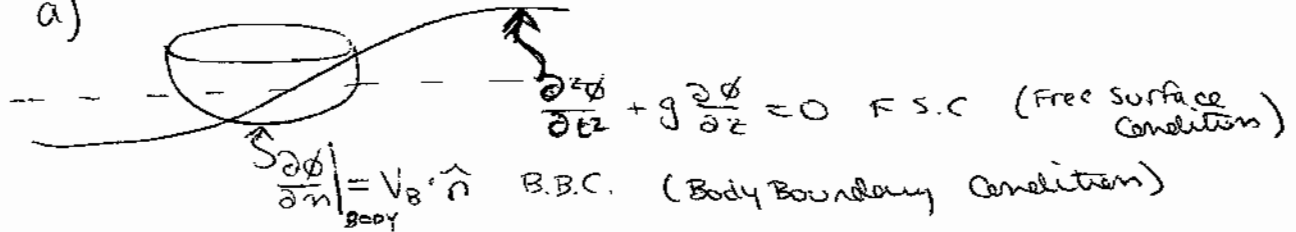
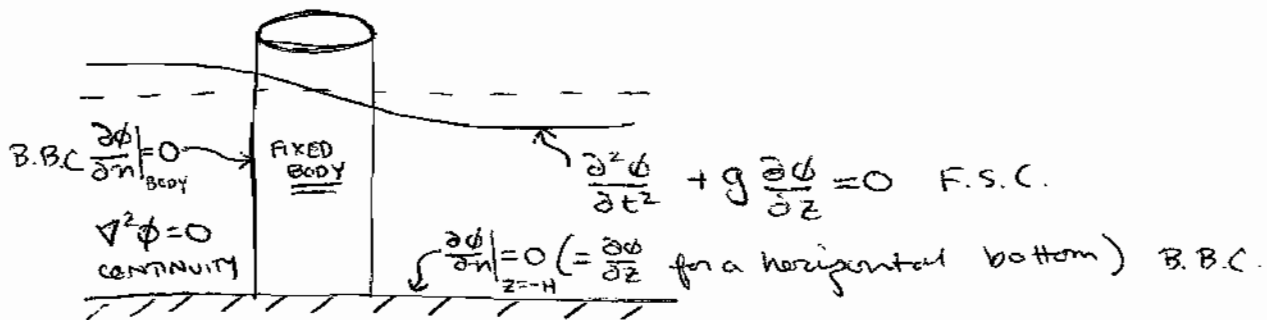
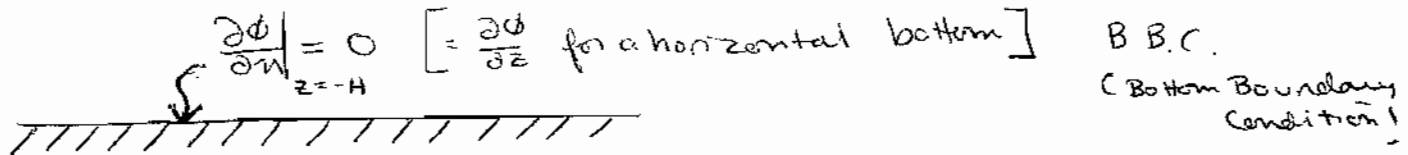


1) a)



$$\nabla^2 \phi = 0 \quad \text{CONTINUITY}$$



b) FOR THE FLOATING BODY:

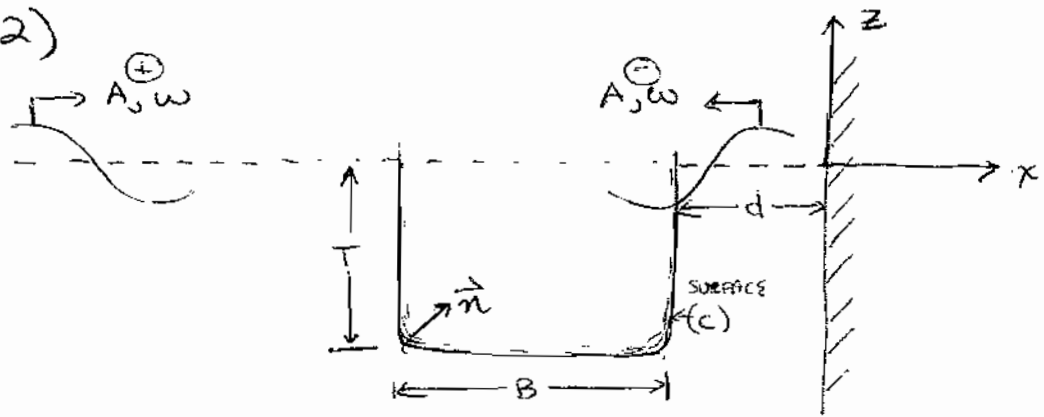
$$\phi_T = \phi_I + \phi_D + \phi_R$$

WHERE  $\phi_T \equiv$  TOTAL POTENTIAL $\phi_I \equiv$  INCIDENT WAVE POTENTIAL, DEPENDS ONLY ON THE INCIDENT WAVES $\phi_D \equiv$  DIFFRACTION POTENTIAL, DEPENDS ON HOW THE INCIDENT WAVES ARE DIFFRACTED BY THE BODY $\phi_R \equiv$  RADIATION POTENTIAL, DEPENDS ON WAVES CREATED BY MOVEMENT OF THE BODY

FOR THE FIXED BODY:

$$\phi_T = \phi_I + \phi_D \quad , \quad \phi_R = 0$$

## Problem 2)



- a) In order to derive the F-K exciting forces, you need to define the "ambient wave" on the body. Due to the presence of the pier, this "ambient wave" is the standing wave created by the wall. I have defined the axes so that the wall is located at  $x=0$ . The velocity potential is that of two waves of equal amplitude ( $A$ ) that propagate in opposite directions:

$$\phi_I = \text{Re} \left\{ \frac{igA}{\omega} e^{kz - ikx + i\omega t} + \frac{igA}{\omega} e^{kz + ikx + i\omega t} \right\}$$

[You could verify here that  $\frac{\partial \phi_I}{\partial x} \Big|_{x=0} = 0$ , which is the fixed body boundary cond.]

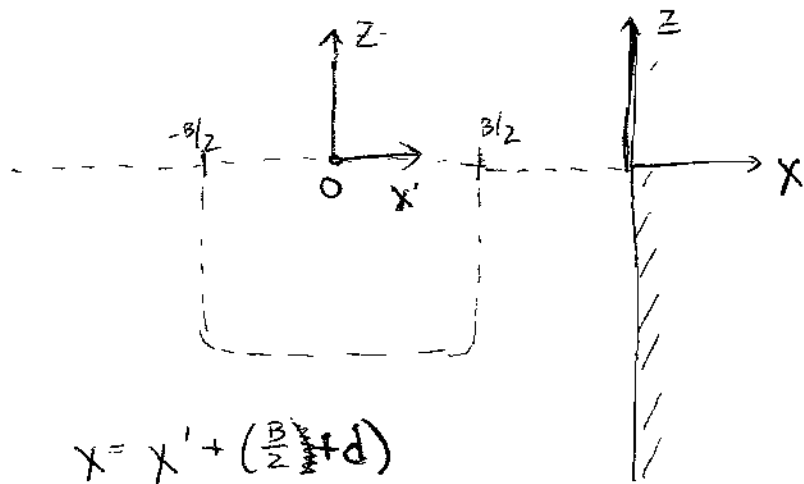
So the F-K EXCITING FORCES IN HEAVE AND SWAY CAN BE DESCRIBED BY THE FOLLOWING EXPRESSION

$$F_i = \int_C -p n_i ds \quad \text{where } n_2 \equiv \text{SWAY} \\ \text{and } n_3 \equiv \text{HEAVE} \\ \text{and } p = -\rho \frac{\partial \phi_I}{\partial t}$$

PROBLEM 2 (CONT.)

b)

NOTE THAT HERE  
"X" DIRECTION  
IS SWAY



$$x = x' + \left(\frac{B}{2} + d\right)$$

DEFINE ANOTHER SET OF AXES AS PICTURED ABOVE, WITH THE ORIGIN CENTERED ON THE 2D BARGE SECTION

THE INCIDENT WAVE POTENTIAL BECOMES:

$$\phi_I = \text{Re} \left\{ \frac{igA}{\omega} e^{kz} e^{-ik(x' + \frac{B}{2} + d) + i\omega t} + \frac{igA}{\omega} e^{kz} e^{ik(x' + \frac{B}{2} + d) + i\omega t} \right\}$$

$$= \text{Re} \left\{ \frac{igA}{\omega} e^{kz} \left[ e^{-ikx' - ik(\frac{B}{2} + d)} + e^{ikx' + ik(\frac{B}{2} + d)} \right] e^{i\omega t} \right\}$$

HEAVE:  $F_3 = \int_{-B/2}^{B/2} p_{I0} dx'$  where  $p_I = -\rho \frac{\partial \phi_I}{\partial t}$

USE TRIGONOMETRIC IDENTITY:  $e^{-i\alpha} + e^{i\alpha} = 2 \cos \alpha$

$$\text{let } \alpha = kx' + k\left(\frac{B}{2} + d\right)$$

$$\therefore e^{-i\left(kx' + k\left(\frac{B}{2} + d\right)\right)} + e^{i\left(kx' + k\left(\frac{B}{2} + d\right)\right)} = 2 \cos\left(kx' + k\left(\frac{B}{2} + d\right)\right)$$

ALSO USE TRIG IDENTITY:  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$\therefore 2 \cos\left(kx' + k\left(\frac{B}{2} + d\right)\right) = 2 \cos(kx') \cos\left(k\left(\frac{B}{2} + d\right)\right) - 2 \sin(kx') \sin\left(k\left(\frac{B}{2} + d\right)\right)$$

$$\therefore \phi_I = \text{Re} \left\{ \frac{igA}{\omega} e^{kz} 2 \left[ \cos(kx') \cos\left(k\left(\frac{B}{2} + d\right)\right) - \sin(kx') \sin\left(k\left(\frac{B}{2} + d\right)\right) \right] e^{i\omega t} \right\}$$

PROBLEM 2 (CONT.)

b. CONT.)

$$P_I = -\rho \frac{\partial \phi_I}{\partial t} = -\rho \operatorname{Re} \left\{ \frac{i g A}{\omega} \cdot i \omega \cdot e^{kz} \cdot 2 \left[ \cos(kx') \cos\left(k\left(\frac{B}{2} + d\right)\right) - \sin(kx') \sin\left(k\left(\frac{B}{2} + d\right)\right) \right] \cdot e^{i\omega t} \right\}$$

NOTE: Because I am integrating w.r.t  $x'$  over the interval  $-\frac{B}{2}$  to  $\frac{B}{2}$ , the  $\sin(kx')$  term drops out  $\therefore$

$$F_3 = \int_{-B/2}^{B/2} \rho \operatorname{Re} \left\{ 2gA e^{kz} \cos(kx') \cos\left(k\left(\frac{B}{2} + d\right)\right) e^{i\omega t} \right\} dx'$$

$$= 2\rho g A e^{k(-T)} \cos\left(k\left(\frac{B}{2} + d\right)\right) \cos \omega t \underbrace{\int_{-B/2}^{B/2} \cos(kx') dx'}_{= \frac{2}{k} \sin \frac{kB}{2}}$$

$$F_3 = \frac{4\rho g A}{k} e^{-kT} \cos \omega t \cos\left(k\left(\frac{B}{2} + d\right)\right) \sin \frac{kB}{2}$$

SO THIS IS THE MEAN F-K FORCE. IT WILL VANISH WHEN

$$\text{EITHER } \cos\left(k\left(\frac{B}{2} + d\right)\right) = 0 \quad \text{OR} \quad \sin \frac{kB}{2} = 0.$$

$$\text{FOR } \cos\left(k\left(\frac{B}{2} + d\right)\right) = 0, \quad k\left(\frac{B}{2} + d\right) = \frac{\pi}{2} (2n+1) \quad \text{for } n = \pm 1, 2, \dots$$

$$k = \frac{\frac{\pi}{2} (2n+1)}{\frac{1}{2}(B+2d)} = \frac{\pi (2n+1)}{B+2d} \quad \text{and} \quad \lambda = \frac{2\pi}{k} = \frac{2\pi (B+2d)}{\pi (2n+1)} = \frac{2(B+2d)}{2n+1}$$

$$\text{FOR } \sin \frac{kB}{2} = 0, \quad \frac{kB}{2} = m\pi \quad \text{for } m = \pm 1, 2, \dots$$

$$k = \frac{2m\pi}{B} \quad \text{and} \quad \lambda = \frac{2\pi}{k} = \frac{2\pi B}{2m\pi} = \frac{B}{m}$$

SO THE MEAN F-K FORCE

$$\lambda = \frac{2(B+2d)}{2n+1} \quad \text{OR} \quad \lambda = \frac{B}{m} \quad \text{for } n = \pm 1, 2, \dots$$

Problem 2b cont.)

SWAY: THE SWAY F-K EXCITING FORCE DEPENDS ONLY ON THE PRESSURE DISTRIBUTION ON THE TWO VERTICAL SIDES

GOING THROUGH A SIMILAR PROCESS AS THE HEAVE SOLUTION ABOVE, YOU FIND THAT:

$$F_z = \rho g A \operatorname{Re} \left\{ \int_{-T}^0 dz e^{kz} (i) \sin \frac{kB}{2} \left[ e^{-ik(\frac{B}{2}+d)} - e^{ik(\frac{B}{2}+d)} \right] e^{i\omega t} \right\}$$

$$= \rho g A \operatorname{Re} \left\{ \int_{-T}^0 dz e^{kz} (i) \sin \frac{kB}{2} (-2i) \sin(k(\frac{B}{2}+d)) e^{i\omega t} \right\}$$

∴ THE SWAY F-K EXCITING FORCE WILL DISAPPEAR WHEN:

$$\sin \frac{kB}{2} = 0 \quad \text{OR} \quad \sin(k(\frac{B}{2}+d)) = 0.$$

$$\text{FOR } \sin \frac{kB}{2} = 0, \quad \frac{kB}{2} = n\pi \quad \text{for } n = \pm 1, 2, \dots$$

$$k = \frac{2n\pi}{B} \quad \text{and} \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{2n\pi}{B}} = \frac{B}{n}$$

$$\text{FOR } \sin(k(\frac{B}{2}+d)) = 0, \quad k(\frac{B}{2}+d) = m\pi \quad \text{for } m = \pm 1, 2, \dots$$

$$k = \frac{m\pi}{(\frac{B}{2}+d)} \quad \text{and} \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{m\pi}{(\frac{B}{2}+d)}} =$$

$$\lambda = \frac{2(\frac{B}{2}+d)}{m} = \frac{B+2d}{m}$$

SO THE SWAY F-K VANISHES AT

$$\lambda = \frac{B}{n} \quad \text{OR} \quad \lambda = \frac{B+2d}{n} \quad \text{for } n = \pm 1, 2, \dots$$