13.42 Design Principles for Ocean Vehicles

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1. Gaussian Distribution

Distributions of random variables are often gaussian in shape, or can be approximated as such. The gaussian density function is described by the probability density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} \tag{1}$$

which is symmetric about $\bar{x}$. Given this pdf the cumulative probability of x is

$$F(x) = \frac{1 + erf\left(\frac{x-\bar{x}}{\sigma}\right)}{2} \tag{2}$$

where erf is the error function:

$$erf(\zeta) = \frac{1}{2\pi} \int_0^{\zeta} e^{-y^2/2} dy \tag{3}$$

For an approximately normal function (with Gaussian distribution) then

68% of events fall within 1σ

95% of events fall within 2σ

97.7% of events fall within 3σ

2. Poisson distribution

Discrete events occur randomly in time with the following probability as $\delta t \to 0$:

$$\left\{ \begin{array}{cc} \lambda\delta t & \text{to have 1 occurence in time interval } \delta t \\ 1-\lambda\delta t & \text{to have 0 occurrences in time } \delta t \\ 0 & \text{to have more than one occurence in time } \delta t \end{array} \right\} \tag{4}$$

Thus the probability to have k occurrences within a finite time t can be shown to be

$$P(k \text{ in } t) = e^{-\lambda t} \frac{(\lambda t)^k}{k!} \tag{5}$$

This can be useful when designing platforms that require less than k occurrences of an event in a certain time t (e.g. less than ten times water on deck in one day).