13.80/2.06 2002

Homework No. 1

Review Questions

2.3
State the quantities corresponding to m, c, k, and x for a torsional system.

For a torsional system, the analogous quanties to the translational systems's mass m, damping constant c, spring stiffness k, and displacement x are respectively:

- The mass moment of inertia with units (N m s^2/rad)
- The torsional damping constant with units (N m s/rad)
- The torsional spring stiffness with units (N m/rad)
- The angular displacement with units (rad)

2.4
What effect does a decrease in mass have on the frequency of a system?

The natural frequency of a SDOF system is $\omega_n = \sqrt{\frac{k}{m}}$. Thus if the mass is increased, the natural frequency drops.

2.5
What effect does a decrease in the stiffness of the system have on the natural period?

The natural period is the reciprocal of the natural frequency given above, thus an increase in stiffness would increase the natural frequency and thereby decrease the natural period.

2.6
Why does the amplitude of free vibration gradually diminish in practical systems?

The free vibration amplitude diminishes, because every real system has some form of damping. Vibrating systems undergo energy conversions between kinetic and potential energy, and during these conversions energy dissipation takes place. The second law of thermodynamics partially states that the usable energy in a closed system diminishes over time, and this is seen in real systems.

2.15
What happens to the energy dissipated by damping?

The energy dissipated by damping is typically converted to heat or sound energy, and is transferred to the system's environment.

1.12
How do you connect several springs to increase the overall stiffness?

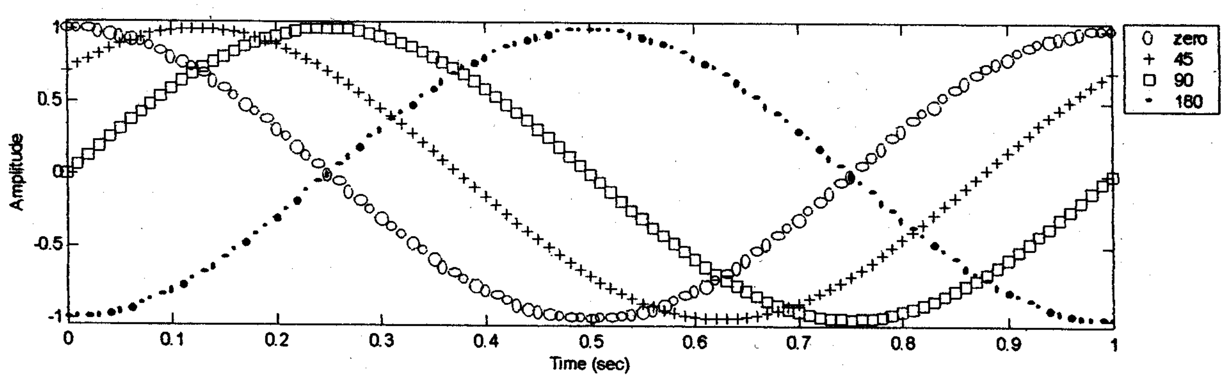
You connect the springs in parallel, so that each spring experiences the same deflection.

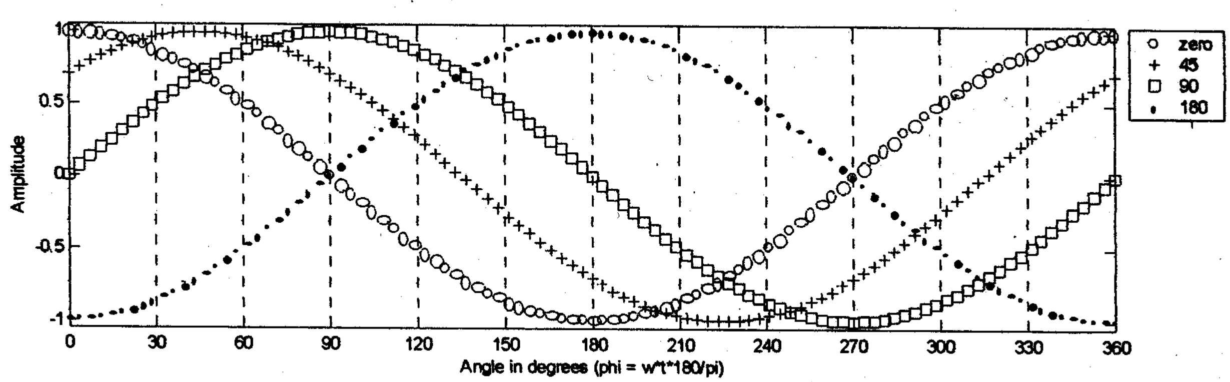
1.15
What is the difference between harmonic motion and periodic motion?

Harmonic motion is a vibration composed of one frequency, whereas periodic motion is any motion that repeats itself in time (it can have multiple frequency components, for instance a square wave).

Problems

1a. Plot $\cos(\omega t-\phi)$ with $\phi = 0, 45, 90, \textit{and}\ 180$ degrees.





The above plots where generated using Matlab. The upper plot is a 1 Hz cosine function plotted against time, whereas the bottom plot is the same 1 Hz cosine function plotted against the argument angle in degrees. ($\theta = \omega t$)
Below is the m-file code that was used to generate the plots.

```
% Homework No.1 prb1 Matlab Code

f = 1;                      %The frequency of oscillation in Hz
w = 2*pi*f;                 %The frequency of oscillation in rad/s

phi1 = 0*pi/180;        %Phase angles in radians
phi2 = 45*pi/180;
phi3 = 90*pi/180;
phi4 = 180*pi/180;

t = 0:0.01:1;           %Vector of time values in seconds

figure(1)
subplot(2,1,1), plot(t,cos(w*t),'ko',t,cos(w*t-phi2),'k+',t,cos(w*t-phi3),...
'ks',t,cos(w*t-phi4),'k.');
axis([0 1 -1.05 1.05]);
xlabel('Time (sec)');
ylabel('Amplitude');
legend('zero','45','90','180');

subplot(2,1,2), plot(t*w*180/pi,cos(w*t),'ko',t*w*180/pi,cos(w*t-phi2),'k+',...
t*w*180/pi,cos(w*t-phi3),'ks',t*w*180/pi,cos(w*t-phi4),'k.');
axis([0 360 -1.05 1.05]);
xlabel('Angle in degrees (phi = w*t*180/pi)');
ylabel('Amplitude');
legend('zero','45','90','180');
```

1a. cont...

For what values of ϕ is $\cos(\omega t - \phi) = \pm\sin(\omega t)$

For what values of ϕ is $\sin(\omega t - \phi) = \pm\cos(\omega t)$

$$\cos(\omega t \pm 90^\circ) = \mp\sin(\omega t)$$

$$\sin(\omega t \pm 90^\circ) = \pm\cos(\omega t)$$

1b.

Express $3\cos(\omega t)+5\sin(\omega t)$ as $A\cos(\omega t-\phi)$ and as $\mathrm{Re}(Be^{i(\omega t-\phi)})$ where "Re" denotes the real part.

The equation $A_1\sin(\omega t)+A_2\cos(\omega t)$ can be expressed as $A\cos(\omega t-\phi)$ where $A=\sqrt{A_1^2+A_2^2}$ and $\phi=\tan^{-1}(\frac{A_1}{A_2})$. In this case $A=\sqrt{5^2+3^2}=\sqrt{34}$ and $\phi=\tan^{-1}(\frac{5}{3})$. Thus $3\cos(\omega t)+5\sin(\omega t)=\sqrt{34}\cos(\omega t\text{-}1.03)$. where ϕ is expressed in radians.

From Euler's equation $e^{i\theta}=\cos(\theta)+i\sin(\theta)$, and therefore the $\mathrm{Re}(Be^{i(\omega t-\phi)})$ is $B\cos(\omega t-\phi)$. This is the same expression as above, and therefore:
$3\cos(\omega t)+5\sin(\omega t) = \mathrm{Re}(e^{i(\omega_n t-1.03)}) = \sqrt{34}\cos(\omega t\text{-}1.03)$.

1b. cont...

A simple Mass-Spring-Dashpot system has an initial displacement of 0.1 m and an initial velocity of 2.0 m/s. For free vibration of this system without damping express the motion x(t) as $Ae^{i(\omega t-\phi)}$.

The solution to a SDOF undamped system is:

$$x(t)=\sqrt{x_o{}^2+\left(\frac{\dot{x}_o}{\omega_n}\right)^2}\cos(\omega_n t-\tan^{-1}\left(\frac{\dot{x}_o}{\omega_n\, x_o}\right))$$

Thus plugging in the given values for x_o and $\dot{x}_o$ the motion can be expressed as $x(t)=\mathrm{Re}\left(\sqrt{0.1^2+\left(\frac{2.0}{\omega_n}\right)^2}\, e^{i(\omega_n t-\tan^{-1}\left(\frac{2.0}{\omega_n\, 0.1}\right))}\right)$ where $\omega_n=\sqrt{k/m}$.

The Phase Angle Question

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$
$$\dot{x}(t) = \omega A\cos(\omega t) - \omega B\sin(\omega t)$$
$$x(0) = B$$
$$\dot{x}(0) = \omega A$$

So now simply choose how you would like to re-write x(t). for example:

$$x(t) = C\cos(\omega t - \phi)$$
$$\dot{x}(t) = -\omega C\sin(\omega t - \phi)$$
$$x(0) = C\cos(-\phi) = C\cos(\phi)$$
$$\dot{x}(0) = -\omega C\sin(-\phi) = \omega C\sin(\phi)$$
$$\frac{x(0)}{\dot{x}(0)} = \frac{B}{\omega A} = \frac{C\cos(\phi)}{\omega C\sin(\phi)} = \frac{1}{\omega\tan(\phi)}$$

Thus $\phi = \tan^{-1}\left(\frac{A}{B}\right)$

This simple procedure can be repeated for whichever form you would like to convert x(t). Thus given $x(t) = A\sin(\omega t) + B\cos(\omega t)$ we can re-write it as:

Form	Phase Angle
$x(t) = C\cos(\omega t - \phi)$	$\phi = \tan^{-1}\left(\frac{A}{B}\right)$
$x(t) = C\cos(\omega t + \phi)$	$\phi = -\tan^{-1}\left(\frac{A}{B}\right)$
$x(t) = C\sin(\omega t - \phi)$	$\phi = -\tan^{-1}\left(\frac{B}{A}\right)$
$x(t) = C\sin(\omega t + \phi)$	$\phi = \tan^{-1}\left(\frac{B}{A}\right)$