1) a,b) See pp 1–3 of 13.42 OCEAN WAVE SPECTRA RDG.

c) SIG. WAVE HT IS VERY CLOSE TO THE WAVEHEIGHT WHICH EXPERIENCED MARINERS WOULD ESTIMATE FROM THEIR OBSERVATIONS

d) B-S USED FREQUENTLY SINCE IT REQUIRES ONLY SIG. WAVE HT AND MODAL FREQUENCY AS INPUTS, AND, UNLIKE P-M, IT CAN BE USED IN SITUATIONS OTHER THAN FULLY DEVELOPED SEAS.

2) B-S: $S_\eta^+(\omega) = \frac{1.25}{4}\frac{\omega_m^4}{\omega^5}\zeta^2 e^{\left\{-1.25\left(\frac{\omega_m}{\omega}\right)^4\right\}}$

$\zeta = 5.75\ m \qquad \omega_m = 0.68\ \text{rad}/s$

$\eta(t) \longrightarrow \boxed{|H(\omega)|} \longrightarrow \eta_p(t)$

$S_{\eta_p}(\omega) = S_\eta(\omega)\,|H(\omega)|^2 \qquad |H(\omega)| = \frac{(0.4-\omega^3)^2+0.25}{(0.4-\omega^3)^2+0.15}$

$$S_{\eta_p}(\omega) = \frac{1.25}{4}\frac{(0.68)^4}{\omega^5}(5.75)^2 e^{\left\{-1.25\left(\frac{0.68}{\omega}\right)^4\right\}}\left[\frac{(0.4-\omega^3)^2+0.25}{(0.4-\omega^3)^2+0.15}\right]^2$$

Ave freq of upcrossings $\bar{\eta}(h) = \frac{1}{2\pi}\sqrt{\frac{M_2}{M_0}}\,e^{-h^2/2M_0}$

and $M_0 = \int_0^\infty S_{\eta_p}(\omega)\,d\omega \approx 4.326$

$M_2 = \int_0^\infty \omega^2 \cdot S_{\eta_p}(\omega)\,d\omega \approx 3.139$

} I solved using MATHCAD.

$\therefore\ \bar{\eta}(h) = \frac{1}{2\pi}\sqrt{\frac{3.139}{4.326}}\,e^{-h^2/2\cdot 4.326}$ SET THIS $\leq \frac{2\ \text{times}}{1\text{hr}} * \frac{1\text{hr}}{3600\text{sec}} = \frac{1\ \text{time}}{1800\text{sec}}$

$$e^{-h^2/8.652} \leq \left(\frac{1}{1800}\right)(2\pi)\sqrt{\frac{4.326}{3.139}}$$

$$-h^2 \leq 8.652\left[\ln\left(\frac{2\pi}{1800}\sqrt{\frac{4.326}{3.139}}\right)\right]$$

$$\boxed{h \geq 6.897\ m}$$

Homework 6 Solutions, 13.42, Spring 2005

Problem 2:

$$w\_m := 0.68 \qquad \zeta := 5.75 \qquad g := 9.81$$

$$S(w) := \frac{1.25}{4}\cdot\frac{w\_m^4}{w^5}\cdot\zeta^2\cdot e^{\left[-1.25\cdot\left(\frac{w\_m}{w}\right)^4\right]}\cdot\left[\left[\frac{\left(0.4-w^3\right)^2+0.25}{\left(0.4-w^3\right)^2+0.15}\right]^2\right]$$

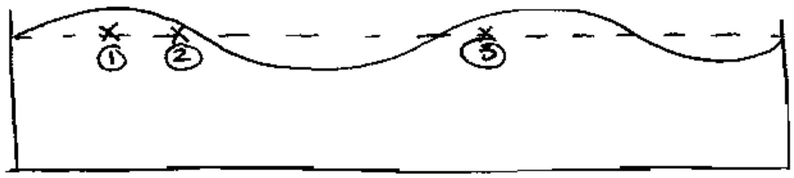
$$M\_o := \int_0^{\infty} S(w)\,dw \qquad M\_2 := \int_0^{\infty} w^2 S(w)\,dw$$

$$M\_o = 4.326 \qquad M\_2 = 3.139$$

$$n(h) := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{M\_2}{M\_o}}\cdot e^{\frac{-h^2}{2\cdot M\_o}}$$

$$h := \sqrt{-2\cdot M\_o\cdot\left(\ln\left(2\frac{\pi}{1800}\cdot\sqrt{\frac{M\_o}{M\_2}}\right)\right)} \qquad h = 6.897 \quad \text{meters}$$

3) a)



$$V_p = \frac{\text{DISTANCE BETWEEN PROBES (1) and (2)}}{\text{TIME IT TAKES A WAVE CREST TO GO FROM (1) TO (2)}}$$

$$V_g = \frac{\text{DISTANCE BETWEEN PROBES (1) and (3)}}{\text{TIME IT TAKES PACKET OF WAVES TO GO FROM (1) TO (3)}}$$

NOTE: $V_g = \frac{1}{2} V_p$ FOR DEEP WATER

$V_g = V_p$ FOR SHALLOW WATER

$A = \frac{1}{2}$ [measured distance between ~~high~~ crest + trough of a wave at a single probe]

$$\lambda = \frac{\text{time between consequtive wave peaks at a single probe}}{V_p}$$

b) $h = 1.5\text{ m}$ FOR DEEP WATER: $\frac{\lambda}{2} \leq h$

$\lambda \leq 2h$ and $k = \frac{2\pi}{\lambda}$ $\therefore$ $\frac{2\pi}{k} \leq 2h$ $\therefore$ $\frac{\pi}{h} \leq k$

IN Deep water $\omega^2 = kg$, $k = \frac{\omega^2}{g}$

$\frac{\omega^2}{g} \geq \frac{\pi}{h}$ $\therefore$ $\omega^2 \geq \frac{\pi g}{h}$ $\therefore$ $\omega \geq \sqrt{\frac{\pi g}{h}}$

$\therefore$ $\omega \geq \sqrt{\frac{\pi \cdot 9.81\,\text{m/s}^2}{1.5\text{m}}} \geq 4.533$ rad/s

$\therefore$ $\boxed{\omega \geq 4.533 \text{ rad/s}}$

c) TAKE WAVE ELEVATION AND HEAVE MOTION DATA IN THE TIME DOMAIN AND DO F.F.T. TO FREQUENCY DOMAIN. RAO [i.e. TRANSFER FUNCTION] IS THEN THE SQUARE ROOT OF THE OUTPUT SPECTRUM DIVIDED BY

Problem 4: Part 1:

$\zeta 1 := 1 \ \text{meter} \qquad T1 := 4 \ \text{sec} \qquad w1\_m := \frac{2\cdot\pi}{T1} \qquad w1\_m = 1.571 \ \frac{\text{rad}}{\text{s}}$

$\zeta 2 := 1.4 \ \text{meter} \qquad T2 := 4.8 \ \text{sec} \qquad w2\_m := \frac{2\cdot\pi}{T2} \qquad w2\_m = 1.309 \ \frac{\text{rad}}{\text{s}}$

$\zeta 3 := 2.2 \ \text{meter} \qquad T3 := 5.4 \ \text{sec} \qquad w3\_m := \frac{2\cdot\pi}{T3} \qquad w3\_m = 1.164 \ \frac{\text{rad}}{\text{s}}$

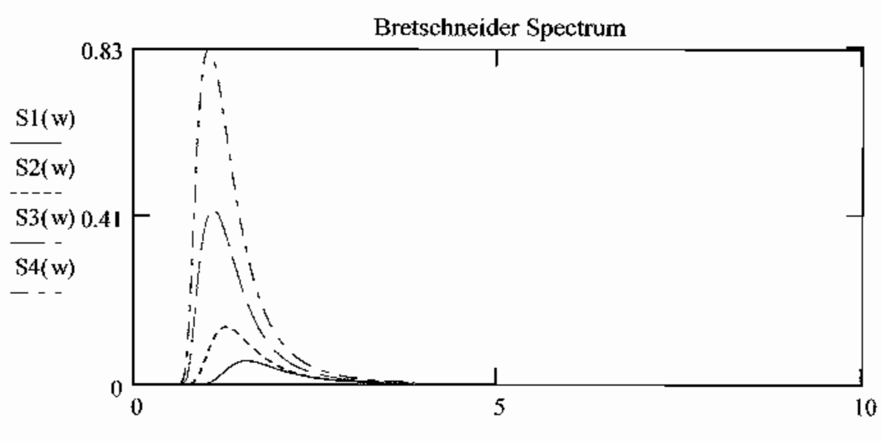
$\zeta 4 := 2.9 \ \text{meter} \qquad T4 := 5.6 \ \text{sec} \qquad w4\_m := \frac{2\cdot\pi}{T4} \qquad w4\_m = 1.122 \ \frac{\text{rad}}{\text{s}}$

$$S1(w) := \frac{1.25}{4}\cdot\frac{w1\_m^4}{w^5}\cdot\zeta 1^2\cdot e^{\left[-1.25\cdot\left(\frac{w1\_m}{w}\right)^4\right]}\cdot\left[\left[\frac{\left(0.4-w^3\right)^2+0.25}{\left(0.4-w^3\right)^2+0.15}\right]\right]^2$$

$$S2(w) := \frac{1.25}{4}\cdot\frac{w2\_m^4}{w^5}\cdot\zeta 2^2\cdot e^{\left[-1.25\cdot\left(\frac{w2\_m}{w}\right)^4\right]}\cdot\left[\left[\frac{\left(0.4-w^3\right)^2+0.25}{\left(0.4-w^3\right)^2+0.15}\right]\right]^2$$

$$S3(w) := \frac{1.25}{4}\cdot\frac{w3\_m^4}{w^5}\cdot\zeta 3^2\cdot e^{\left[-1.25\cdot\left(\frac{w3\_m}{w}\right)^4\right]}\cdot\left[\left[\frac{\left(0.4-w^3\right)^2+0.25}{\left(0.4-w^3\right)^2+0.15}\right]\right]^2$$

$$S4(w) := \frac{1.25}{4}\cdot\frac{w4\_m^4}{w^5}\cdot\zeta 4^2\cdot e^{\left[-1.25\cdot\left(\frac{w4\_m}{w}\right)^4\right]}\cdot\left[\left[\frac{\left(0.4-w^3\right)^2+0.25}{\left(0.4-w^3\right)^2+0.15}\right]\right]^2$$



Problem 4: Part 2:

Part a: $U := 7.20222$ m per sec, = 14 kts $x := 51856$ m, = 28 nm

$$x\_bar := \frac{g \cdot x}{U^2} \qquad a := 0.076 \cdot x\_bar^{-0.22}$$

$$\sigma(w) := \begin{cases} 0.07 & \text{if } w \le w1\_m \\ 0.09 & \text{if } w > w1\_m \end{cases}$$

$$\delta(w) := \frac{-(w - w1\_m)^2}{2 \cdot (\sigma(w))^2 \cdot (w1\_m)^2}$$

$\gamma := 3.3$ This is a standard value of gamma.

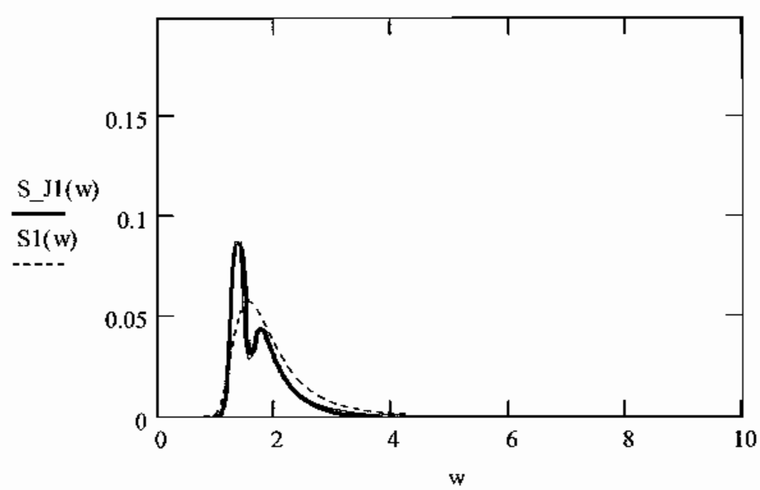
$$S\_J1(w) := \frac{a \cdot g^2}{w^5} \cdot e^{\frac{-5}{4} \cdot \left(\frac{w1\_m}{w}\right)^4} \cdot \gamma^{\delta(w)}$$

$a = 0.01$

$g = 9.81$

$w1\_m = 1.571$

$\gamma = 3.3$ δ = function



b: $\zeta 1 := 1$ meter $\quad T1 := 4$ sec

$$w1\_m1 := \frac{1 \cdot \pi}{T1} \quad w1\_m1 = 0.785 \quad \frac{rad}{s} \qquad w1\_m3 := \frac{3 \cdot \pi}{T1} \quad w1\_m3 = 2.356 \quad \frac{rad}{s}$$

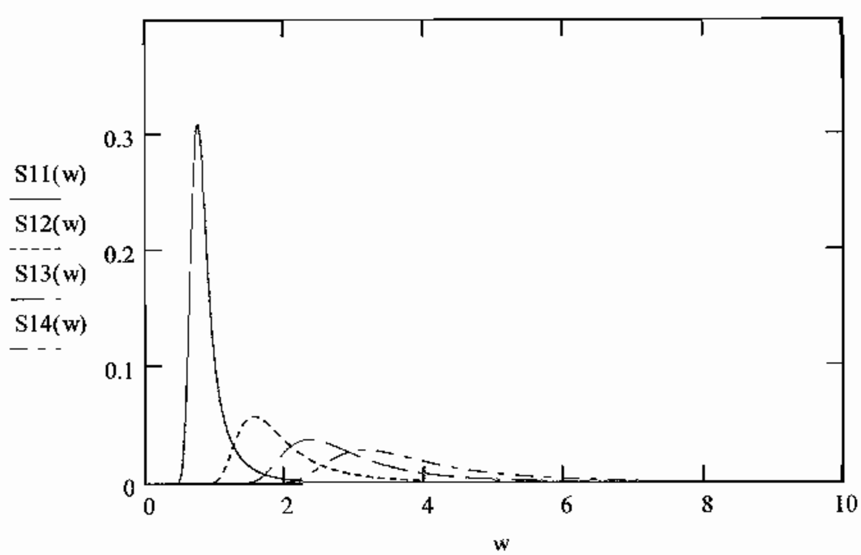
$$w1\_m2 := \frac{2 \cdot \pi}{T1} \quad w1\_m2 = 1.571 \quad \frac{rad}{s} \qquad w1\_m4 := \frac{4 \cdot \pi}{T1} \quad w1\_m4 = 3.142 \quad \frac{rad}{s}$$

$$S11(w) := \frac{1.25}{4} \cdot \frac{w1\_m1^4}{w^5} \cdot \zeta 1^2 \cdot e^{\left[-1.25 \cdot \left(\frac{w1\_m1}{w}\right)^4\right]} \cdot \left[\left[\frac{\left(0.4 - w^3\right)^2 + 0.25}{\left(0.4 - w^3\right)^2 + 0.15}\right]^2\right]$$

$$S12(w) := \frac{1.25}{4} \cdot \frac{w1\_m2^4}{w^5} \cdot \zeta 1^2 \cdot e^{\left[-1.25 \cdot \left(\frac{w1\_m2}{w}\right)^4\right]} \cdot \left[\left[\frac{\left(0.4 - w^3\right)^2 + 0.25}{\left(0.4 - w^3\right)^2 + 0.15}\right]^2\right]$$

$$S13(w) := \frac{1.25}{4} \cdot \frac{w1\_m3^4}{w^5} \cdot \zeta 1^2 \cdot e^{\left[-1.25 \cdot \left(\frac{w1\_m3}{w}\right)^4\right]} \cdot \left[\left[\frac{\left(0.4 - w^3\right)^2 + 0.25}{\left(0.4 - w^3\right)^2 + 0.15}\right]^2\right]$$

$$S14(w) := \frac{1.25}{4} \cdot \frac{w1\_m4^4}{w^5} \cdot \zeta 1^2 \cdot e^{\left[-1.25 \cdot \left(\frac{w1\_m4}{w}\right)^4\right]} \cdot \left[\left[\frac{\left(0.4 - w^3\right)^2 + 0.25}{\left(0.4 - w^3\right)^2 + 0.15}\right]^2\right]$$



Problem 4, Part 3:

$$M\_0n := \int_0^{\infty} S1(w)\,dw \qquad M\_0n = 0.064 \qquad M\_0 := \left(\frac{\zeta 1}{4}\right)^2 \qquad M\_0 = 0.063$$

$$M\_2n := \int_0^{\infty} w^2 \cdot S1(w)\,dw \qquad M\_2n = 0.308 \qquad M\_2 := 1.982 \cdot \left(\frac{\zeta 1 \cdot w1\_m}{4}\right)^2 \qquad M\_2 = 0.306$$

$$M\_4n := \int_0^{\infty} w^4 \cdot S1(w)\,dw \qquad M\_4 := 7.049 \cdot M\_0 \cdot w1\_m^4 \qquad M\_4 = 2.682$$

M 4 integral does not converge. The values for M_0 and M_2 are similar to approximations.

$$\varepsilon := \sqrt{1 - \frac{M\_2^2}{M\_0 \cdot M\_4}} \qquad \varepsilon = 0.665$$ Bandwidth

$N := 10$

$$a\_10 := \sqrt{M\_0} \cdot \sqrt{2 \cdot \ln\left(\frac{2 \cdot \sqrt{1 - \varepsilon^2}}{1 + \sqrt{1 - \varepsilon^2}} \cdot N\right)} \qquad a\_10 = 0.518$$ meters

$N := 50$

$$a\_50 := \sqrt{M\_0} \cdot \sqrt{2 \cdot \ln\left(\frac{2 \cdot \sqrt{1 - \varepsilon^2}}{1 + \sqrt{1 - \varepsilon^2}} \cdot N\right)} \qquad a\_50 = 0.685$$ meters

$N := 100$

$$a\_100 := \sqrt{M\_0} \cdot \sqrt{2 \cdot \ln\left(\frac{2 \cdot \sqrt{1 - \varepsilon^2}}{1 + \sqrt{1 - \varepsilon^2}} \cdot N\right)} \qquad a\_100 = 0.746$$ meters

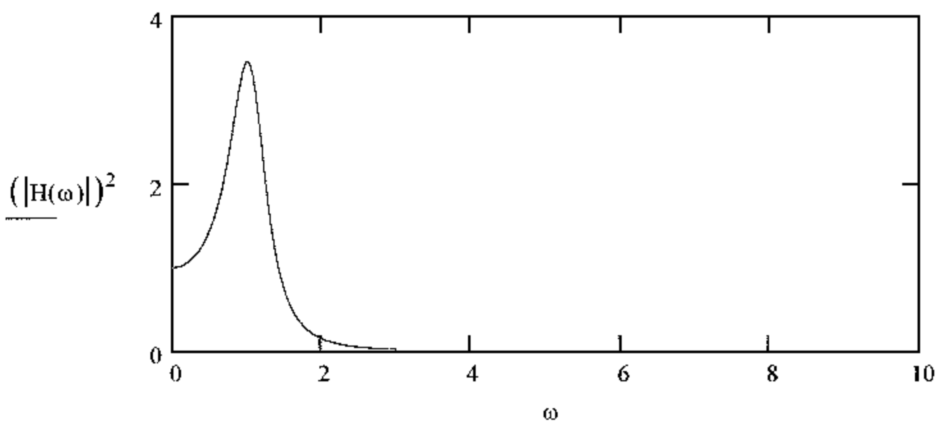
Problem 5:

Part a:

$\omega\_n := 1.1$ rad/s $\qquad \beta := 0.28 \qquad i := \sqrt{-1}$

$$H(\omega) := \frac{\omega\_n^2}{-\omega^2 + 2\cdot i\cdot \beta\cdot \omega\_n\cdot \omega + \omega\_n^2} \qquad H(\omega) \rightarrow \frac{1.21}{-\omega^2 + .616\cdot i\cdot \omega + 1.21}$$

$$(|H(\omega)|)^2 \rightarrow \frac{1.4641}{\left(-\omega^2 + 1.21\right)^2 + .379456\cdot \omega^2}$$

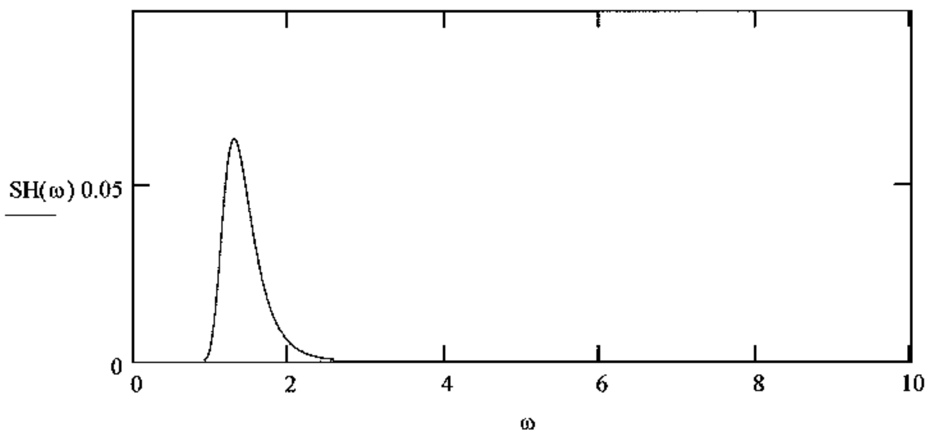


Part b:

$$S1(\omega) \rightarrow 1.9531250000000000000\cdot 10^{-2}\cdot \frac{\pi^4}{\omega^5}\cdot \exp\left(-7.8125000000000000000\cdot 10^{-2}\cdot \frac{\pi^4}{\omega^4}\right)\cdot \frac{\left[\left(.4 - \omega^3\right)^2 + .2\right.}{\left[\left(.4 - \omega^3\right)^2 + .1\right.}$$

$$SH(\omega) := S1(\omega)\cdot (|H(\omega)|)^2$$

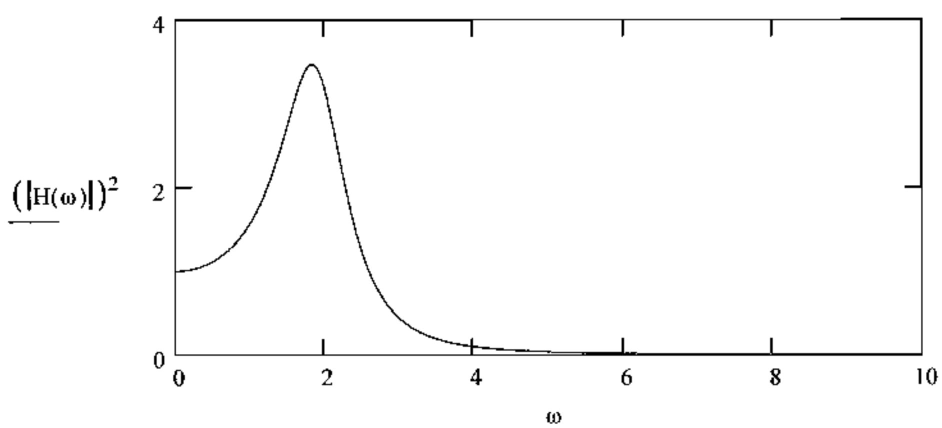
$$SH(\omega) \rightarrow 2.8595703125000000000\cdot 10^{-2}\cdot \frac{\pi^4}{\omega^5}\cdot \exp\left(-7.8125000000000000000\cdot 10^{-2}\cdot \frac{\pi^4}{\omega^4}\right)\cdot \frac{\left[\left(.4\right.\right.}{\left[\left(.4 - \omega^3\right)^2 + .15\right]^2}$$



Part c: $\omega\_n := 2$ rad/s Repeat above steps with a higher frequency.

$$H(\omega) := \frac{\omega\_n^2}{-\omega^2 + 2\cdot i\cdot\beta\cdot\omega\_n\cdot\omega + \omega\_n^2} \qquad H(\omega) \rightarrow \frac{4}{-\omega^2 + 1.12\cdot i\cdot\omega + 4}$$

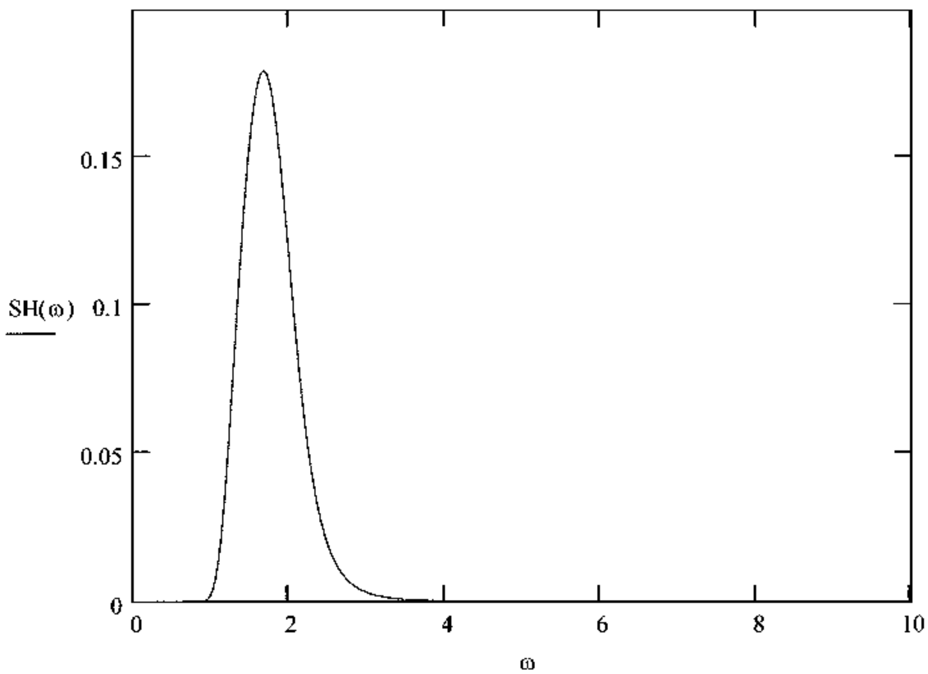
$$(|H(\omega)|)^2 \rightarrow \frac{16}{\left(-\omega^2 + 4\right)^2 + 1.2544\cdot\omega^2}$$



$$S1(\omega) \rightarrow 1.95312500000000000000\cdot 10^{-2}\cdot\frac{\pi^4}{\omega^5}\cdot\exp\left(-7.81250000000000000000\cdot 10^{-2}\cdot\frac{\pi^4}{\omega^4}\right)\cdot\frac{\left[\left(.4 - \omega^3\right)^2 + .2\right]}{\left[\left(.4 - \omega^3\right)^2 + .1\right]}$$

$$SH(\omega) := S1(\omega)\cdot(|H(\omega)|)^2$$

$$SH(\omega) \rightarrow .31250000000000000000 \cdot \frac{\pi^4}{\omega^5} \cdot \exp\left(-7.8125000000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^4}\right) \cdot \frac{\left[\left(.4 - \omega^3\right)\right.}{\left[\left(.4 - \omega^3\right)^2 + .15\right]^2 \cdot \left[\left(-\omega\right.\right.}$$

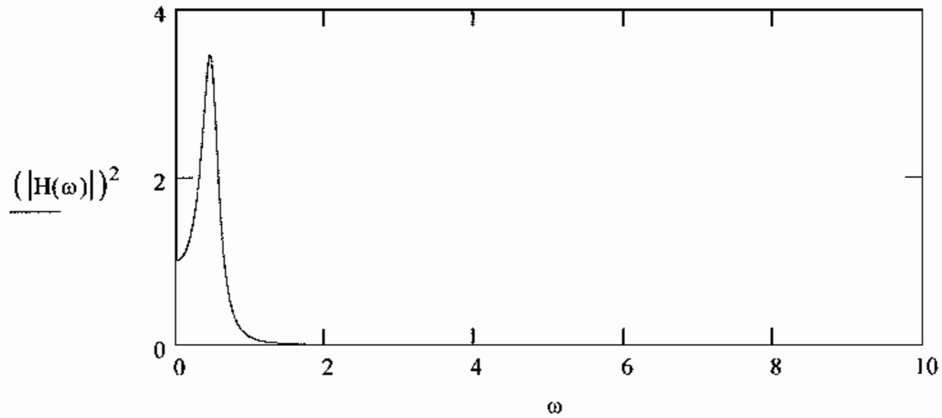


The spectrum is taller with the higher frequency.

Part c cont $\omega\_n := .5$ rad/s Repeat above steps with a lower frequency.

$$H(\omega) := \frac{\omega\_n^2}{-\omega^2 + 2 \cdot i \cdot \beta \cdot \omega\_n \cdot \omega + \omega\_n^2} \qquad H(\omega) \rightarrow \frac{.25}{-\omega^2 + .280 \cdot i \cdot \omega + .25}$$

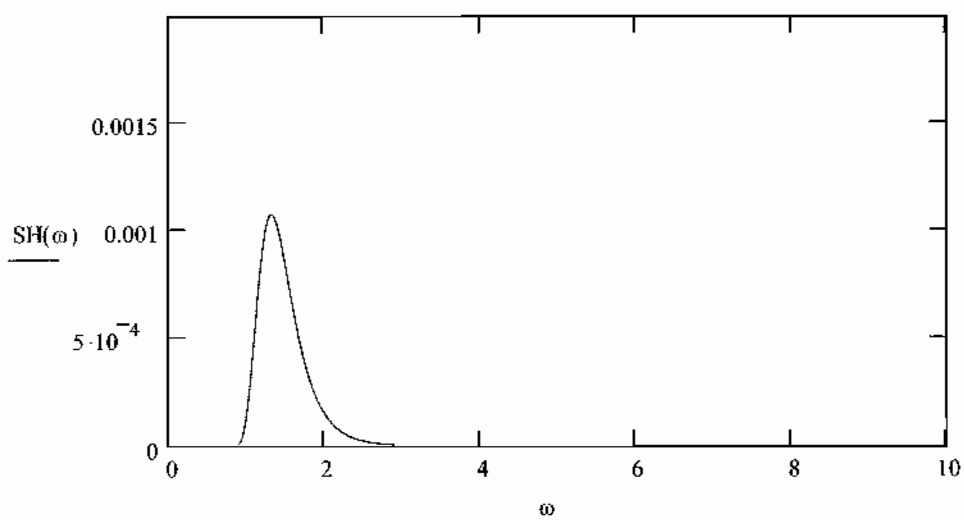
$$\left(|H(\omega)|\right)^2 \rightarrow \frac{6.25 \cdot 10^{-2}}{\left(-\omega^2 + .25\right)^2 + 7.8400 \cdot 10^{-2} \cdot \omega^2}$$



$$S1(\omega) \to 1.95312500000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^5} \cdot \exp\left(-7.81250000000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^4}\right) \cdot \frac{\left[\left(.4 - \omega^3\right)^2 + .\right.}{\left[\left(.4 - \omega^3\right)^2 + .\right.}$$

$$SH(\omega) := S1(\omega) \cdot (|H(\omega)|)^2$$

$$SH(\omega) \to 1.22070312500000000000 \cdot 10^{-3} \cdot \frac{\pi^4}{\omega^5} \cdot \exp\left(-7.81250000000000000000 \cdot 10^{-2} \cdot \frac{\pi^4}{\omega^4}\right) \cdot \frac{[}{\left[\left(.4 - \omega^3\right)^2 + .15\right.}$$



The spectrum is much shorter for the this frequency.

6) FOR A SHIP WITH FORWARD SPEED

$$\omega_e = \omega + \frac{U_s \cos\theta}{g}\omega^2$$ $\omega_e \equiv$ ENCOUNTER FREQUENCY

FOR HEAD SEAS, $\theta = 0$ so $\cos\theta = 1$

$$\omega_e = \omega + \frac{U_s}{g}\omega^2$$

The AMPLITUDE of heave motion is unchanged from the stationary to moving ship. However, the ship moving into head seas experiences this motion at a faster frequency.