace{m_{ij}}_{\substack{\text{added}\\ \text{mass}}} \dot{U}_j - \underbrace{d_{ij}}_{\substack{\text{wave}\\ \text{radiation}\\ \text{damping}}} U_j$$

Important parameters

$$\left.\begin{array}{l}(1) K_c = \frac{UT}{l} = 2\pi\frac{A}{l} \\ \\ (2) \text{diffraction parameter } \frac{l}{\lambda}\end{array}\right\} \begin{array}{l}\text{interrelated since maximum wave steepness: } \frac{A}{\lambda} \leq 0.07 \\ \left(\frac{A}{l}\right)\left(\frac{l}{\lambda}\right) \leq 0.07\end{array}$$

- If $K_c \leq 1$: no appreciable flow separation, viscous effect confined to b.l. (hence small), solve problem via potential theory. In addition, depending on the value of the ratio $\frac{l}{\lambda}$:
  - If $\frac{l}{\lambda} << 1$, ignore diffraction , wave effects in radiation problem (i.e., $d_{ij} \approx 0, m_{ij} \approx m_{ij}$ infinite fluid added mass). F-K approximation might be used, calculate $\vec{F}_{FK}$.
  - If $\frac{l}{\lambda} >> 1/5$, must consider wave diffraction, radiation ($\frac{A}{l} \leq \frac{0.07}{l/\lambda} \leq 0.035$).

- If $K_c >> 1$: separation important, viscous forces can not be neglected. Further on if:

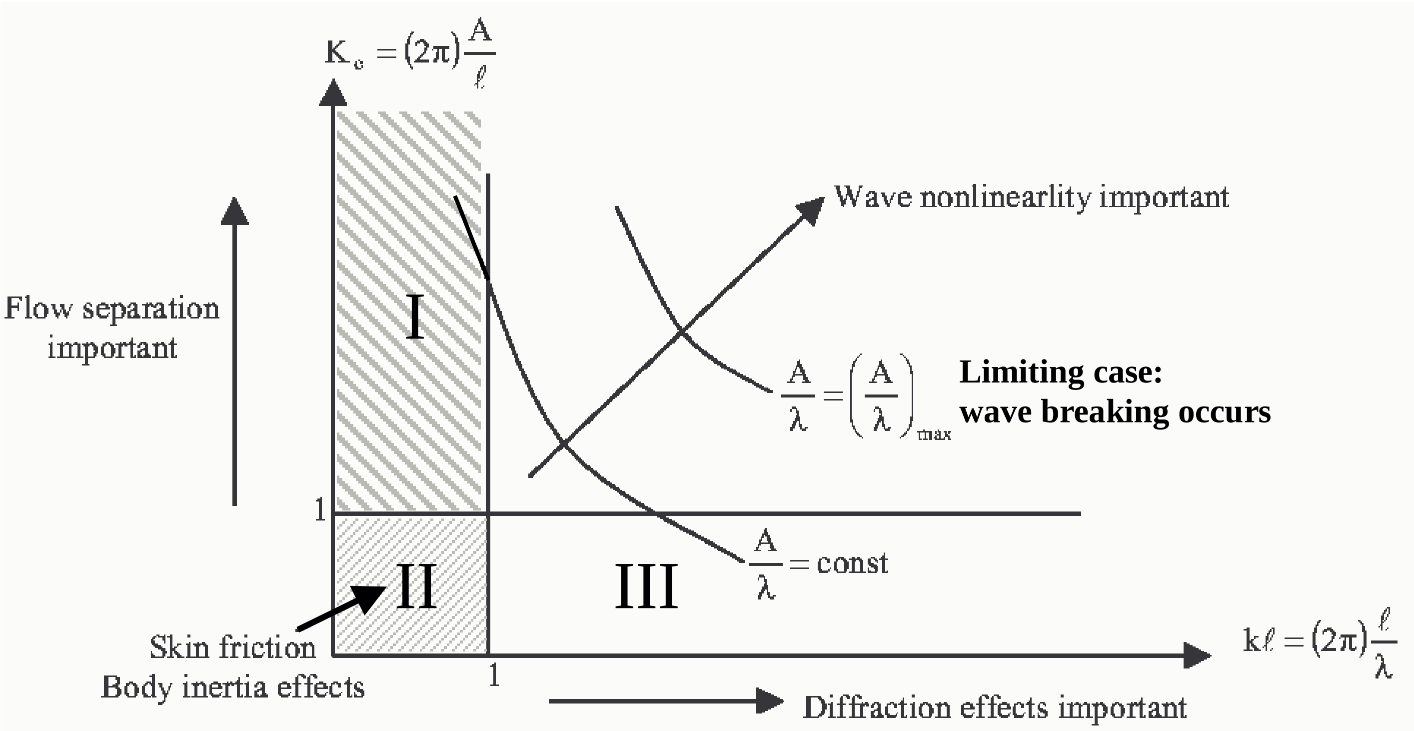
$$\frac{l}{\lambda} \leq \frac{0.07}{A/l} \text{ so } \frac{l}{\lambda} << 1 \text{ ignore diffraction and F-K approximation might be used}$$

$$F = \frac{1}{2}\rho l^2 \underbrace{U(t)}_{\substack{\text{relative} \\ \text{velocity}}} |U(t)| C_D(R)$$

- Intermediate K_c - both viscous and inertial effects important, use Morrison's formula.

$$F = \frac{1}{2}\rho l^2 U(t)|U(t)|C_D(R) + \rho l^3 \dot{U} C_m(R, K_c)$$

Summary:



I. Use: C_D and $F-K$ approximation.
II. Use: C_F and $F-K$ approximation.
III. C_D is not important and $F-K$ approximation. is not valid.