14.02 Principles of Macroeconomics
Problem Set 2 *Solution*
Fall 2004

Part I. True/False/Uncertain
Justify your answer with a short argument.

1. Paradox of saving occurs when the attempts by people to save more lead to a decline in output & an increase in saving.

   *False.* Y ↓ S (no change)  (page 60)

   \[ S = Y_0 - C \]
   \[ S = Y - T - C \]

   Private Saving:      Eqm. Condition:  
   \[ S = Y_0 - C \]  \[ Y = Z \]  \[ S = Y - T - C \]  \[ Y = C + I + G \]

   Why Y ↓ ?
   \[ S = Y - T - C \]
   \[ S = - C_0 + (1 - c_1)(Y - T) \]

   when \( C_0 \) ↓

   1. \([-c_0]\) ↑ → S ↑
   2. \( C_0 \) ↓ → C ↓ → Z ↓ → Y ↓ → S ↓

   * Consumers' decision to save more can't affect I, G, or T. (by assumption)

   i.e. We know S did not change

   (see eqn S.12) → book

2. When mpc increases and investment decreases, goods market equilibrium output increases.

   *Uncertain* (graph 3-2)

   Goods Market: \[ Z = C + I + G \]
   \[ Z = C_0 + c_1(Y - T) + I + G \]

   \[ mpc : \text{marginal propensity to consume} \quad (c_1) \]

   \[ Z \]

   \[ c_1 \quad \text{slope of ZZ} \]

   \( c_1 \) ↑ makes ZZ steeper

   \( S_0, Y \) ↑

   \[ I \] ↓ → ZZ shifts down

   \[ S_0, Y \] ↓

   Depends on how much \( \Delta I \) & \( \Delta c_1 \)
3. If investment is really sensitive to changes in the interest rate (b₁ large), then IS is flatter and fiscal policy is more effective.

**False.** When investment is really sensitive to changes in the interest rate, then IS is flatter but fiscal policy is less effective. This is because there will be more crowding out of investment, and therefore an increase in government spending will be less effective.

\[
I = I(Y, i) \\
I = b₁Y - b₂i \\
\text{investment sensitivity}\hspace{1cm}\text{to } \Delta i
\]

\[\Delta I\]

\[\Delta i\]

\[\text{IS}\]

\[Y_0\]

\[Y\]

\[\text{let } b₁ \text{ be large.}\]

\[\text{start at } A.\]

\[\text{If } b₁ \text{ is large, then } I \text{ rises}\]

\[I \text{ rises} \rightarrow \epsilon \text{ rises} \rightarrow Y \text{ rises}\]

\[b₁ \text{ large means for a given } \Delta i, \Delta I \text{ is large and thus } \Delta Y \text{ is large.}\]

\[\text{IS is Flatter.}\]

4. The price of bonds increases when the interest rate rises.

**False** (page 74–75)

**Price of Bonds** $\frac{\$100}{1+i}$ if $\$100$ was the face value of a bond

\[i \downarrow \rightarrow P_b \uparrow\]

\[i \uparrow \rightarrow P_b \downarrow\]

* If a bond promises to pay $\$100$ in a year, its face value is $\$100 \& P_b \leq 100$ if $i \geq 0.$
5. Monetary contraction and fiscal expansion increase equilibrium output and interest rate.

Uncertain \( i \uparrow \) but \( \Delta Y \) uncertain (chapter 6)

Monetary Contraction

\[ M^s \downarrow \]

When the Fed \( M^s \downarrow \), \( i \uparrow \).

Given a level of \( Y \), \( i \) is higher so \( LM \) shifts up and left.

* 3 possible outcomes...

Fiscal Expansion

\[ \Delta G \]

When \( \Delta G \uparrow \rightarrow E \uparrow \rightarrow Y \uparrow \)
at a given \( i \), higher \( Y \)

So IS shifts up and right

(Because at a higher \( i \) people demand less money)

\( \Delta Y \)

option 1

\( i \uparrow \)

option 2

\( i \uparrow \)

option 3
6. The money multiplier is always less than 1.

**False.**

\[ 0 < \Theta < 1 \]
\[ \Theta = \text{reserve ratio} \]

\[ 0 < c < 1 \]
\[ c = \text{some constant} \]
\[ c \text{ is the proportion of } M^d \text{ (money demand) people hold as } CU^d \text{ (currency). Since people hold both } CU^d \text{ (currency) and } D^d \text{ (deposits), } c \text{ is between 0 and 1.} \]

Because \[ 0 < \Theta < 1 \] and \[ 0 < c < 1 \], money multiplier \[ \frac{1}{c + \Theta(1 - c)} \] is always greater than 1.

(see chapter 4)
Part II. THE MONEY MARKET
(all units are trillions of US $)

Money Demand: \( M^d = SY (0.2 - i) \)
Nominal Income: \( SY = 2000 \)
Money Supply: \( M^s = 300 \)

1. Find \( M^d \) for \( i = 10\% \) and \( i = 5\% \).
   \[
   i = 10\% \quad \rightarrow \quad M^d = 200 = 2000 (0.2 - 0.1)
   
   i = 5\% \quad \rightarrow \quad M^d = 300 = 2000 (0.2 - 0.05)
   \]

2. What is the relationship between \( i \) and \( M^d \).
   
   \( i \) and \( M^d \) are inversely related. A higher interest rate leads to a lower demand for money, and vice versa. This can be expressed as:
   
   \[ M^d = M^d (Y, i) \]

3. Graph \( M^s \) and \( M^d \).
   
   

4. Alan Greenspan decreases \( M^s \) by 50. What happens to money market equilibrium? (solve & graph)

   \[ E_{eq} \rightarrow M^s = M^d \]

   \[
   250 = 2000 (0.2 - i) \\
   i = 0.075
   
   \[ i = 7.5\% \]
   \]
5. Describe how the Fed changes \( i \) in the U.S.

The Fed can \( \uparrow i \) by \( \downarrow M^s \) (money supply).
The Fed can \( \downarrow i \) by \( \uparrow M^s \).

\[
\begin{align*}
M^s_{\text{new}} &\quad \text{M}^s_{\text{old}} \\
\text{M} &\quad \text{M} \\
i &\quad \text{i} \\
i &\quad \text{i} \
id &\quad \text{\uparrow} \quad \text{M}^s &\quad \text{\downarrow}
\end{align*}
\]

Part III. Money Multiplier

Checkable deposits: \( D^d = $900 \) billion
Total money supply: \( M^s = $1800 \) billion
Reserve ratio: \( \theta = 0.2 \)

1. Find \( CU^d \), \( R^d \) and \( D^d \) in equilibrium.

\[
\begin{align*}
M^d &= CU^d + D^d \\
M^s &= M^d \quad \text{(in eqm)} \\
1800 &= CU^d + 900 \\
CU^d &= 900
\end{align*}
\]

\[
\begin{align*}
R^d &= \theta D^d \\
R^d &= 0.2(900) \\
R^d &= 180
\end{align*}
\]

\[
\begin{align*}
D^d &= D^d \\
D^d &= 900
\end{align*}
\]

note: \( (1-c) M^d = P^d \)

2. Find the money multiplier.

\[
\begin{align*}
H^d &= CU^d + R^d \\
H^d &= CM^d + \theta (1-c) M^d \\
H^d &= [C + \theta (1-c)] M^d \\
H^d &= \left[ \frac{1}{C + \theta (1-c)} \right] M^d
\end{align*}
\]

\[
\begin{align*}
\text{money multiplier} &\quad \text{mm} = \left[ \frac{1}{C + \theta (1-c)} \right] M^d \\
\text{mm} &= \frac{1}{0.5 + 0.2(0.5)} \\
\text{mm} &= 1.67
\end{align*}
\]

*when the Fed \( \uparrow M^s \) by \$100, 
the overall \( M^s \) \( \uparrow \) by \$167
(See page 82 & 83)
3. Describe 2 different ways the Fed can decrease money supply.

1. The Fed can sell bonds thru open market operations.
   This ↓ Ms. (This decreases the Ms and increases i.)

2. ↑θ (The Fed can raise reserve ratio)

4. If the Fed wants to decrease the money supply by $500 million (in order to raise i), what amount of bonds would it have to sell/buy?

   \[ mm = 1.67 \text{ (from part 2)} \]
   \[ (mm = \text{money multiplier}) \]
   If the Fed wants the overall money supply to ↓ by 500, it initially needs to ↓ Ms (by selling bonds) by less than 500 due to the money multiplier.

   Initially, the Fed will sell about $300 million worth of bonds.

   \[ $300 \times mm = $300 \times 1.67 \approx $500 \text{ million}. \]

* Make sure you can explain how the money multiplier works.
  (Page 82-83)
Part IV. IS - LM
(All units are millions of US dollars)

\[
C = 200 + (0.25)Y_D \\
I = 150 + 0.25Y - 1000i \\
T = 200 \\
G = 250 \\
(M/P)^s = 1600 \\
(M/P)^d = 2Y - 8000i
\]

1. Find the equation for aggregate demand (Z).

\[
Z = C + I + G \\
= 200 + (0.25)Y_D + 150 + 0.25Y - 1000i + 250 \\
= 600 + 0.25(Y - 200) + 0.25Y - 1000i \\
= 550 + 0.5Y - 1000i
\]

\[
Z = 550 + 0.5Y - 1000i
\]

2. Derive the IS equation.

\[
\text{IS eqn} \quad \rightarrow \quad \text{Goods market eqn} \quad \rightarrow \quad Y = Z
\]

\[
Y = Z \\
= 550 + 0.5Y - 1000i \\
0.5Y = 550 - 1000i \\
Y = 1100 - 2000i \\
i = (1100 - Y)(\frac{1}{2000})
\]

3. Derive the LM equation.

\[
\text{LM eqn} \quad \rightarrow \quad \text{money (financial) market eqn} \quad \rightarrow \quad M^s = M^d
\]

\[
M^s = M^d \\
\frac{(M^s)}{P} = \frac{(M^d)}{P} \\
1600 = 2Y - 8000i \\
2Y = 1600 + 8000i \\
Y = 800 + 4000i \\
i = \frac{Y}{4000} - \frac{1}{5}
\]
4. Solve for equilibrium real output, \( i, Y, C \).

\[ \text{egm} \leftrightarrow \text{where IS & LM intersect} \]

\[ \text{IS: } Y = 1100 - 2000i \]
\[ \text{LM: } Y = 6800 + 4000i \]

\[ Y = 1100 - 2000i = 6800 + 4000i \]
\[ 300 = 6000i \]
\[ 0.05 = i \]

\[ Y = 1100 - 2000(0.05) = 1000 \]

\[ Y^* = 1000 \]
\[ i^* = 0.05 \]
\[ C = 400 \]
\[ I = 350 \]

5. Graph IS-LM of above with correct labels.

when graphing be sure to always have correct axis labels!

6. Monetary expansion:

Let \( M^d \) (nominal money supply) increase to 1840. Find equilibrium \( Y, i, C \) and \( I \). What happens to \( Y, i, C \) and \( I \) when the Fed increases money supply thru open market operations?

\[ \text{Money mkt eqm} \rightarrow \quad M^d = M^d \]
\[ 1840 = 2Y - 8000i \]
\[ 2Y = 1840 + 8000i \]
\[ Y = 920 + 4000i \]

\[ \text{IS-LM eqm} \rightarrow \quad 920 + 4000i = 1100 - 2000i \]
\[ 6000i = 180 \]
\[ i = 3\% \]

\[ Y = 1040 \]
\[ C = 410 \]
\[ I = 380 \]

Expansionary monetary policy reduces \( i \), increases \( Y, C, \) & \( I \).

* notice that IS stayed the same. only LM eqm changed & shifted.
7. Graph part 6 (a new graph starting from part 5).

[Graph with IS-LM equilibrium]

Expansionary
Monetary policy

A = old equilibrium
B = new equilibrium

8. Fiscal expansion: (Continue from part 5)
   Let G increase to 400. Find equilibrium Y, i, C and I. What happens to equilibrium Y, i, C and I when government spending increases?

   Goods mkt equilibrium
   \[ Y = \bar{Y} \]
   \[ Y = 1400 - 2000i \]

   IS-LM
   \[ 1400 - 2000i = 800 + 4000i \]
   \[ 600 = 6000i \]
   \[ 10\% = i \]

   *note: with fiscal expansion, nothing is happening to LM.

9. Graph part 8 (a new graph starting from part 5).

[Graph with IS-LM equilibrium]
10. There is a sudden drop in consumer confidence and $c_0$ drops from 200 to 100. How can the government counterbalance the drop in GDP using government spending as a policy instrument?

\[ c_0 \downarrow \rightarrow c \downarrow \rightarrow z \downarrow \rightarrow IS \text{ shifts down and to the left} \]

The government can:

\[ z \uparrow \text{ by } 0 \uparrow \text{ by 100} \rightarrow z \uparrow \rightarrow y \uparrow \]

\[ \Theta \downarrow \text{ by } T \rightarrow Y_d \uparrow \rightarrow c \uparrow \rightarrow z \uparrow \rightarrow y \uparrow \]