

## 14.02 Quiz 1 Solution

**QUESTION 1: TRUE OR FALSE.** Explain your answer fully 30 points (6 pts. each)

1. Assume that government spending is a linear function of total output: i.e.  $G = g_0 - g_1 Y$ , where  $g_0$  is positive. The Keynesian multiplier is larger when  $g_1$  is positive than when  $g_1$  is equal to zero.

**Solution. False:** the equilibrium on the goods market implies that  $Y = c_0 + c_1(Y - T) + g_0 - g_1 G$  or  $Y = 1/(1 - c_1 + g_1) * (c_0 + g_0 - c_1 T)$  The Keynesian multiplier is  $1/(1 - c_1 + g_1)$  which is smaller than  $1/(1 - c_1)$ . The positive impact of an increase in autonomous spending is partially offset by the subsequent drop in government spending.

2. In an economy where individuals demand half of their money as currency and half as checkable deposits, an increase in high-powered money by the Central Bank has a larger effect on interest rates and output than in an economy where individuals hold all of their money as cash.

**Solution. True.** The money multiplier in an economy where individuals demand half of their money in currency and half as checkable deposits is  $1/(.5 + .5q)$  which is higher than 1 as it would be in an economy where individuals would hold all their money as cash. Therefore an increase in high-powered money by the Central Bank has a larger effect (provided that banks do not keep all their money in reserves,  $q < 1$ , in which case the money multiplier would always be 1).

3. The Bank of Japan (BoJ: central bank of Japan) has published the following nominal GDP growth figures for 2000, 2001 and 2002:  $g_{2000} = 1.1\%$ ,  $g_{2001} = -2.5\%$  and  $g_{2002} = -0.7\%$ . In the same publication the real GDP growth figures are:  $gr_{2000} = 3.2\%$ ,  $gr_{2001} = -1.2\%$  and  $gr_{2002} = 1.6\%$ . These figures imply that the rate of inflation was negative in 2000, positive in 2001 and again negative in 2002.

**Solution. False:** the rate of inflation will be positive if nominal output grows faster than real output, and negative otherwise. In every year, the nominal GDP growth was lower than the real GDP growth, implying that inflation was negative all three years (and not positive in 2001 as stated). Japan experienced deflation three years in a row.

4. Fiscal policy cannot affect the level of GDP if money demand does not depend on the interest rate

**Solution. True:** *when money demand does not depend on the interest rate, the LM curve is vertical, meaning that output is set on the financial market, no matter what the conditions on the goods market are. Fiscal policy moves the IS curve, and this will have no effect on output with a vertical LM curve: any increase in demand due to increase in government spending is totally offset by an increase in the interest rate (which depresses investment).*

5. Consider the simple goods market model, where  $Z = C + I + G$  and both  $I$  and  $G$  are fixed. Let the consumption function be  $C = c_0 + c_1(Y-T)$ . If  $c_1$  rises, the private saving rate, defined as  $S/Y$ , decreases.

**Solution. True:** *if  $c_1$  increases, the equilibrium output increases. On the other hand, private savings must be equal to investment minus public savings, which is fixed. Therefore, private savings are not affected (the savings paradox). If  $S$  is unchanged and  $Y$  goes up, then  $S/Y$  decreases.*

**QUESTION 2: THE GOODS MARKET.** 20 points (5 pts. each)

Consider the following equations describing the components of demand and equilibrium in the goods market:

$$C = 120 + 0.5(Y - T)$$

$$I = 40$$

$$G = 20$$

$$T = 40$$

1. Solve for the equilibrium level of output in this economy.

**Solution:** *The aggregate demand of this economy is given by*

$$\begin{aligned} Z &= C + I + G \\ Z &= 120 + 0.5(Y - T) + I + G \end{aligned}$$

*The equilibrium condition is that aggregate demand is equal to output:  $Z = Y$*

$$\begin{aligned} Y^* &= 120 + 0.5(Y^* - T) + I + G \\ Y^* &= 1/(1 - 0.5)(120 + I + G - 0.5T) \\ Y^* &= 2(120 + 40 + 20 - 0.5 \cdot 40) \\ Y^* &= 320 \end{aligned}$$

2. There is a permanent increase in government spending of 10 units (so, now  $G=30$ ). Solve for the new equilibrium  $Z$  and  $Y$ .

**Solution:** *In this very simple goods market model, an increase in government spending of 10 units leads to an increase in equilibrium output of  $10 \cdot 1/(1-0.5)$  units (10 units times the Keynesian multiplier).*

$$Y^* = 320 + 10 \cdot 1/(1-0.5) = 320 + 20 = 340$$

*Since we must have aggregate demand equal to output,  $Z = Y^* = 340$*

Now, assume that it takes one period for the firms to adjust production in order to meet demand. In this economy, the equilibrium condition is  $Y_t = Z_{t-1}$ .

3. Assume that  $Y_0 = Y_{-1}$  is equal to the equilibrium output you found in part 1. At time  $t = 0$ , there is a permanent increase in government spending by 10 units (so, now  $G=30$ ). Solve for  $Z_0$ .

**Solution:** At time  $t = 0$ , government spending increases. Aggregate demand therefore increases:

$$\begin{aligned} Z_0 &= C_0 + I + G + 10 \\ Z_0 &= 120 + 0.5(Y_{-1} - T) + I + G + 10 \\ Z_0 &= 120 + 0.5(320 - 40) + 40 + 20 + 10 \\ Z_0 &= 330 \end{aligned}$$

The increase in government spending leads to a one for one increase in the aggregate demand.

4. Solve for  $Y_1$  and  $Z_1$  and  $Y_2$ .

**Solution:** We know that  $Y_1 = Z_0 = 330$ .

$$\begin{aligned} Z_1 &= C_1 + I + G \\ Z_1 &= 120 + 0.5(Y_0 - T) + I + G + 10 \\ Z_1 &= 120 + 0.5(330 - 40) + 40 + 20 + 10 \\ Z_1 &= 335 \end{aligned}$$

We know that  $Y_2 = Z_1 = 335$ .

The increase in aggregate demand at  $t = 1$  leads to an increase in output. This implies an increase in disposable income. Out of this additional disposable income, people consume a fraction equal to their marginal propensity to consume (0.5). This leads to a further increase in aggregate demand, and a subsequent increase in output.

**Bonus question (10 points):**

Solve for  $Y_n$  and for  $Y_\infty$ . Compare this result to your answer in question 2.

**Solution:** the first increment in output is 10 units. The second one is  $0.5 \cdot 10$ . The third one will be  $10 \cdot 0.5^2$ . The  $n$ -th one will be  $10 \cdot 0.5^n$ . The total increase after  $n$  periods will be:

$$\begin{aligned} Y_n &= 10 \cdot (1 + 0.5^1 + 0.5^2 + \dots + 0.5^n) \\ Y_n &= 10 \cdot (1 - 0.5^{n+1}) / (1 - 0.5) \end{aligned}$$

As the number of periods goes to infinity,  $0.5^{n+1}$  goes to zero, and therefore the total increment in output following a permanent increase of government spending of 10 units will be:

$$Y_\infty = 1 / (1 - 0.5) \cdot 10 = 20$$

*This is the same increase as in question 2. The increase is equal to the increase in government spending, 10 units, multiplied by the Keynesian multiplier  $1/(1 - 0.5)$ . The Keynesian multiplier comes from the addition of all the successive rounds of increase in demand (through increases in income and therefore of consumption).*

**QUESTION 3: IS-LM 50 points**

Consider an economy described by the following short run model.

$$\begin{aligned} C &= c_1(Y-T) \\ I &= b_0 + b_1Y - b_2i \\ M^d/P &= C - a^*i \\ G &= G_0 \\ T &= T_0 \\ M^s &= M_0 \end{aligned}$$

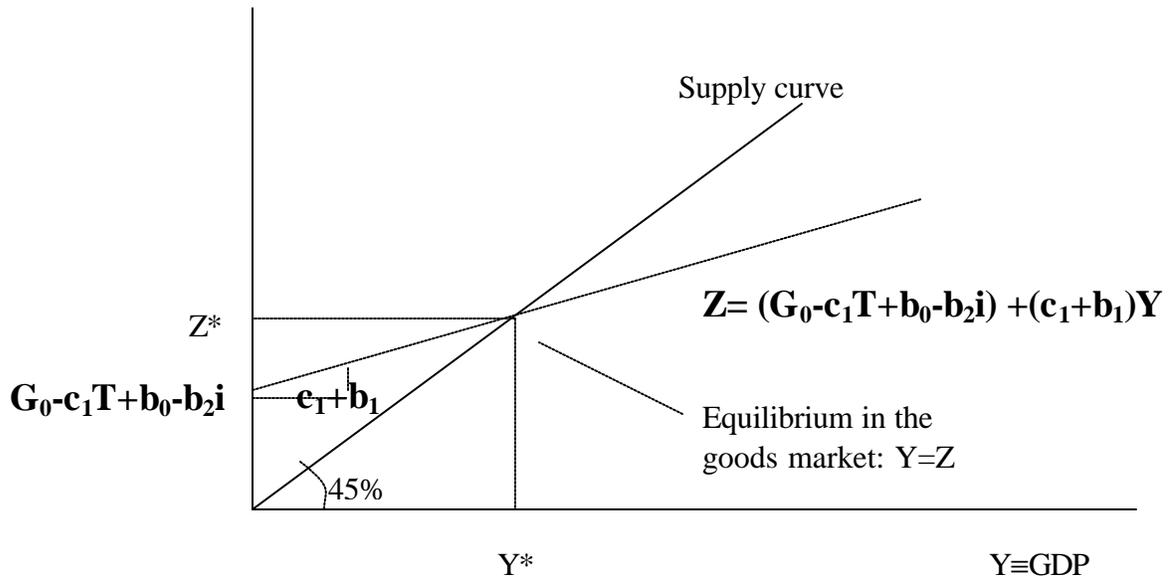
All the parameters ( $a, b_0, b_1, b_2, c_1$ ) in the model are positive, and  $b_1 + c_1 < 1$ . We assume for simplicity that  $P=1$ .

**Important:** Note that unlike in the short run model we have seen in class, money demand depends on consumption, and not on total output.

**Important:** always give a brief explanation of what you are doing (for all the questions).

1. Find the equation for the aggregate demand ( $Z$ ). Draw the aggregate demand curve and the equilibrium in the goods market (45 degree line) on a graph with output ( $Y$ ) on the horizontal axis, and aggregate demand ( $Z$ ) on the vertical axis.

$$Z = C + I + G = c_1(Y - T_0) + b_0 + b_1Y - b_2i + G_0 = (G_0 - c_1T_0 + b_0 - b_2i) + (c_1 + b_1)Y$$



2. Find the equilibrium on the goods market, and derive the equation for the IS curve.

$$\text{In equilibrium: } Y = Z \quad \mathbf{P} \quad Y = (G_0 - c_1 T_0 + b_0 - b_2 i) + (c_1 + b_1) Y$$

$$\Rightarrow i = \frac{1}{b_2} (G_0 + b_0 - c_1 T_0) - \frac{(1 - (c_1 + b_1))}{b_2} Y \equiv a_1 - a_2 Y$$

$$\Leftrightarrow Y = \frac{1}{1 - (c_1 + b_1)} (G_0 + b_0 - c_1 T_0) - \frac{b_2}{1 - (c_1 + b_1)} i$$

3. Find the equilibrium on the financial market. Derive the equation for the LM curve.

$$M^d = C - a^* i = c_1 (Y - T_0) - a^* i$$

$$M^s = M_0$$

$$\text{In equilibrium: } M^s = M^d$$

$$\mathbf{P} \quad M_0 = c_1 (Y - T_0) - a^* i$$

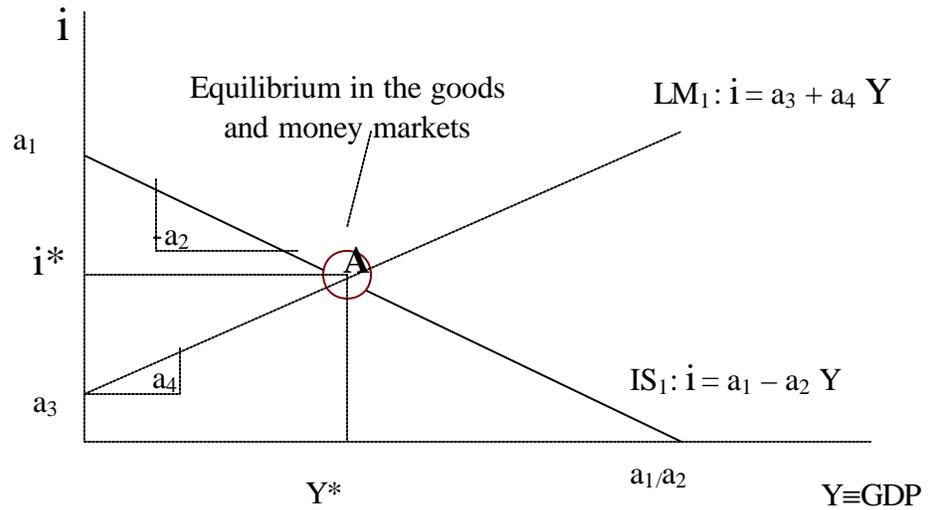
$$\Rightarrow i = -\frac{1}{a} (M_0 + c_1 T_0) + \frac{c_1}{a} Y \equiv a_3 + a_4 Y$$

$$\Leftrightarrow Y = \frac{1}{c_1} (M_0 + c_1 T_0) + \frac{a}{c_1} i$$

4. Show graphically the equilibrium in both the goods market and the financial markets. Call the equilibrium output  $Y_0$ , the IS curve  $IS_0$  and the LM curve  $LM_0$ . Do not do any algebra, but make sure you put all the available information on your graph.

The IS curve gives the pairs of  $(Y,i)$  that support equilibrium in the goods market. The LM curve gives the pairs of  $(Y,i)$  that support equilibrium in the money market.

The intersection of both gives  $(Y,i)$  that support a simultaneous equilibrium in both market.



5. There is a sudden drop in business confidence, and firms decide suddenly to invest less for any level of output and interest rates. From this question onward, the new behavior of firms obeys the following equation:

$$I = b_0 - \beta + b_1 Y - b_2 i \text{ with } \beta > 0$$

Find the new IS relation (call it  $IS_1$ ) and the new LM relation (call it  $LM_1$ ). Show graphically how the two curves shift, and state what happens to equilibrium output and interest rates. Call the new equilibrium output  $Y_1$ .

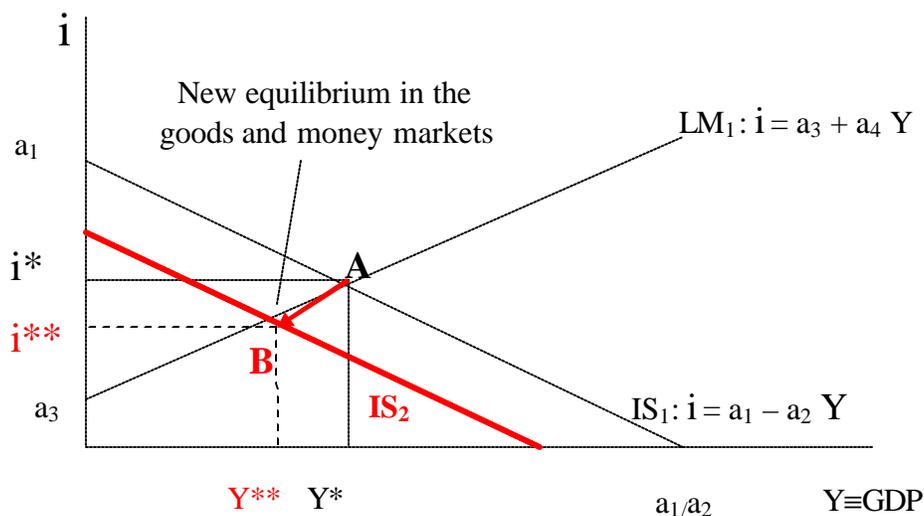
The IS relation is affected through the reduction in  $b_0$ , which affects the intersection ( $-a_1$ ), and causes the IS curve to shift left-down (?):

$$\Rightarrow i = \frac{1}{b_2} (G_0 + b_0 - \Delta - c_1 T_0) - (1 - (c_1 + b_1)) Y \equiv a_1' - a_2 Y$$

$$\Leftrightarrow Y = \frac{1}{1 - (c_1 + b_1)} (G_0 + b_0 - \Delta - c_1 T_0) - \frac{b_2}{1 - (c_1 + b_1)} i$$

The LM curve is not affected.

Therefore, in the new equilibrium ( $Y^-, i^-$ ), as shown in the following graph:



6. Now suppose the government wants to counterbalance the effect of the drop in investors' confidence on GDP using **government spending** as a policy instrument. Should the government increase or decrease  $G$ ? What new level of government spending  $G_1$  should the government choose to completely offset the effect of lower business confidence?

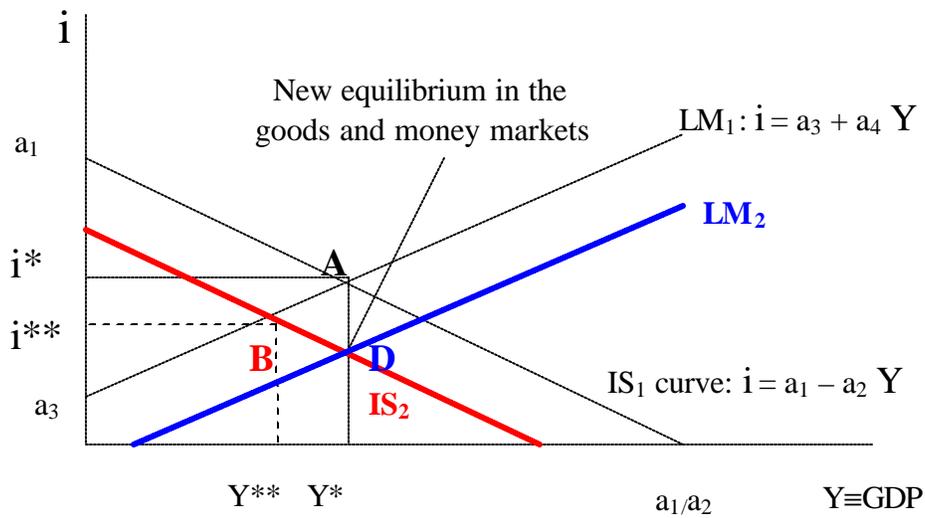
*A change in  $G$  affects only the IS curve through its effect on the intersection ( $a_1$ ). The government can counterbalance the effect of the drop in investors' confidence on GDP by increasing  $G$  by exactly ? (i.e., by the same drop in  $b_0$ ), which will shift IS back to its original position:*

$$\Rightarrow i = \frac{1}{b_2} (G_0 + \Delta + b_0 - \Delta - c_1 T_0) - (1 - (c_1 + b_1)) Y \equiv a_1' - a_2 Y$$

$$\Rightarrow i = \frac{1}{b_2} (G_0 + b_0 - c_1 T_0) - (1 - (c_1 + b_1)) Y \equiv a_1 - a_2 Y$$

7. Suppose now the government cannot use fiscal instruments (G or T) because Congress would not allow it. The Central Bank decides to use **monetary policy** to restore the original equilibrium output  $Y_0$ . Should the Central Bank increase or decrease  $M^s$ ? Draw the new IS (call it  $IS_2$ ) and the new LM curve (call it  $LM_2$ ) after the change in money supply.

*M should be increased, so the LM curve shifts down-right (?) and crosses IS at  $Y=Y^*$ , which is at point D in the graph below:*



8. For each of the two sub-questions 6 and 7, state qualitatively how  $i$  and  $I$  move, compared to sub-question 4, in response to the change in economic policy:?  
(Qualitatively means whether a variable goes up, down, or stays unchanged relative to sub-question 4)

*A government expansion (section 6): The interest rate returns to its value in section 4, as it is clear from the graph of section 6. By considering the investment equation, and noting that the interest rate and GDP are the same in section 4 and 6, however the constant in the investment equation is lower in section 6, then it must be that the investment in section 6 is lower than in section 4. Here government investment replaces private investment.*

*A monetary expansion: by inspection the graph in section 7, we see that the interest rate is lower in section 6 than in section 4. So the investment will be higher than in section 6. By how much? We know that in equilibrium  $Y = C + I + G$ .  $Y$  is the same as in question 4,  $C$  as well (same disposable income), and  $G$  as well. Therefore,  $I$  must be the same as in question 4: the drop in interest rate has convinced the investors to go back to their original level of investment.*

9. Suppose the government wants both to return to the original level of output  $Y_0$  **and** to achieve as high a level of investment as possible. Of the two options considered in sub-questions 6 and 7 (government spending or monetary policy) which economic policy instrument should the government use? Briefly justify your answer.

*Since in both cases, the economy returns to the same level of  $Y$ , the government should look for the policy with the lowest interest rate in the new equilibrium. Comparing the above graphs, we find that monetary expansion brings about the lowest interest rate. With a monetary expansion, investment is ? higher than with a fiscal expansion, and output is the same.*

10. Suppose the government decides to increase **taxes**. What is the effect on equilibrium output and equilibrium interest rate? Draw the new IS curve (call it IS<sub>3</sub>) and the new LM curve (call it LM<sub>3</sub>). You must justify your answer.

*A change in T affects both IS and LM:*

**IS:** - T<sup>↑</sup> - a<sub>1</sub> shifts the IS curve up-right (?):

$$\Rightarrow i = \frac{1}{b_2} (G_0 + b_0 - c_1 T_0 \uparrow) - (1 - (c_1 + b_1)) Y \equiv \downarrow a_1 - a_2 Y$$

**LM:** - T<sup>↑</sup> - a<sub>3</sub> shifts the LM curve up-left (?).

$$\Rightarrow i = -\frac{1}{a} (M_0 + c_1 T_0 \uparrow) + \frac{c_1}{a} Y \equiv \downarrow a_3 + a_4 Y$$

In equilibrium: IS=LM

$$\begin{aligned} \Rightarrow \frac{1}{b_2} (G_0 + b_0 - c_1 T_0 \uparrow) - (1 - (c_1 + b_1)) Y &= -\frac{1}{a} (M_0 + c_1 T_0 \uparrow) + \frac{c_1}{a} Y \\ \Rightarrow \left( \frac{c_1}{a} + 1 - (c_1 + b_1) \right) Y &= \frac{1}{b_2} (G_0 + b_0 - c_1 T_0 \uparrow) + \frac{1}{a} (M_0 + c_1 T_0 \uparrow) \\ \Rightarrow \left( \frac{c_1}{a} + 1 - (c_1 + b_1) \right) Y &= \left( \frac{1}{b_2} (G_0 + b_0) + \frac{1}{a} M_0 \right) + \left( \frac{1}{a} - \frac{1}{b_2} \right) c_1 T_0 \uparrow \\ \Rightarrow Y &= \frac{\left( \frac{1}{b_2} (G_0 + b_0) + \frac{1}{a} M_0 \right)}{\left( \frac{c_1}{a} + 1 - (c_1 + b_1) \right)} + \frac{\left( \frac{1}{a} - \frac{1}{b_2} \right) c_1}{\left( \frac{c_1}{a} + 1 - (c_1 + b_1) \right)} T_0 \uparrow \equiv \mathbf{d}_0 + \mathbf{d}_1 T_0 \uparrow \end{aligned}$$

*Note that the sign of  $\mathbf{d}_1$  depends on the relative values of  $a$  and  $b_2$ . if the sign of  $\mathbf{d}_1$  is positive, that is if  $b_2$  higher than  $a$ , then the intersection of the two curves will be moving to the right as we increase taxes. Therefore, in this case, we need to increase taxes to restore the original GDP, and the new equilibrium interest rate will be lower than the original (see the graph below for this case). The opposite when the sign is negative.*

*The intuition is simple: if  $b_2$  is high compared to  $a$ , then changing taxes affects the IS curve a lot (investment reacts to changes in interest rate), but the demand for money*

*reacts relatively little (little impact of changes in consumption on money demand) and the LM curve does not move much.*

