**Roll Motion**

Coupled with sway & yaw.

Linear damping due to radiated waves is small  
so Roll Motion can be LARGE.

- perfect circular shape  
  - with motion about its center  
  - No waves are created

- Non circular shape  
  - Wave making is relatively small

Where does additional damping come from?

- Friction  
  - Eddy-generation

- Surface friction \( \rightarrow \) Viscous

- Eddy-generation

- Separation on appendages such as fins or keels  
  - Also Eddy-generation

**Friction**  
\[ F_f = C_f \frac{1}{2} \rho U^2 S \]  
\( F \propto U^2 \rightarrow \text{NON-LINEAR} \)
Roll Force \rightarrow F_4 = \phi \frac{\dot{x}_4}{|\dot{x}_4|} \quad \phi = \text{constant}

\[ x_4 = \phi_0 \cos \omega t \quad \phi_0 = \text{roll amplitude} \]

**CAN WE FIND AN EQUIVALENT LINEARIZATION OF ROLL FORCE?**

\[ F_4^L = 2\dot{x}_4 \]

Where coefficient \( \varepsilon \) is determined such that the energy per cycle spent by \( F_4 \) is the same as from the linearized \( F_4^L \) →

\[ \int_0^T F_4 \dot{x}_4 \, dt = \int_0^T F_4^L \dot{x}_4 \, dt \]

\[ \int_{\text{integral over one period}} \dot{x}_4 = \phi_0 \omega \sin \omega t \]

\[ \phi \omega^3 \phi_0^3 \int_0^{2\pi} \sin^2 \theta |\sin \theta| \, d\theta = \varepsilon \omega^3 \phi_0^2 \int_0^{2\pi} \sin^2 \theta \, d\theta \]

\[ \int_0^{2\pi} \sin^2 \theta |\sin \theta| \, d\theta = 2 \int_0^{2\pi} \sin^3 \theta \, d\theta = 4 \cdot \frac{4}{3} \]

\[ \int_0^{2\pi} \sin^2 \theta \, d\theta = \frac{2\pi}{2} \]