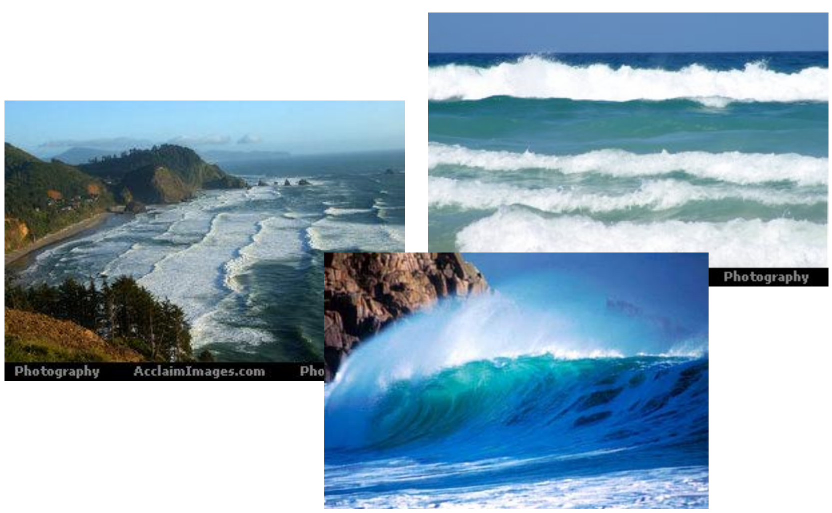
13.42 Lecture:
Ocean Waves
Spring 2005

Alexandra H. Techet

MIT Ocean Engineering

Ocean Waves



OCEAN WAVE GENERATION

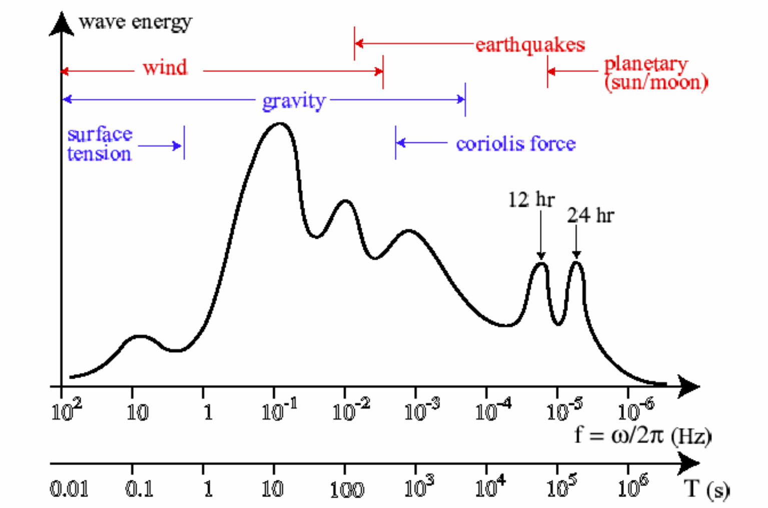


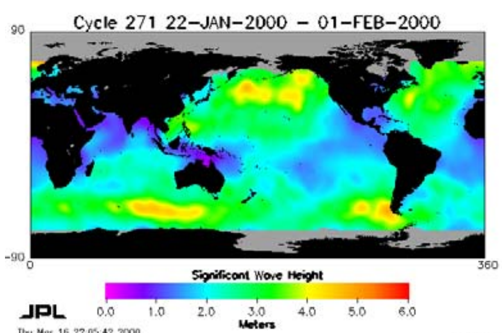
FIGURE 1. Wave energy spectra. Red text indicates wave generation mechanisms and blue text indicates damping/restoring forces.

Wave and Sea State

- Idea of sea state is vague since it does not indicate wave period.
- However it is widely used so we deal...

World Meteorological Org. Sea State Codes

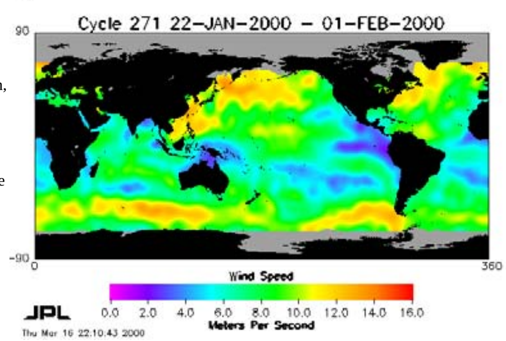
Sea State Code	Significant Wave Height: Range	Significant Wave Height: Mean	Description
0	0 (meters)	0 (meters)	Calm (glassy)
1	0-0.1	0.05	Calm (rippled)
2	0.1-0.5	0.3	Smooth (mini-waves)
3	0.5-1.25	0.875	Slight
4	1.25-2.5	1.875	Moderate
5	2.5-4.0	3.25	Rough
6	4.0-6.0	5.0	Very Rough
7	6.0-9.0	7.5	High
8	9.0-14.0	11.5	Very High
9	> 14.0	> 14.0	Huge

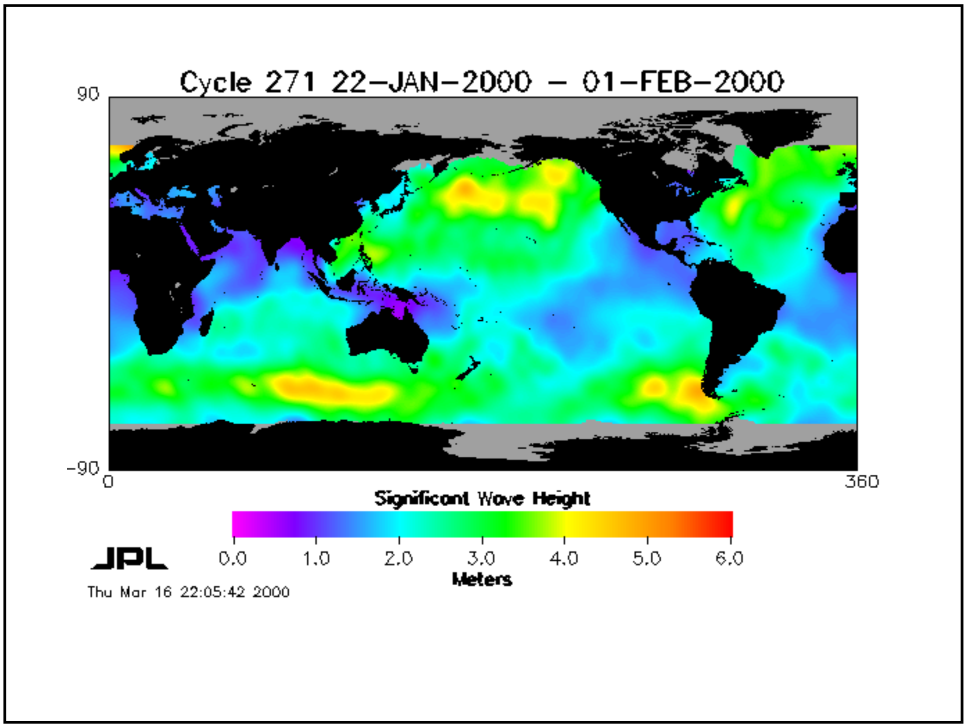


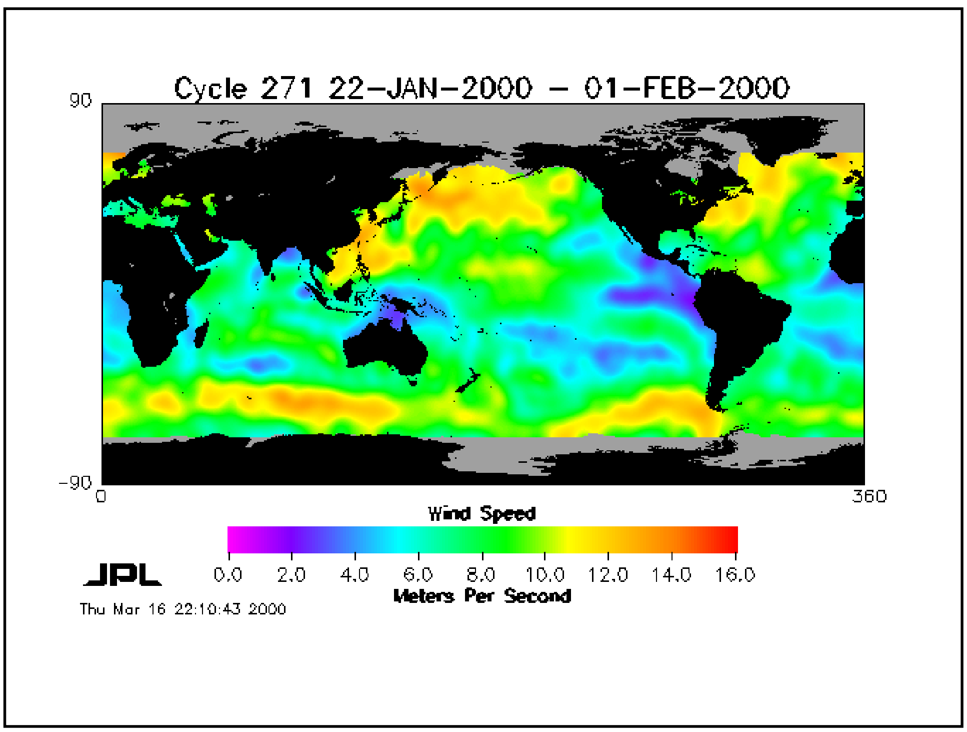
The highest waves generally occur in the Southern Ocean, where waves over six meters in height (shown as red in images) are found. The strongest winds are also generally found in this region. The lowest waves (shown as purple in images) are found primarily in the tropical and subtropical oceans where the wind speed is also the lowest.

In general, there is a high degree of correlation between wind speed and wave height.

The highest winds generally occur in the Southern Ocean, where winds over 15 meters per second (represented by red in images) are found. The strongest waves are also generally found in this region. The lowest winds (indicated by the purple in the images) are found primarily in the tropical and subtropical oceans where the wave height is also the lowest.




Cycle 271 22-JAN-2000 – 01-FEB-2000
90
-90
0
360
Significant Wave Height
0.0
1.0
2.0
3.0
4.0
5.0
6.0
Meters
JPL
Thu Mar 16 22:05:42 2000


Cycle 271 22-JAN-2000 – 01-FEB-2000
90
-90
0
360
Wind Speed
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
Meters Per Second
JPL
Thu Mar 16 22:10:43 2000

Wind Generated Waves

- Wind blows over long distance and long period time before sea state is fully developed.
- When wind speed matches wave crest phase speed the phase speed is maximized. Thus the limiting frequency is dependent on the wind speed due to the dispersion relationship.

$$U_{wind} \approx C_p = \omega / k = g / \omega$$

Limiting frequency: $\omega_c \approx g / U_{wind}$

Wave development and decay

- Fetch is the distance wind must blow to achieve fully developed seas (usually given in standard miles).
- For a storm with wind speed U_w the effects of the storm can be felt a distance away, *R*.
- The number of wave cycles between the storm and the observation location is $N = R/\lambda$.
- The amplitude of the waves decay exponentially as

$$a(t) = e^{-\gamma t}$$

where $\gamma = 2\nu k^2 = 2\nu\omega^4 / g^2$

(From Landau and Lifshitz)

Typical Spectrum

$$S^{+}(\omega) = \frac{A}{\omega^5}\, e^{-B/\omega^4}$$

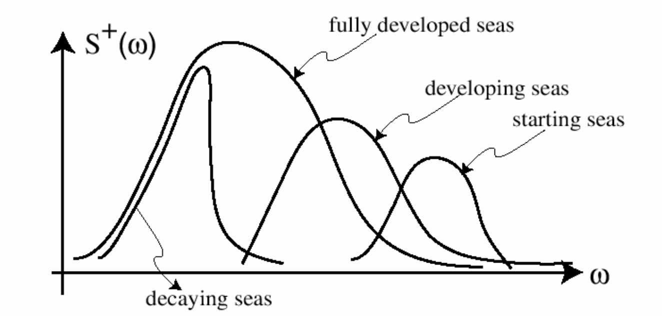
Based on measured spectra and theoretical results, several standard forms have been developed.

Limitations on Empirical Spectra

- Fetch limitations
- State of development or decay
- Seafloor topography
- Local Currents
- Effect of distant storms (swells)

Wave Spectra

- Many spectra are *strictly* valid for FULLY DEVELOPED SEAS.
- Developing seas have a broader spectral peak. Decaying seas have a narrower peak.



Pierson-Moskowitz Spectrum

Developed by offshore industry for fully developed seas in the North Atlantic generated by local winds. One parameter spectrum.

Mathematical form of $S^{+}(\omega)$ in terms of the significant wave height, $H^{1/3}$. ($H^{1/3}=\zeta$)

$$S^{+}(\omega) = \frac{8.1}{10^3}\,\frac{g^2}{\omega^5}\,e^{-0.032\,(g/\zeta\omega^2)^2}$$

Spectrum Assumptions

- Deep water
- North Atlantic data
- Unlimited fetch
- Uni-directional seas
- No swell

Bretschneider Spectrum

Replaced P-M spectrum since need for fully developed seas is too restrictive. Two parameter spectrum.

$$S^+(\omega) = \frac{1.25}{4} \frac{\omega_m^4}{\omega^5} \zeta^2 e^{-1.25\,(\omega_m/\omega)^4}$$

Significant wave height

$$\zeta \equiv H^{1/3} = 4\sqrt{M_o}$$

Modal frequency

$$\omega_m = 0.4\sqrt{g/\zeta}$$

$$\int_{-\infty}^{\infty} S^+(\omega)\, d\omega = M_o = \left(\frac{\zeta}{4}\right)^2$$

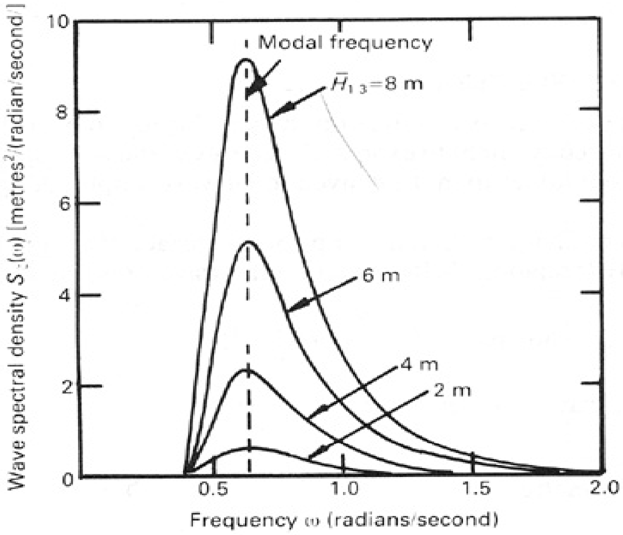


Fig. 4.9 — Bretschneider wave energy spectra; modal period T_0 = 10 seconds.

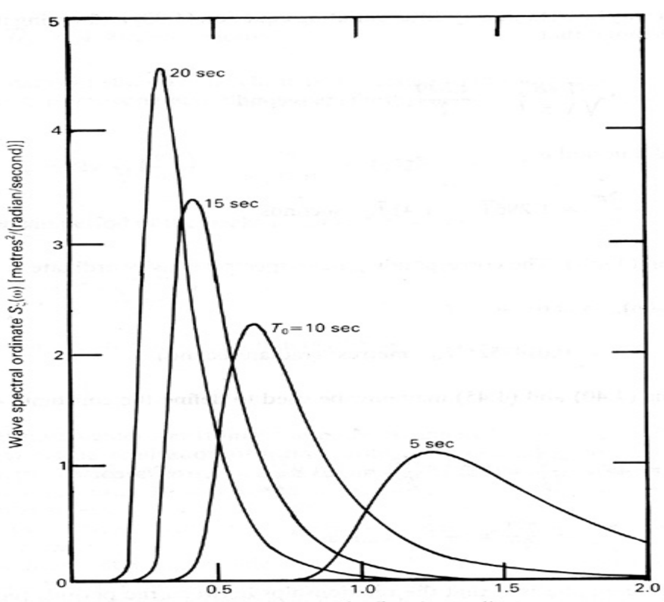


Fig. 4.8 — Bretschneider wave energy spectra; characteristic wave height 4 metres.

JONSWAP Spectrum

JONSWAP spectrum was developed for the *limited fetch* North Sea by the offshore industry and is used extensively.

$$S^{+}(\omega) = \frac{ag^2}{\omega^5}\, e^{-\frac{5}{4}\left(\frac{\omega_m}{\omega}\right)^4} \gamma^{\delta}$$

$$a = 0.076\, \overline{x}^{\,(-0.22)}$$

$$\overline{x} = \frac{gx}{U^2}$$

$$\delta = -\frac{(\omega - \omega_m)^2}{2\sigma^2\omega_m^2}$$

$$\sigma = \begin{cases} 0.07; & \omega \le \omega_m \\ 0.09; & \omega > \omega_m \end{cases}$$

The JONSWAP spectrum is thus a distortion of the Bretschneider spectrum specified in terms of the characteristic wave height and the modal period. Fig. 4.10

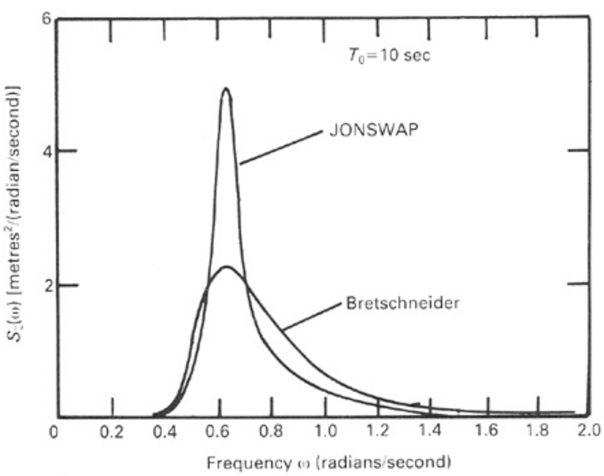


Fig. 4.10 — JONSWAP and Bretschneider spectra; significant wave height 4 metres.

Amplitude spectrum

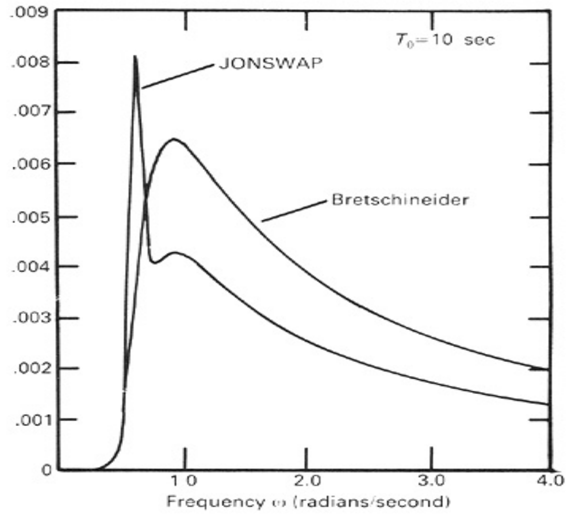


Fig. 4.11 — Wave slope spectra; significant wave height 4 metres.

For wave slope spectra these two do not match as well

Ochi Spectrum

Ochi spectrum is an extension of the BS spectrum, allowing to make it wider, λ small, for developing seas, or narrower, λ larger, for swell. Three parameter spectrum.

$$S^{+}(\omega) = \frac{1}{4} \frac{\left(\frac{4\lambda+1}{4}\,\omega_m^4\right)^{\lambda}}{\Gamma(\lambda)} \frac{\zeta^2}{\omega^{4\lambda+1}} exp\left\{-\left(\frac{4\lambda+1}{4}\right)\left(\frac{\omega_m}{\omega}\right)^4\right\}$$

λ determines the width of the spectrum

Γ(x) = the gamma function of x

Storm and Swell

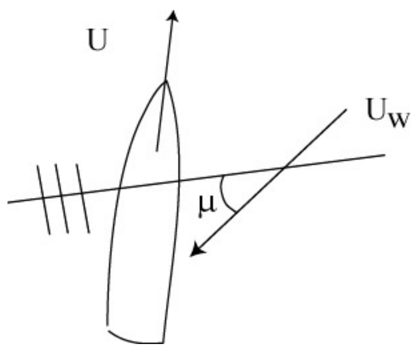
- Two spectra can be superimposed to represent a local storm and a swell.

$$S^+(\omega) = \underbrace{S_1^+(\omega)}_{\text{storm}} + \underbrace{S_2^+(\omega)}_{\text{swell}}$$

Directionality in waves

- In reality, waves are three-dimensional in nature and different components travel in different directions.
- Measurements of waves are difficult and thus spectra are made for “uni-directional” waves and corrected for three-dimensionality.

Correction to uni-directionality



$$S^+(\omega,\mu) = S^+_{BS}(\omega)\ M(\mu)$$

$$\int_{-\pi/2}^{\pi/2} M(\mu)\ d\mu = 1$$

$$M(\mu) = \frac{2}{\pi}\cos^2\mu$$

$$-\frac{\pi}{2} < \mu < \frac{\pi}{2}$$

M(μ) spreads the energy over a certain angle contained within (- π/2, π/2) from the wind direction

Short Term Statistics

- Short term statistics are valid only over a period of time up to a few days, while a storm retains its basic features
- During this period the sea is described as a stationary and ergodic random process with a spectrum $S^+(\omega)$ parameterized by (ω_m, ζ).
- Wave spreading and swell are two additional parameters of importance. Fetch also plays an important role.

Long Term Statistics

- Over the long term the sea is not stationary.
- We can represent long term stats as the sum of several short term statistics by piecing together a group of storms with different durations and significant wave heights.

Storm Statistics

- For each storm (i) we use the significant wave height and average period to construct a spectrum and then find the short term statistics.
- For structural analysis the failure level is a large quantity compared to the rms value, so we use the rate of exceeding some level a_o.

Observed Wave Heights

Sea conditions reported by sailors estimating the average wave height and period. It was found that this is VERY close to the significant wave height.

Hogben and Lumb (1967)

$H^{1/3} = 1.06\ H_v$ (meters)

$\overline{T} = 1.12\ T_v$ (seconds)

$T_z = 0.73\ T_v$ (seconds)

Nordenstrom (1969)

$H^{1/3} = 1.68\ (H_v)^{0.75}$ (meters)

$\overline{T} = 2.83\ (T_v)^{0.44}$ (seconds)

Use these...