Due to unsteady flow, forces $X(t)$ and $Y(t)$ vary with time.

Force coefficients:

$$C_x = \frac{X(t)}{\frac{1}{2} \rho U^2 d}$$

$$C_y = \frac{Y(t)}{\frac{1}{2} \rho U^2 d}$$

Force Time Trace

![Force Time Trace Diagram]
Large responses may occur when:

\[ \omega_n = 2 \pi f_v - \omega_n = \sqrt{\frac{k}{m + m_a}} \]

What is \( m_a \) a function of?

**EXPERIMENT:** Force motion \( y(t) \) and measure force \( Y(t) \)

\[
y(t) = a \cos(\omega t) \\
\dot{y}(t) = -ao \sin(\omega t) \\
\ddot{y}(t) = -ao^2 \cos(\omega t) \\
Y(t) = Y_1 \cos(\omega t - \psi) + Y_r(t)
\]

Lift force is sinusoidal component and residual force. Filtering the recorded lift data will give the sinusoidal term which can be subtracted from the total force.

**Lift Force:**

\[
Y_1(t) = Y_1 \cos(\omega t - \psi) \\
= (Y_1 \cos \psi) \cos \omega t + (Y_1 \sin \psi) \sin \omega t
\]

\[
Y_1(t) = \left( -\frac{Y_1}{ao} \cos \psi \right) \ddot{y}(t) + \left( -\frac{Y_1}{ao} \sin \psi \right) \dot{y}(t)
\]

Added Mass: \( M_a(\omega, a) = \frac{Y_1}{ao^2} \cos \psi \)

Lift in-phase with velocity: \( Y_{1s}(\omega, a) = - \frac{Y_1}{ao} \sin \psi \)
Total Force:

\[ Y_f(t) = -M_L(\omega, a) \ddot{y}(t) + Y_1(\omega, a) \dot{y}(t) \]

\[ = -\left(\frac{\rho d^2}{2}\right) C_{ma}(\omega, a) \ddot{y}(t) + \frac{1}{2}\rho d U^2 C_{Lv}(\omega, a) \dot{y}(t) \]

If \( C_{Lv} > 0 \) then the fluid force amplifies the motion instead of opposing it. This is self-excited oscillation.

\( C_{ma}, C_{Lv} \) are dependent on \( \omega \) and \( a \).

Lift in phase with velocity

Gopalkrishnan (1993)

Drag Amplification

VIV tends to increase the effective drag coefficient. This increase has been investigated experimentally.

\[ \bar{C}_d = 1.2 + 1.1 \left( \frac{a}{d} \right) \]

\[ \tilde{C}_d \text{ occurs at twice the shedding frequency.} \]
Amplitude Estimation

Blevins (1990)

\[
a/d \approx \frac{1.29}{\left[1 + 0.43 S_d\right]^{1.35}}
\]

\[
S_d \approx 2 \pi \frac{f_n}{\rho d^2} \frac{2 \bar{m}}{\rho d^2} \frac{2 \bar{n}}{\rho d^2} \frac{2 \bar{u}}{\rho d^2} \frac{2 \bar{v}}{\rho d^2}
\]

\[
\zeta = \frac{b}{2 \sqrt{k(m^2 + m')}}
\]

\[
m' = \rho \cdot C_{ma} \quad \text{where} \quad C_{ma} = 1.0
\]

Three Dimensional Effects

Shear layer instabilities as well as longitudinal (braid) vortices lead to transition from laminar to turbulent flow in cylinder wakes.

Longitudinal vortices appear at \( R_d = 230 \).

Longitudinal Vortices

The presence of longitudinal vortices leads to rapid breakdown of the wake behind a cylinder.

C.H.K. Williamson (1992)
Longitudinal Vortices

\[ S_t = \frac{\bar{f} d}{U} \]

where \( d \) is the average cylinder diameter.

Oscillating Tapered Cylinder

\( U(0) = U_0 \)

Spanwise Vortex Shedding from 40:1 Tapered Cylinder

- \( R_e = 400; \quad S_t = 0.198; \quad A/d = 0.5 \)
- \( R_e = 1500; \quad S_t = 0.198; \quad A/d = 0.5 \)
- \( R_e = 1500; \quad S_t = 0.198; \quad A/d = 1.0 \)

\( d_{\text{max}} \)
Flow Visualization Reveals:
A Hybrid Shedding Mode

- ‘2P’ pattern results at the smaller end
- ‘2S’ pattern at the larger end
- This mode is seen to be repeatable over multiple cycles

DPIV of Tapered Cylinder Wake

Digital particle image velocimetry (DPIV) in the horizontal plane leads to a clear picture of two distinct shedding modes along the cylinder.

Rd = 1500; St = 0.198; A/d = 0.5

Evolution of the Hybrid Shedding Mode

Rd = 1500; St = 0.198; A/d = 0.5
Evolution of the Hybrid Shedding Mode

‘2P’    ‘2S’
\[ z/d = 7.9 \quad \text{and} \quad z/d = 22.9 \]
\[ R_d = 1500; \, S_t = 0.198; \, A/d = 0.5 \]
**Objectives:**
- Confirm numerically the existence of a stable, periodic hybrid shedding mode $2S_2P$ in the wake of a straight, rigid, oscillating cylinder.

**Principal Investigator:**
- Prof. George Em Karniadakis, Division of Applied Mathematics, Brown University

**Approach:**
- DNS - Similar conditions as the MIT experiment (Triantafyllou et al.)
- Harmonically forced oscillating straight rigid cylinder in linear shear inflow
- Average Reynolds number is 400

**Methodology:**
- Parallel simulations using spectral/hp methods implemented in the incompressible Navier-Stokes solver NEKTAR

**Results:**
- Existence and periodicity of hybrid mode confirmed by near wake visualizations and spectral analysis of flow velocity in the cylinder wake and hydrodynamic forces

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**VIV Suppression**

- Helical strake
- Shroud
- Axial slats
- Streamlined fairing
- Splitter plate
- Ribboned cable
- Pivoted guiding vane
- Spoiler plates

---

**VIV Suppression by Helical Strakes**

- Helical strakes are a common VIV suppression device.
Trip Wire Experiments

Trip wires cause the shear layer to transition early and thus retard separation. The wires can reduce amplitude of vibration but this reduction is very dependant on angle on incoming flow.

Lift Characteristics of Rigid Cylinder

- Mean lift coefficient
- Phase angle between oscillating lift force and cylinder motion.

Tandem Rigid Cylinders-Amplitude

- Single cylinder
- One offset
- 10 lateral offset

Leading cylinder
- Trailing cylinder 10D
- Trailing cylinder 5D
Oscillating Cylinders

Parameters:

\[ y(t) = a \cos(\omega t) \]
\[ y'(t) = -a \omega \sin(\omega t) \]

\[ V_m = \frac{a}{\omega} \]

\[ v = \frac{\mu}{\rho} ; \ T = \frac{2\pi}{\omega} \]

\[ Re = \frac{V_m d}{v} \quad \text{Reynolds #} \]
\[ b = \frac{d^2}{vT} \]
\[ KC = \frac{V_m T}{d} \quad \text{Keulegan-Carpenter #} \]
\[ St = \frac{f_v d}{V_m} \quad \text{Strouhal #} \]

Reynolds # vs. KC #

\[ Re = \frac{V_m d}{v} = \frac{\omega d}{v} = 2\pi \left( \frac{a}{d} \right) \left( \frac{d}{vT} \right) \]

\[ KC = \frac{V_m T}{d} = 2\pi \frac{a}{d} \]

\[ Re = KC \times b \]

\[ b = \frac{d^2}{vT} \]

Also effected by roughness and ambient turbulence
Forced Oscillation in a Current

\[ y(t) = a \cos(\omega t) \]
\[ \omega = 2\pi f = 2\pi / T \]

Parameters: \( a/d, \rho, \nu, \theta \)
Reduced velocity: \( U_r = U/fd \)
Max. Velocity: \( V_m = U + a\omega \cos \theta \)
Reynolds #: \( Re = V_m d / \nu \)
Roughness and ambient turbulence

Wall Proximity

At \( e/d > 1 \) the wall effects are reduced.
\( C_d, C_m \) increase as \( e/d < 0.5 \)
Vortex shedding is significantly affected by the wall presence.
In the absence of viscosity these effects are effectively non-existent.

Galloping

Galloping is a result of a wake instability.

Resultant velocity is a combination of the heave velocity and horizontal inflow.

If \( \omega_n << 2\pi f \), then the wake is quasi-static.
Lift Force, \( Y(\alpha) \)

\[
C_y = \frac{Y(t)}{\frac{1}{2} \rho U^2 A_p}
\]

Galloping motion

\[
m\ddot{y} + b\dot{y} + ky = Y(t)
\]

\[
Y(t) = \frac{1}{2} \rho U^2 a C_y - m_y \ddot{y}(t)
\]

\[
C_y(\alpha) = C_y(0) + \frac{\partial C_y}{\partial \theta}(0) + \ldots
\]

Assuming small angles, \( \alpha \):

\[
\alpha - \tan \alpha = \frac{\dot{y}}{U} \quad \beta = \frac{\partial C_y}{\partial \theta}(0) \quad V - U
\]

Instability Criterion

\[
(m+m_y)\ddot{y} + (b + \frac{1}{2} \rho U^2 a \frac{P}{U}) \dot{y} + ky \approx 0
\]

\[
\text{If} \quad b + \frac{1}{2} \rho U^2 a \frac{P}{U} < 0
\]

*Then the motion is unstable!*

*This is the criterion for galloping.*
β is shape dependent

<table>
<thead>
<tr>
<th>Shape</th>
<th>$\frac{\partial C_y}{\partial \alpha}$</th>
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<tr>
<td>1</td>
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Instability:

$$\beta = \frac{\partial C_y}{\partial \alpha} < \frac{-b}{\frac{1}{2} \rho U a}$$

Critical speed for galloping:

$$U > \frac{b}{\frac{1}{2} \rho a \left( -\frac{\partial C_y}{\partial \alpha} \right)}$$

Torsional Galloping

Both torsional and lateral galloping are possible. FLUTTER occurs when the frequency of the torsional and lateral vibrations are very close.
Galloping vs. VIV

• Galloping is low frequency
• Galloping is NOT self-limiting
• Once $U > U_{critical}$ then the instability occurs irregardless of frequencies.

References