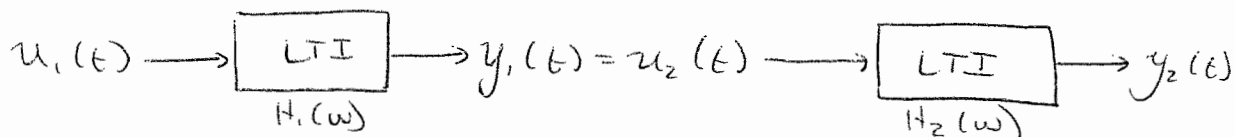


PROBLEM 1:



Given $u_1(t) = u_0 \cos(\omega_0 t)$ and $H_1(\omega)$ and $H_2(\omega)$.

Can solve this several ways: convolution integrals in time domain, or multiplication in frequency domain.

The advantage of going to the frequency domain is the multiplication of input by each filter to get output.

$$\text{i.e. } y_2(\omega) = u_1(\omega) H_1(\omega) H_2(\omega)$$

Finite exponentials have the unique feature that they can be multiplied by transfer functions directly so rewrite $u_1(t)$ as $u_1(t) = \text{Re} \{ u_0 e^{i\omega_0 t} \}$

$$y_2(t) = \text{Re} \{ u_0 e^{i\omega_0 t} H_1(\omega) H_2(\omega) \}$$

where $H_1(\omega)$ and $H_2(\omega)$ have magnitudes $|H_1(\omega)|$ and $|H_2(\omega)|$ and phase $\angle H_1(\omega)$ and $\angle H_2(\omega)$

$$y_2(t) = u_0 |H_1(\omega)| |H_2(\omega)| \cos(\omega_0 t + \angle H_1(\omega) + \angle H_2(\omega))$$

You could also have done convolution integrals or FT's to frequency domain and then taken IFT $[y_2(\omega) = u_1(\omega) H_1(\omega) H_2(\omega)]$.

Problem 2:

a) NOT LTI

b) YES LTI

Problem 3:

a) $f(t) = u_0(t - \tau)$

$$FT(f(t)) = F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} u_0(t - \tau) e^{-i\omega t} dt$$

$$\left[\text{USE: } \int_{-\infty}^{\infty} f(t) u_0(t - a) dt = f(a) \text{ where } f(t) = e^{-i\omega t} \right]$$

$$F(\omega) = e^{-i\omega \tau}$$

b) $\hat{f}(\alpha) = FT[f(x)] = \int_{-\infty}^{\infty} f(x) e^{-i\alpha x} dx$

$$FT\left[\frac{df}{dx}\right] = \int_{-\infty}^{\infty} \frac{df}{dx} e^{-i\alpha x} dx$$

$$\text{let } u = e^{-i\alpha x} \quad du = -i\alpha e^{-i\alpha x} dx$$

$$dv = \frac{df}{dx} dx \quad v = f$$

DO INTEGRATION BY PARTS:

$$= \left[e^{-i\alpha x} \cdot f \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f \cdot (-i\alpha e^{-i\alpha x} dx)$$

$$= 0 + i\alpha \int_{-\infty}^{\infty} f e^{-i\alpha x} dx = \boxed{i\alpha \hat{f}(\alpha)}$$

c) $\int_{-\infty}^{\infty} \frac{d^2 f}{dx^2} e^{-i\alpha x} dx$

$$\text{let } u = e^{-i\alpha x} \quad du = -i\alpha e^{-i\alpha x} dx$$

$$dv = \frac{d^2 f}{dx^2} dx \quad v = \frac{df}{dx}$$

AGAIN, DO INTEGRATION BY PARTS:

$$= \left[\frac{df}{dx} \cdot e^{-i\alpha x} \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{df}{dx} \cdot (-i\alpha e^{-i\alpha x} dx)$$

$$= 0 + (i\alpha) \int_{-\infty}^{\infty} \frac{df}{dx} e^{-i\alpha x} dx = \boxed{(i\alpha)^2 \hat{f}(\alpha)} = \boxed{-\alpha^2 \hat{f}(\alpha)}$$

from part b)

this equals $i\alpha \hat{f}(\alpha)$

Problem 4:

- a) LTI system with transfer function $H(\omega)$
and sinusoidal input:

$$u(t) = u_0 e^{i(\omega_0 t + \psi_0)}$$

if the output is $y(t)$ then:

$$y(t) = y_0 e^{i(\omega_0 t + \psi)} \quad \text{where } y_0 = |H(\omega)|_{\omega=\omega_0} \cdot u_0$$

$$\text{and } \psi = \psi_0 + \angle H(\omega)|_{\omega=\omega_0}$$

$$\therefore \boxed{y(t) = u_0 |H(\omega_0)| e^{i(\omega_0 t + \psi_0 + \angle H(\omega_0))}}$$

- b) Yes, you can find the transfer function by

- sending a series of monochromatic sinusoidal waves down the tank
- measuring the platform response at each frequency
- compute the magnitude and phase of the transfer function at each frequency:

if input is $u(t) = u_0 \cos(\omega_0 t + \psi_0)$

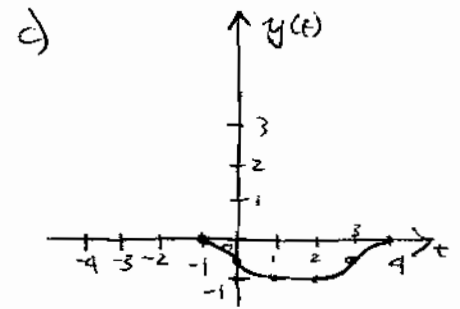
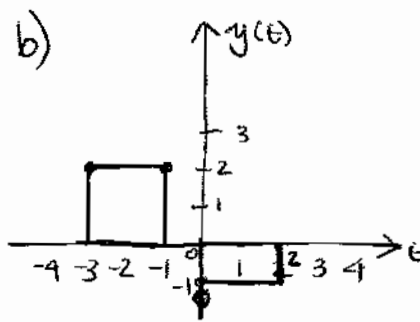
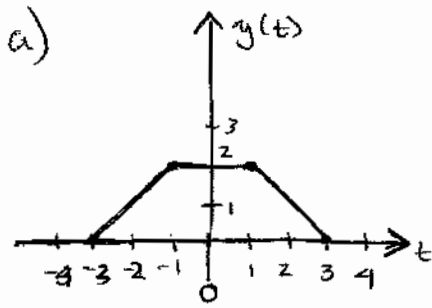
and output is $y(t) = y_0 \cos(\omega_0 t + \psi_1)$,

u_0, ω_0, ψ_0 are known inputs and y_0, ψ_1 are measured,

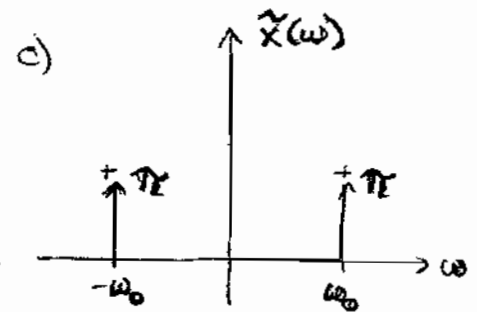
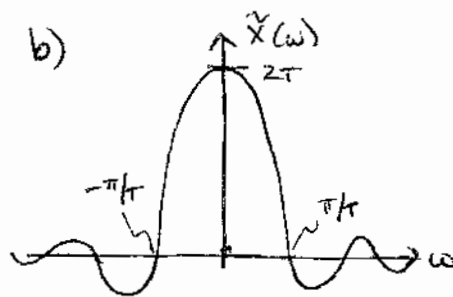
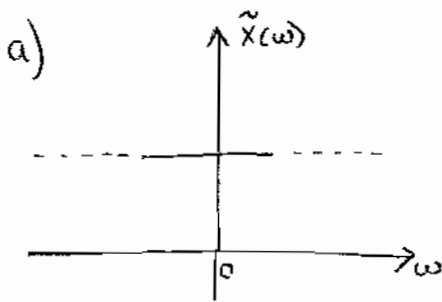
then $|H(\omega)|_{\omega=\omega_0} = y_0/u_0$

and $\angle H(\omega)|_{\omega=\omega_0} = \psi_1 - \psi_0$

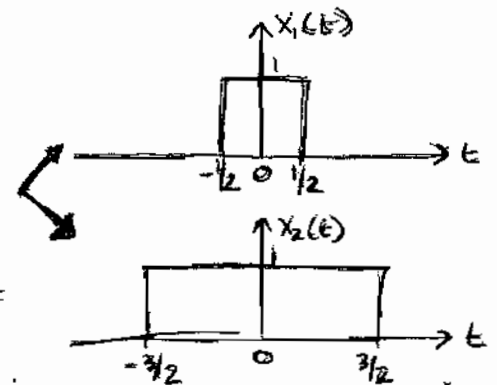
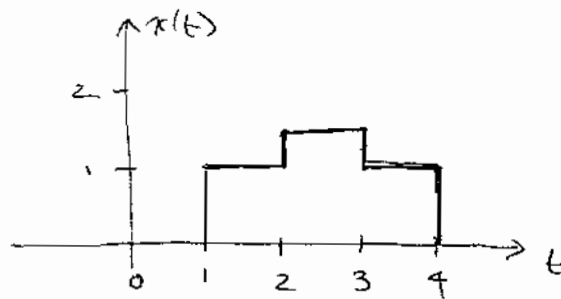
Problem 5: Use graphical "Flip & slide" method.



Problem 6:



Problem 7:



Decompose $x(t)$ into: $x(t) = \frac{1}{2} x_1(t - 2.5) + x_2(t - 2.5)$

USE F.T. OF A RECTANGULAR PULSE SIGNAL $x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$ $X(\omega) = \int_{-T_1}^{T_1} e^{-i\omega t} dt = 2 \frac{\sin \omega T_1}{\omega}$

$$\therefore X_1(\omega) = \frac{2 \sin(\omega/2)}{\omega} \quad \text{and} \quad X_2(\omega) = \frac{2 \sin(3\omega/2)}{\omega}$$

USING THE LINEARITY AND TIME SHIFTING PROPERTIES OF THE F.T.

$$X(\omega) = \left\{ \frac{\sin(\omega/2)}{\omega} + \frac{2 \sin(3\omega/2)}{\omega} \right\} e^{-i \frac{5\omega}{2}}$$

13.42 HW2 SOLUTIONS

SPRING 2005

PROBLEM 8: ASSUME DEEP WATER

$$\eta(x, t) = a \cos(kx - \omega t)$$

$$\phi(x, z, t) = \frac{ga}{\omega} e^{kz} \sin(kx - \omega t)$$

$$\left[\text{USE } \eta(x, t) = -\frac{1}{g} \frac{\partial \phi}{\partial t} \Big|_{z=0} \right]$$

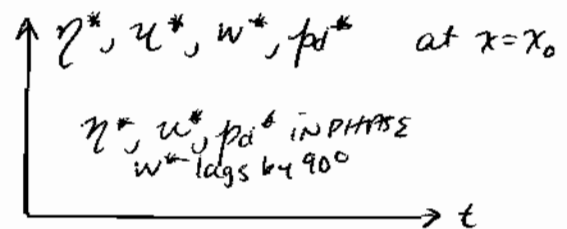
$$u(x, z, t) = \frac{\partial \phi}{\partial x} = \omega a e^{kz} \cos(kx - \omega t)$$

$$w(x, z, t) = \frac{\partial \phi}{\partial z} = \omega a e^{kz} \sin(kx - \omega t)$$

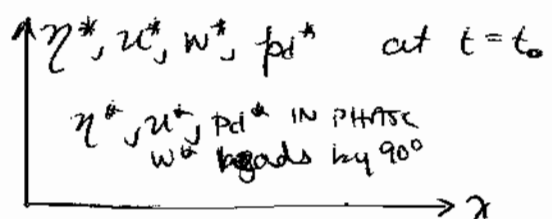
$$p_d = -\rho \frac{\partial \phi}{\partial t} = \rho g a e^{kz} \cos(kx - \omega t)$$

a) See attached plots of the form

* " indicates NONDIMENSIONALIZED VALUE



b) See attached plots of the form



$$c) P_{\text{TOTAL}} = P_d + P_{\text{static}} = -\rho \frac{\partial \phi}{\partial t} - \rho g z = \rho g [a e^{kz} \cos(kx - \omega t) - z] \Big|_{z=z_n}$$

where $z_n = -16n$ meters

$$\text{WAVE CREST: } \cos(kx - \omega t) = 1 \quad \therefore P_{\text{TOT}} = \rho g [a e^{kz_n} - z_n] \text{ @ CREST}$$

$$\text{WAVE TROUGH: } \cos(kx - \omega t) = -1 \quad \therefore P_{\text{TOT}} = \rho g [-a e^{kz_n} - z_n] \text{ @ TROUGH}$$

$$\text{WAVE NODE: } \cos(kx - \omega t) = 0 \quad \therefore P_{\text{TOT}} = -\rho g z_n \text{ @ NODE}$$

PROBLEM 8 CONT.:

d) FLUID ACCELERATION $\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v}$

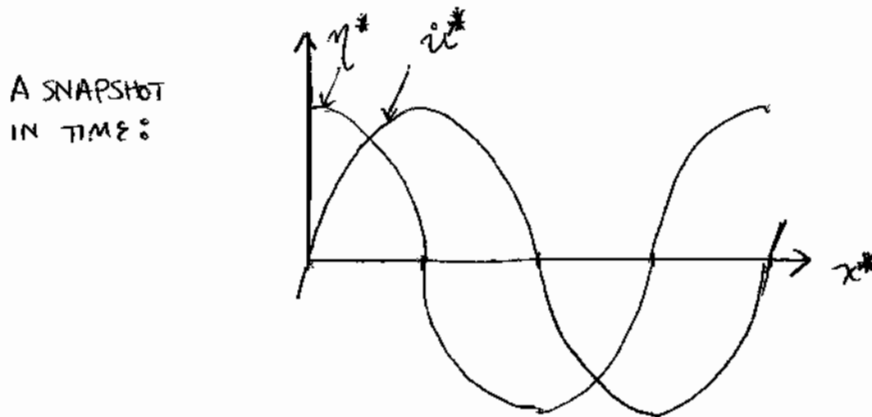
BY LINEAR APPROXIMATION $\frac{D\vec{v}}{Dt} \approx \frac{\partial \vec{v}}{\partial t}$

F.A.M. \propto FLUID ACCELERATION

\therefore HORIZONTAL ADDED MASS FORCE IS $\begin{cases} \text{MAXIMUM} \\ \text{MINIMUM} \\ \text{ZERO} \end{cases}$ WHEN FLUID HORIZONTAL ACCELERATION IS.

HORIZONTAL FLUID ACCELERATION $= \frac{\partial u}{\partial t} = +\omega^2 a e^{kz} \sin(kx - \omega t)$

THIS ACHIEVES A $\begin{cases} \text{MAX} \\ \text{MIN} \\ \text{ZERO} \end{cases}$ WHEN $\sin(kx - \omega t)$ IS $\begin{cases} 1 \\ -1 \\ 0 \end{cases}$ AND $\cos(kx - \omega t)$ IS $\begin{cases} 0 \\ 0 \\ \pm 1 \end{cases}$



u and η are 90° out of phase

THE HORIZONTAL ADDED MASS FORCE IS ZERO AT BOTH CREST AND TROUGH OF THE WAVE ELEVATION.

THE HORIZONTAL ADDED MASS FORCE ALTERNATES BETWEEN MAXIMUM AND MINIMUM AT CONSECUTIVE NODES.

e) HORIZONTAL VISCIOUS FORCE $\propto |u|$.

u IS IN PHASE WITH η SO $|\text{HORIZ. VISCIOUS FORCE}|$ IS MAXIMUM UNDER CRESTS & TROUGHS.

VERTICAL VISCIOUS FORCES $\propto |w|$, AND w IS 90° OUT OF PHASE WITH η SO $|\text{VERT. VISCIOUS FORCE}|$ IS MAXIMUM UNDER NODES.

