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## 14.02 Principles of Macroeconomics

### Problem Set 5 Solutions

Fall 2004

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#### Part I. True/False/Uncertain

Justify your answer with a short argument.

1. A higher saving rate alone can sustain higher growth of output forever.

False. A higher saving rate alone cannot sustain higher growth of output forever. When the saving rate increases, output per worker increases until a new steady state is reached. Once the higher level of output per worker is reached (the new steady state), growth is equal to zero. (See pages 225-228)

2. The golden-rule level of capital tells us that the highest level of consumption in steady-state is achieved when the saving rate is equal to 0.

False. An economy in which the saving rate is zero is an economy in which capital is equal to zero. In this case, output is also equal to zero, and so is consumption. As saving rate increases, values of capital per worker, output per worker, and consumption per worker will also increase. However, if the saving rate is equal to 1, people save all their income, and consumption is also equal to zero. Therefore, the saving rate that maximizes the steady-state level of consumption is somewhere between 0 and 1. (See pages 229-230)

3. A flexible exchange rate regime is superior to a fixed exchange rate regime.

Uncertain. (See chapter 21)

Reasons against fixed exchange rate regime:

First, countries that operate under fixed exchange rates and perfect capital mobility give up two macroeconomic instruments, the interest rate and the exchange rate (they give up monetary policy). This not only reduces their ability to respond to shocks, but may also lead to exchange rate crises. Second, the anticipation that a country that operates under a fixed exchange rate may have to devalue leads investors to ask for very high interest rates, making the economic situation worse, and putting more pressure on the country to devalue.

Reasons for fixed exchange rate regime:

When a group of economies are highly integrated, then adopting a common currency (fixed exchange rate regime) may be the right thing to do. Even though countries' give up independent monetary policy (control of its own domestic interest rate), adopting a common currency (like the EU) maybe be advantageous since it cuts transaction costs of exchanging money. In addition, international trade is much easier for buyers and sellers since goods are priced in the same currency. This may spur competition among firms which benefits the consumers. Another reason for adopting a fixed exchange rate is when a country needs to limit its own ability to use monetary policy in order to convince the market (the investors) to invest in within its borders.

When a currency depreciates a lot, then, investors lose out since the return to their investment denominated in that currency loses value. So, in order to attract investment, a country with a high inflation (which leads to real depreciation) may want to adopt a fixed exchange rate scheme such as a hard peg.

Reasons against flexible exchange rate regime:

Under a flexible exchange rate regime, the exchange rate may move a lot which may destabilize the economy too much. Also, because of the J-curve effect, the adjustment process to medium-run (long-run) equilibrium may take a while. So, monetary or fiscal policies' effects may be delayed. For example, when the central bank increases the nominal money supply to increase output, in the short-run,  $Y$  may actually decrease before it starts to increase since  $NX$  may drop initially.

Reasons for flexible exchange rate regime:

Independent monetary policy which allows the country to control its own domestic interest rate.

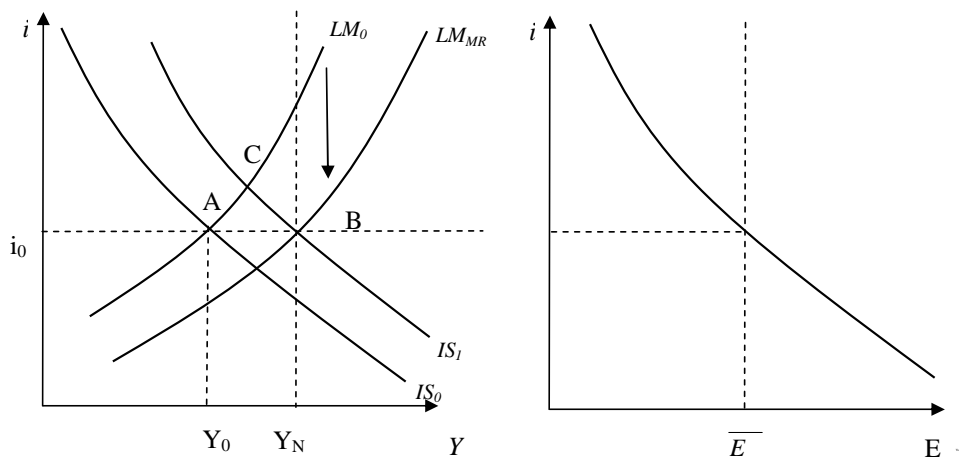
## Part II. Open-Economy AS-AD

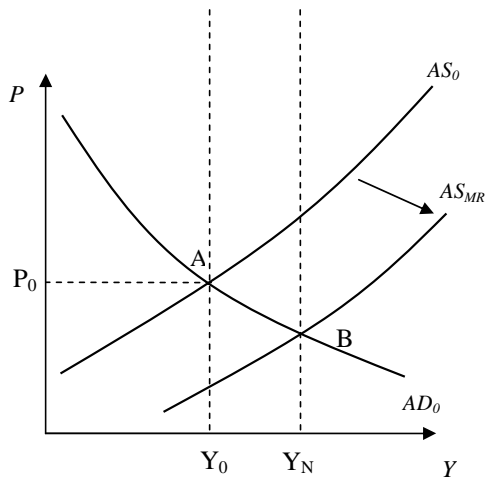
Real exchange rate: 
$$\varepsilon = \frac{EP^*}{P}$$

i- rate parity condition: 
$$i = i^* + \left[ \frac{E^e}{E} - 1 \right]$$

IS: 
$$Y = C(Y, T) + I(Y, i) + G + NX(Y, Y^*, \varepsilon)$$

- Suppose the economy is at point A where  $Y_0 < Y_N$ . If  $E = \bar{E}$ , what happens to  $Y$ ,  $NX$ ,  $P$ , and  $\varepsilon$  over time? Explain the intuition and also show graphically using IS-LM, AS-AD, and interest-rate parity curves.





At point A,  $Y_0 < Y_N$ , so  $u_0 > u_N$ . This means that the economy is not performing at its capacity. So, people expect the price level to decrease in the near future. When  $P^e$  decreases,  $W$  decreases since the wage setting relation tells us that  $W = P^e F(u, z)$ . A decrease in  $W$  decreases  $P$  since the price setting relation tells us that  $P = (1 + \mu) W$ . As  $P^e$  decreases AS shifts to the right and down.

What is happening to the IS-LM curves?

When  $P$  decreases, real exchange rate increases if  $E$  is fixed. Real depreciation leads to a NX increase. This shifts the IS curve to the right. Notice that at point C, the domestic interest rate has increased. This means that  $E$  will have to decrease if interest-parity condition holds. However, the original premise of the question was that we were in a fixed exchange rate regime. So, something must happen in order to preserve  $E$  at  $\bar{E}$  when domestic demand increases as a result of real depreciation.

A decrease in  $P$  also shifts LM curve down since a decrease in  $P$  increases the real money supply. (If the increase in real money supply due to a decrease in  $P$  is not enough to offset the increase in interest rate, then the central bank has to increase the nominal money supply in order to shift LM to  $LM_{MR}$ .)

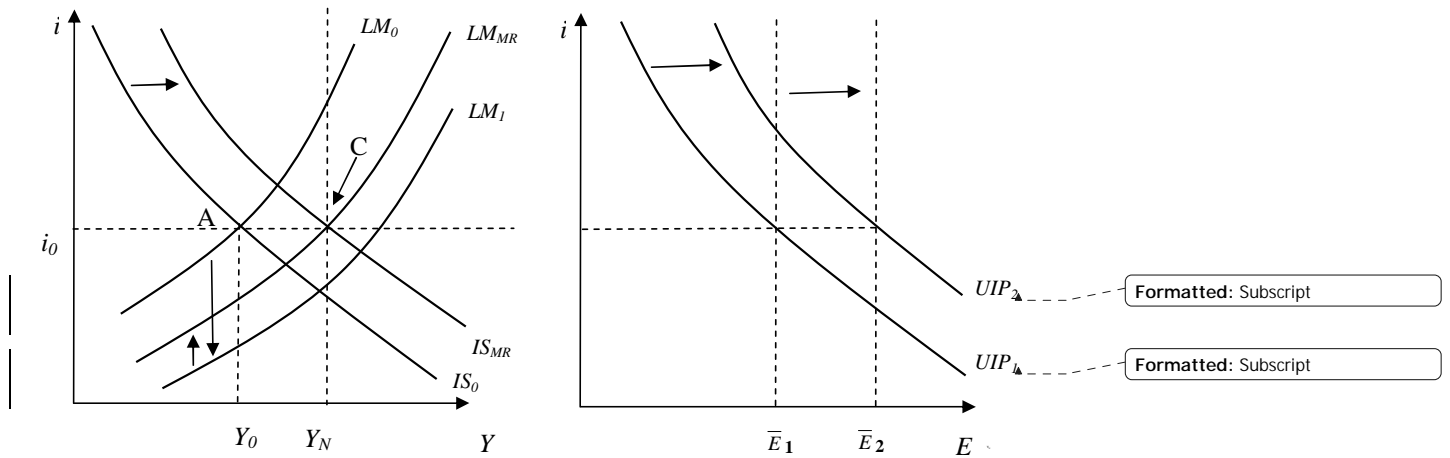
This process continues until the economy reaches the MR/LR equilibrium where  $Y = Y_N$  and  $u = u_N$ . Point B is the medium-run (long-run) equilibrium. Note that NX increase in this case. A decrease in  $P$  is the same as an increase in the real exchange rate. By Marshall-Lerner condition, an increase in real exchange rate increases NX.

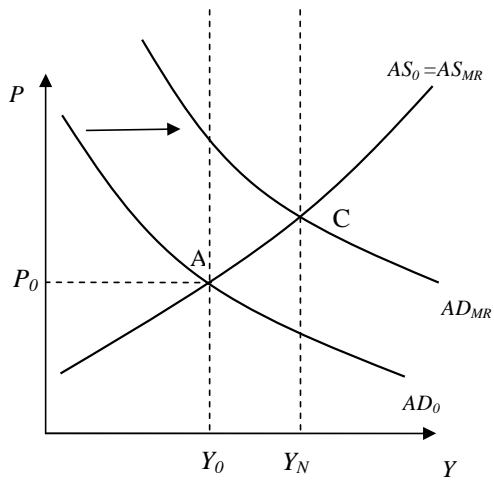
This is what happens if the central bank or the government does nothing (or only accommodates the change in IS) to speed up the process of adjustment to the medium-run equilibrium. The problem with this is that this adjustment may take a long time. So, in the meanwhile, the economy is performing at under-capacity and the unemployment is high. Therefore, there is an incentive for the central bank and/or the government to act so that the adjustment process is faster.

2. Now, suppose the central bank wants to intervene to speed up the adjustment process from point A to point C, the medium-run (long-run) equilibrium. What can it do? Explain with words and also show graphically.

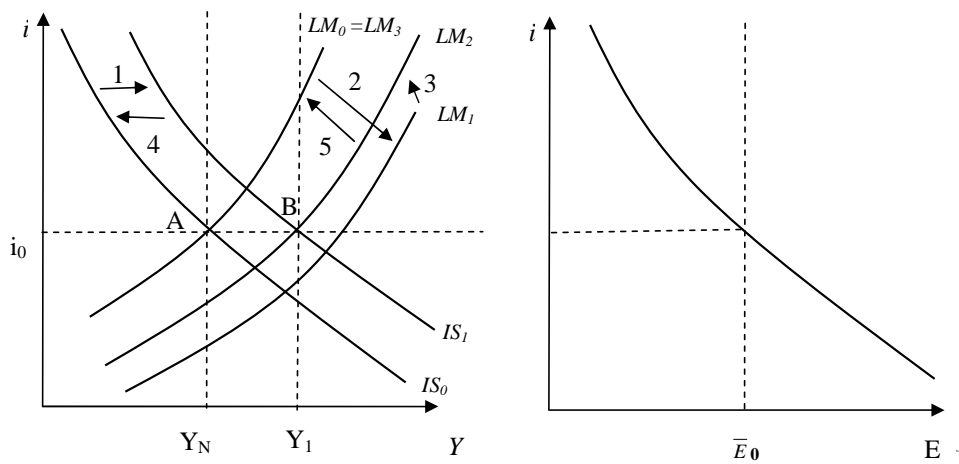
If a country has a credible fixed exchange rate system, then its announcement of a one-time devaluation is also credible. So, the central bank can announce a one-time devaluation of its currency and is believed by the investors. Credibility means that investors expect the exchange rate of this country to increase and be fixed at its new level. So,  $E^e$  increases by the amount of the announcement. This shifts the interest-rate parity condition curve to the right/up. Since the central bank has not changed its money supply, today's exchange rate increases to  $\bar{E}_2$ .

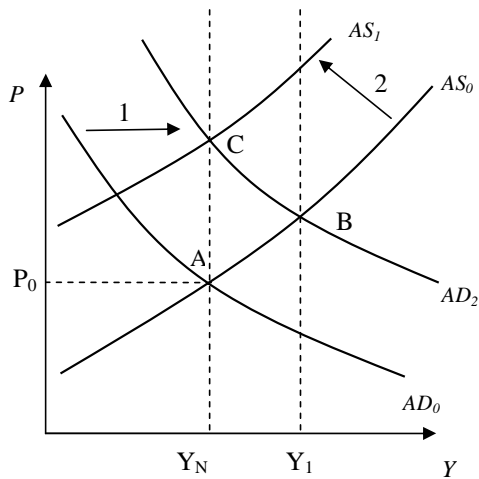
When the exchange rate increases, then NX increases due to the Marshall-Lerner condition. This means that the IS curve will shift to the right. This also shifts the AD curve to the right and P starts to rise. In a fixed exchange rate regime, monetary policy must accommodate. The central bank now must increase the money supply so that the exchange rate does not change from  $\bar{E}_2$ . Notice how the central bank actually increases the money supply, so that the LM curve shifts to  $LM_1$ , but the increase in P reduces the real money supply to  $LM_{MR}$  curve level. (Note that if the central bank does nothing to the money supply, then the interest rate will change.)





3. Now, assume that this economy is where  $P=P^e$  and  $E=\bar{E}$ . What happens to  $Y$ ,  $i$ , and  $P$  if the government increases  $G$  in the short-run and the medium-run (long-run)? Explain with words and also show graphically.





Start at point A where  $P=P^e$  and  $Y=Y_N$ . When  $G$  increases, IS shifts to the right. In order to maintain a fixed exchange rate at  $E_0$ , the central bank must increase  $M^s$  when  $G$  increases. There are two separate things happening here. The central bank increases the nominal money supply. This shifts the LM curve to the right/down to  $LM_1$ . But, the IS curve is shifting to the right, which also shifts AD to the right. This increases  $P$ . This in turn decreases the real money supply and shifts the LM to the left a little (to  $LM_2$ ). So, the short-run equilibrium is at point B where the  $IS_1$  and  $LM_2$  curves intersect.

At point B (the short-run equilibrium), the economy is at over-capacity since  $Y_N < Y_1$ . This means that people expect the price level to increase. This, in turn, increases  $W$ , which increases  $P$ . When  $P^e$  increases, 3 separate things happen all at once. First, the AS curve shifts up and to the left. Second, the LM curve shifts up and to the left also since the real money supply decreases as  $P$  increases. (The central bank has to decrease the nominal money supply if the decrease in the real money supply due to an increase in  $P$  is not enough to keep the domestic interest from changing.) Third, the real exchange rate decreases (US dollar increases in value) even though  $E$  (the nominal exchange rate) is fixed when  $P$  increases. By the Marshall-Lerner condition, we know that  $NX$  decreases. This shifts the IS curve to the left. (Note: This shift of the IS curve does not affect AD since it was caused by the change in  $P$ .  $P$  is on the axis of the AS-AD diagram, so this is a movement along the AD curve.). So, the medium-run equilibrium is at point C where the  $AD_1$  and  $AS_1$  curves intersect and the  $IS_0$  and  $LM_0$  curves intersect.

### Part III. Solow Model of Growth

Suppose that the production function is given by  $Y_t = 0.5 \sqrt{K_t} \sqrt{N}$ .

Assume that the size of the population, the participation rate, and the unemployment rate are all constant.

1. Is this production function characterized by constant returns to scale? Explain.

Yes.

If  $\lambda Y = F(\lambda K, \lambda N)$ , where  $\lambda$  is an arbitrary parameter, then we say that the production function is characterized by constant returns to scale. This means that if we triple both the number of worker and the amount of capital in the economy, the output will triple also.

$$F(K, N) = 0.5 \sqrt{K} \sqrt{N}$$

$$F(\lambda K, \lambda N) = 0.5 \sqrt{\lambda K} \sqrt{\lambda N} = \lambda 0.5 \sqrt{K} \sqrt{N} = \lambda F(K, N) = \lambda Y$$

$$F(\lambda K, \lambda N) = \lambda Y$$

2. Transform the production function into a relationship between output per worker and capital per worker.

$$Y_t = F(K_t, N) = 0.5 \sqrt{K_t} \sqrt{N}$$

Since this production function is CRS (constant returns to scale),

$$\frac{Y_t}{N} = F\left(\frac{K}{N}, \frac{N}{N}\right) = F\left(\frac{K}{N}, 1\right) = f\left(\frac{K}{N}\right) = 0.5 \sqrt{\frac{K}{N}} \sqrt{\frac{N}{N}} = 0.5 \sqrt{\frac{K}{N}}$$

$$\boxed{\frac{Y_t}{N} = f\left(\frac{K}{N}\right) = 0.5 \sqrt{\frac{K}{N}}}$$

3. Derive the steady state level of capital per worker in terms of the saving rate ( $s$ ) and the depreciation rate ( $\delta$ ).

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = (s) f\left(\frac{K^*}{N}\right) - (\delta) \left(\frac{K^*}{N}\right) \quad (\text{equation 11.3, page 223})$$

This equation describes what happens to capital per worker from year  $t$  to  $(t+1)$ .

In steady state (SS), we know that capital stock does not change.

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = 0 \rightarrow s f\left(\frac{K^*}{N}\right) = \delta \left(\frac{K^*}{N}\right) \rightarrow s \left(\frac{1}{2}\right) \left(\sqrt{\frac{K^*}{N}}\right) = \delta \left(\frac{K^*}{N}\right) \rightarrow \frac{1}{2} \left(\frac{s}{\delta}\right) = \sqrt{\frac{K^*}{N}}$$

$$\rightarrow \boxed{\frac{K^*}{N} = \frac{1}{4} \left(\frac{s^2}{\delta^2}\right)}$$

4. Derive the equations for steady-state output per worker and steady-state consumption per worker in terms of  $s$  and  $\delta$ .

$$\frac{Y_t}{N} = \frac{1}{2} \sqrt{\frac{K_t}{N}} \rightarrow \frac{Y^*}{N} = \frac{1}{2} \sqrt{\frac{K^*}{N}} \rightarrow \frac{Y^*}{N} = \frac{1}{2} \sqrt{\frac{1}{4} \left( \frac{s^2}{\delta^2} \right)} \rightarrow \boxed{\frac{Y^*}{N} = \frac{1}{4} \left( \frac{s}{\delta} \right)}$$

$$\frac{C_t}{N} = \frac{Y_t}{N} - \delta \left( \frac{K_t}{N} \right) \rightarrow \frac{C^*}{N} = \frac{Y^*}{N} - \delta \left( \frac{K^*}{N} \right) \rightarrow \frac{C^*}{N} = \frac{Y^*}{N} - s \left( \frac{Y^*}{N} \right) \rightarrow$$

$$\frac{C^*}{N} = \frac{1}{4} \left( \frac{s}{\delta} \right) - \frac{1}{4} \left( \frac{s^2}{\delta} \right) \rightarrow \boxed{\frac{C^*}{N} = \frac{1}{4} \left[ \frac{s}{\delta} (1-s) \right]}$$

5. Let  $\delta = 0.08$  and  $s = 0.16$ . Calculate the steady-state output per worker, capital per worker, and consumption per worker.

$$\frac{Y^*}{N} = \frac{1}{4} \left( \frac{0.16}{0.08} \right) = \frac{1}{2}$$

$$\frac{K^*}{N} = \frac{1}{4} \left( \frac{0.16}{0.08} \right)^2 = 1$$

$$\frac{C^*}{N} = \frac{1}{4} \left( \frac{0.16}{0.08} \right) (0.84) = 0.41$$

6. Let  $\delta = 0.08$  and  $s = 0.32$ . Calculate the steady-state output per worker, capital per worker, and consumption per worker.

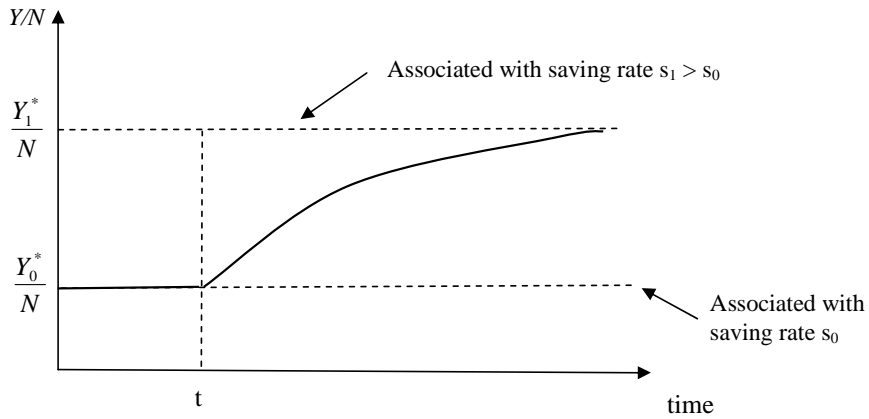
$$\frac{Y^*}{N} = \frac{1}{4} \left( \frac{0.32}{0.08} \right) = 1$$

$$\frac{K^*}{N} = \frac{1}{4} \left( \frac{0.32}{0.08} \right)^2 = 4$$

$$\frac{C^*}{N} = \frac{1}{4} \left( \frac{0.32}{0.08} \right) (0.68) = 0.68$$

7. What is the effect of an increase in the saving rate on output per worker over time? Show the transition from  $s_0$  to  $s_1$  graphically.





8. Explain what happens to the level of output per worker and the growth of output per worker when the saving rate increases from  $s_0$  to  $s_1$ .

When the saving rate increases from  $s_0$  to  $s_1$ , output per worker increases from  $\frac{Y_0^*}{N}$  to  $\frac{Y_1^*}{N}$ . Once output per worker reaches  $\frac{Y_1^*}{N}$ , the new steady-state output per worker, then output per worker stops growing. A higher saving rate cannot sustain growth forever.