13.42 Lecture 2: Vortex Induced Vibrations

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Wake Patterns Behind Heaving Cylinders

- Shedding patterns in the wake of oscillating cylinders are distinct and exist for a certain range of heave frequencies and amplitudes.
- The different modes have a great impact on structural loading.
Transition in Shedding Patterns

\[ A/d = \frac{f}{U^2} = \frac{f}{V_r} \]

\[ f^* = \frac{f}{U} \]

\[ V_r = \frac{U}{f d} \]

Williamson and Roshko (1988)
End Force Cross-Correlation

Hover, Teчет, Trianafyllou (JFM 1998)

$F_c = \text{Force Correlation Coefficient}, \ \mu = \text{correlation standard deviation}$
VIV in the Ocean

- Non-uniform currents effect the spanwise vortex shedding on a cable or riser.
- The frequency of shedding can be different along length.
- This leads to “cells” of vortex shedding with some length, $l_c$. 
Three Dimensional Effects

Shear layer instabilities as well as longitudinal (braid) vortices lead to transition from laminar to turbulent flow in cylinder wakes.

Longitudinal vortices appear at $R_d = 230$. 
Longitudinal Vortices

The presence of longitudinal vortices leads to rapid breakdown of the wake behind a cylinder.

C.H.K. Williamson (1992)
Oscillating Tapered Cylinder

Strouhal Number for the tapered cylinder:

\[ S_t = \frac{f d}{U} \]

where $d$ is the average cylinder diameter.
Spanwise Vortex Shedding from 40:1 Tapered Cylinder

$R_d = 400; \quad St = 0.198; \quad A/d = 0.5$

$R_d = 1500; \quad St = 0.198; \quad A/d = 0.5$

$R_d = 1500; \quad St = 0.198; \quad A/d = 1.0$

No Split: ‘2P’

$d_{max}$

$A/d$

$d_{min}$

Tchét et al. (JFM 1998)
Flow Visualization Reveals: A Hybrid Shedding Mode

- ‘2P’ pattern results at the smaller end
- ‘2S’ pattern at the larger end
- This mode is seen to be repeatable over multiple cycles

Digital particle image velocimetry (DPIV) in the horizontal plane leads to a clear picture of two distinct shedding modes along the cylinder.

DPIV of Tapered Cylinder Wake

\[ \text{Rd} = 1500; \ St = 0.198; \ A/d = 0.5 \]
Evolution of the Hybrid Shedding Mode

‘2P’

\[ \frac{z}{d} = 7.9 \]

‘2S’

\[ \frac{z}{d} = 22.9 \]

\[ \text{Rd} = 1500; \ St = 0.198; \ A/d = 0.5 \]
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Rd = 1500; St = 0.198; A/d = 0.5
Objectives:
• Confirm numerically the existence of a stable, periodic hybrid shedding mode 2S~2P in the wake of a straight, rigid, oscillating cylinder

Principal Investigator:
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Approach:
• DNS - Similar conditions as the MIT experiment (Triantafyllou et al.)
• Harmonically forced oscillating straight rigid cylinder in linear shear inflow
• Average Reynolds number is 400

Methodology:
• Parallel simulations using spectral/hp methods implemented in the incompressible Navier- Stokes solver NEKTAR

Results:
• Existence and periodicity of hybrid mode confirmed by near wake visualizations and spectral analysis of flow velocity in the cylinder wake and of hydrodynamic forces
VIV Suppression

- Helical strake
- Shroud
- Axial slats
- Streamlined fairing
- Splitter plate
- Ribboned cable
- Pivoted guiding vane
- Spoiler plates

Fig. 3-23 Add-on devices for suppression of vortex-induced vibration of cylinders: (a) helical strake; (b) shroud; (c) axial slats; (d) streamlined fairing; (e) splitter; (f) ribboned cable; (g) pivoted guiding vane; (h) spoiler plates.
VIV Suppression by Helical Strakes

Helical strakes are a common VIV suppression device.
Flexible Cylinders

Mooring lines and towing cables act in similar fashion to rigid cylinders except that their motion is not spanwise uniform.

\[ m \frac{d^2y}{dt^2} = T(s) \frac{d^2y}{dt^2} + L(s,t) \]

Tension in the cable must be considered when determining equations of motion.
Long flexible cylinders can move in two directions and tend to trace a figure-8 motion. The motion is dictated by the tension in the cable and the speed of towing.
2-DOF Cylinder Motion

CYLINDER ORBITAL PLOTS

Nominal Reduced Velocity, $V_{rn}$

Rigid cylinder free to move in both transverse ($y$) and longitudinal ($x$) directions. High length:diameter ratio.
FFT Magnitude Contours For Frequency Ratio $\frac{f_x}{f_y} = 1.0$

Transverse Position

In-Line Position
FFT Magnitude Contours For Frequency Ratio $fx/fy = 2.0$

Transverse Position

In-Line Position
Power Balance Along a Riser
Overview & Aim

- Obtain power balance along a riser
  - We need power input from the fluid to the riser as a function of $z$.
  - Input: Cross-flow displacement: $y(z,t)$
    Lift force: $L(z,t)$
    Normalized Lift force: $f(z,t) = \frac{L(z,t)}{\frac{1}{2} \rho ds U^2}$
    Incoming Velocity Profile: $U(z)$
  - Output: $C_{lv_m}(z, \omega_m)$ & $C_{lv}(z)$
  - Power balance is correlated with $Clv(z)$.
Fourier Decomposition

• Obtain the Fourier decomposition of force and displacement as:

\[
y(z, t) = \text{Re} \left[ \sum_{n=1}^{N} \hat{y}_n e^{i\omega_n t} \right]
\]

\[
f(z, t) = \text{Re} \left[ \sum_{n=1}^{N} \hat{f}_n e^{i\omega_n t} \right]
\]

\[\hat{y}_n = \hat{y}_n(z, \omega_n) \quad \text{&} \quad \hat{f}_n = \hat{f}_n(z, \omega_n)\]

• Fourier Coefficients
Choose a Set of Important Frequencies

- Choose the set of frequencies $\omega_m$ which represents
  
  $\hat{y}_m(z, \omega_m)$ & $\hat{f}_m(z, \omega_m)$
  
  – peaks in the span-averaged Fourier coefficients:

![Graph showing mean magnitude of Y & F_y with corresponding frequency & Picked frequencies.](image)
Obtaining Global $C_{lv}$

- Obtain the multi-frequency lift coefficients in phase with velocity and acceleration as:
  
  Lift coefficient in phase with velocity for $\omega_m$:
  $$C_{lv_m}(z, \omega_m) = \left| \hat{f}_m(z, \omega_m) \right| \sin \left( \arg \left( \hat{f}_m \right) - \arg \left( \hat{y}_m \right) \right)$$

  Lift coefficient in phase with acceleration for $\omega_m$:
  $$C_{la_m}(z, \omega_m) = -\left| \hat{f}_m(z, \omega_m) \right| \cos \left( \arg \left( \hat{f}_m \right) - \arg \left( \hat{y}_m \right) \right)$$

- Global lift coefficient in phase with velocity:
  $$C_{lv}(z) = -\frac{\sum_m C_{lv_m} |\omega_m \hat{y}_m|}{\sum_m |\omega_m \hat{y}_m|^2}$$

- $C_{lv}$ is very important as it expresses energy balance.

- $C_{lv}^* U(z)^2$ is representative of the power input from the fluid to the riser.
Input Sheared Velocity Profile
Selected Frequencies

- Chosen set of frequencies $\omega_m$:

$$\omega_m = 2\pi (0.1733 \ 0.1831 \ 0.1929 \ 0.2026 \ 0.2124 \ 0.2222 \ 0.2295 \ 0.2393)$$
• $C_{lv}^* U(z)^2$ is representative of the power input from the fluid to the riser.

• One can clearly observe that there is a positive input of power from the fluid where the incoming velocity is high.

• The power is transferred to the low incoming velocity region, as traveling waves in the riser.
Oscillating Cylinders

Parameters:

\[ \text{Re} = \frac{V_m d}{\nu} \]
\[ b = \frac{d^2}{\nu T} \]
\[ \text{KC} = \frac{V_m T}{d} \]
\[ \text{St} = \frac{f_v d}{V_m} \]

\[ y(t) = a \cos(\omega t) \]
\[ \dot{y}(t) = -a \omega \sin(\omega t) \]

\[ V_m = a \omega \]

\[ \nu = \mu/\rho ; \ T = \frac{2\pi}{\omega} \]
Reynolds # vs. KC #

\[ \text{Re} = \frac{V_m d}{\nu} = \frac{\omega ad}{\nu} = 2\pi \left(\frac{a}{d}\right)\left(\frac{d^2}{\nu T}\right) \]

\[ \text{KC} = \frac{V_m T}{d} = 2\pi \frac{a}{d} \]

\[ \text{Re} = \text{KC} \times b \]

\[ b = \frac{d^2}{\nu T} \]

Also effected by roughness and ambient turbulence
Forced Oscillation in a Current

\[ y(t) = a \cos(\omega t) \]

\[ \omega = 2 \pi f = \frac{2\pi}{T} \]

Parameters: \( a/d, \rho, \nu, \theta \)

Reduced velocity: \( U_r = U/\text{f}d \)

Max. Velocity: \( V_m = U + a\omega \cos \theta \)

Reynolds #: \( \text{Re} = V_m d / \nu \)

Roughness and ambient turbulence
Wall Proximity

At \( e/d > 1 \) the wall effects are reduced.

*\( C_d, C_m \) increase as \( e/d < 0.5 \)

Vortex shedding is significantly affected by the wall presence.

In the absence of viscosity these effects are effectively non-existent.
Galloping

Galloping is a result of a wake instability.

Resultant velocity is a combination of the heave velocity and horizontal inflow.

If $\omega_n << 2\pi f_v$ then the wake is quasi-static.
Lift Force, $Y(\alpha)$

$$C_y = \frac{Y(t)}{\frac{1}{2} \rho U^2 A_p}$$
Galloping motion

\[ m\ddot{z} + b\dot{z} + kz = L(t) \]

\[ L(t) = \frac{1}{2} \rho U^2 a C_{lv} - m_a \ddot{y}(t) \]

\[ C_l(\alpha) = C_l(0) + \frac{\partial C_l(0)}{\partial \alpha} + \ldots \]

Assuming small angles, \( \alpha \):

\[ \alpha \sim \tan \alpha = -\frac{\dot{z}}{U} \quad \quad \beta = \frac{\partial C_l(0)}{\partial \alpha} \quad \quad V \sim U \]
Instability Criterion

\[(m + m_a) \ddot{z} + (b + \frac{1}{2} \rho U^2 a \frac{\beta}{U}) \dot{z} + kz \approx 0\]

If \[b + \frac{1}{2} \rho U^2 a \frac{\beta}{U} < 0\]

Then the motion is unstable!
This is the criterion for galloping.
\( \beta \) is shape dependent

\[ \frac{\partial C_1(0)}{\partial \alpha} \]

\[ \begin{array}{c}
\text{Shape} \\
1 \\
2 \\
4 \\
\end{array} \]

\[ \begin{array}{c}
\text{\( \partial C_1(0) \)} \\
-2.7 \\
0 \\
-3.0 \\
-10 \\
-0.66 \\
\end{array} \]
Instability:

\[ \beta = \frac{\partial C_1}{\partial \alpha} (0) < \frac{-b}{\frac{1}{2} \rho U a} \]

Critical speed for galloping:

\[ U > \frac{b}{\frac{1}{2} \rho a \left( -\frac{\partial C_1}{\partial \alpha} (0) \right)} \]
Both torsional and lateral galloping are possible. FLUTTER occurs when the frequency of the torsional and lateral vibrations are very close.
Galloping vs. VIV

- Galloping is low frequency
- Galloping is NOT self-limiting
- Once $U > U_{\text{critical}}$ then the instability occurs irregardless of frequencies.
References