
14.02 Principles of Macroeconomics
Problem Set 6
Solutions
Fall 2004

Part I. True/False/Uncertain

Justify your answer with a short argument.

1. In an economy with technological progress, the saving rate is irrelevant in the long-run.

False. It is true that the rate of capital and output growth in steady state is independent of the saving rate. However, the saving rate does affect the steady-state level of output per effective worker. Suppose there is an increase in the saving rate. Then, capital per effective worker and output per effective worker will increase for some time as they converge to their new higher level. Their growth rate will eventually return to the sum of the population growth rate and the technological growth rate ($g_A + g_N$). (See pages 250-251.)

2. The rate of technological progress declined in the mid-1970s due to the rise of the service sector.

Uncertain. Why the rate of technological progress declined in the mid-1970s is one of the unanswered questions in economics. The explanation that involves the rise of the service sector is only one of a few different hypotheses that have been proposed. Other explanations include decreased R&D spending (a decline not in the amount, but rather in the fertility of R&D) or measurement error (that, in fact, there was no slowdown in the rate of technological progress at all). (See pages 256-258.)

3. There is much theoretical and empirical support for the idea that faster productivity growth leads to higher unemployment.

False. In the short-run, there is no reason to expect, nor does there appear to be, a systematic relationship between movements in productivity growth and movements in unemployment. In the medium- /long-run, if there is a relation between productivity growth and unemployment, it appears to be an inverse relationship. Lower productivity growth leads to higher unemployment and vice versa. (See pages 277-278.)

4. Increased globalization and technological progress can both be blamed, at least to some extent, for the increase in wage inequality in the United States.

True. An increase in globalization leads to an increase in international trade. Those firms that employ higher proportions of low-skilled workers are increasingly driven out of the market by imports from similar firms in low-wage countries, according to this argument. The other line of argument focuses on skill-biased technological progress. New machines and new methods of production require higher-skilled workers. (See pages 280-281.)

- The Fisher hypothesis states that, in the short-run, the nominal interest rate increases one for one with inflation, while the real interest rate remains unchanged.

False. In the short-run, the increase in nominal money growth leads to an increase in the real money stock. This translates into a decrease in the real, as well as in the nominal, interest rate. (See page 302.) The Fisher hypothesis states that in the medium-/long-run the nominal interest rate increases one for one with inflation, while the real interest rate remains unchanged.

Part II. The Solow Model Revisited

The Republic of Solowakia has the following production function:

$$Y = F(K, N) = AK^\alpha N^{1-\alpha}, \text{ where } \alpha < 1.$$

Let g_A be the growth rate of A, g_N be the growth rate of N, and δ be the rate of depreciation in this economy.

- Interpret the parameter A. Empirically, what has happened to A over time?

A is a technological parameter. It tells us how much output can be produced from given amounts of capital and labor at any time. (See page 244.) Although the book does not directly provide us with data on the parameter A, we see that the rate of growth of A (the rate of technological progress) given in Table 12-2 has been positive at least for years 1950-1987 for France, Germany, Japan, the UK, and the US. Over a longer period of time, the increase in A has been even more dramatic, taking into account the Agricultural Revolution and the Industrial Revolution.

- Verify that the above production function has the property of constant returns to scale.

We must verify that $f(\lambda K, \lambda N) = \lambda f(K, N)$, that is, if you multiply all inputs by a scalar, you will end up multiplying output by the same amount.

$$f(\lambda K, \lambda N) = A(\lambda K)^\alpha (\lambda N)^{1-\alpha} = \lambda^{\alpha+1-\alpha} AK^\alpha N^{1-\alpha} = \lambda AK^\alpha N^{1-\alpha} = \lambda f(K, N).$$

- Verify that the above production function is concave in capital (which means that the second derivative is negative). What does that mean in economics terms?

For this we must take the partial derivative of $F(\cdot)$ with respect to K.

$$\frac{\partial F(K, N)}{\partial K} = A\alpha K^{\alpha-1} N^{1-\alpha}$$

$$\frac{\partial^2 F(K, N)}{\partial K^2} = A\alpha(\alpha-1)K^{\alpha-2} N^{1-\alpha} < 0 \text{ because } \alpha < 1. \text{ Therefore, } F(\cdot) \text{ is concave in K.}$$

This means that this production function exhibits diminishing returns to capital (increases in capital lead to smaller and smaller increases in output as the level of capital increases).

- What is effective labor in this economy?

Let's rewrite the production function in the following fashion:

$$F(K, N) = K^\alpha (A^{\frac{1}{1-\alpha}} N)^{1-\alpha}$$

Let's redefine the technological parameter as $\tilde{A} = A^{\frac{1}{1-\alpha}}$. (Note that we can do that, because A is just a constant in this model.)

Then, effective labor is $\tilde{A}N$.

5. At what rate do output and capital grow in Solowakia in the long run? (Hint: use $g_{\tilde{A}}$ for the transformed g_A .)

We will first calculate the growth rates mathematically (you didn't have to do this). Let's start with the transformed production function, $F(K, N) = K^\alpha (\tilde{A}N)^{1-\alpha}$ and take logarithms of both sides:

$$\log Y = \alpha \log K + (1-\alpha) \log \tilde{A} + (1-\alpha) \log N$$

Taking the time derivative of the above equation, we get the following expression:

$$\frac{\partial \log Y}{\partial t} = \alpha \frac{\partial \log K}{\partial t} + (1-\alpha) \frac{\partial \log \tilde{A}}{\partial t} + (1-\alpha) \frac{\partial \log N}{\partial t}$$

But $\frac{\partial \log Y}{\partial t}$ is the growth rate of output (Y), g_Y ; $\frac{\partial \log \tilde{A}}{\partial t}$ is the growth rate of \tilde{A} , $g_{\tilde{A}}$;
 $\frac{\partial \log K}{\partial t}$ is the growth rate of capital (K), g_K ; and $\frac{\partial \log N}{\partial t}$ is the population growth rate, g_N .

So, we have the following decomposition of output growth into growth driven by inputs and growth driven by technology:

$$g_Y = \alpha g_K + (1-\alpha)(g_{\tilde{A}} + g_N)$$

Because in steady-state we are on a balanced-growth path, we know that g_Y must equal g_K . So we have,

$$g_Y = \alpha g_Y + (1-\alpha)(g_{\tilde{A}} + g_N)$$

$$g_Y (1-\alpha) = (1-\alpha)(g_{\tilde{A}} + g_N)$$

$$g_Y = g_{\tilde{A}} + g_N$$

$$g_K = g_{\tilde{A}} + g_N$$

Output and capital both grow at the rate $g_{\tilde{A}} + g_N$. Intuitively, this is because the growth rate of effective labor equals $g_{\tilde{A}} + g_N$. It cannot be $g_A + g_N$, because we have transformed the A parameter. In the long run steady state, in this economy, what is constant is not output but rather output per effective worker. This implies that output grows at the same rate as effective labor. Same logic applies to capital. (See pages 247-248.)

6. Rewrite the production function in terms of only capital per effective worker.

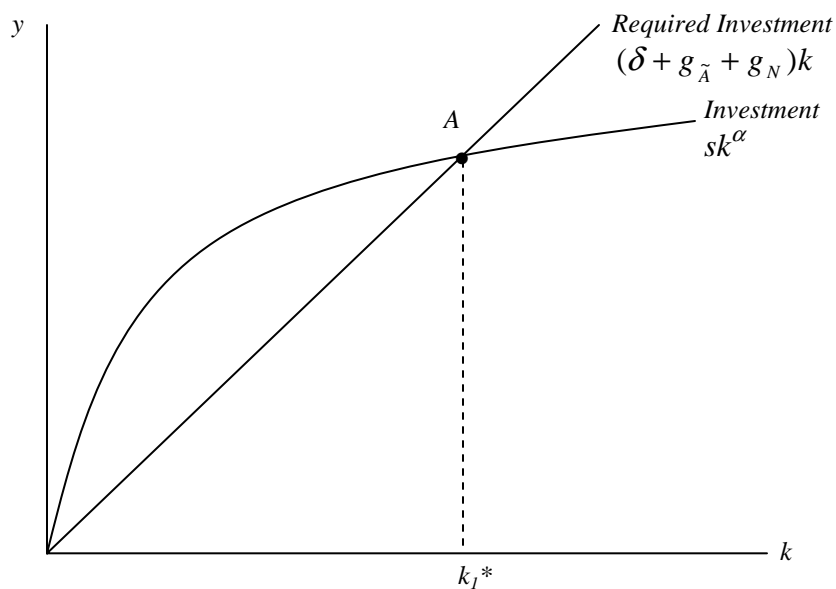
$$F(K,1) = \left(\frac{K}{\tilde{A}N}\right)^\alpha \left(\frac{\tilde{A}N}{\tilde{A}N}\right)^{1-\alpha} = \left(\frac{K}{\tilde{A}N}\right)^\alpha$$

Let $k = \frac{K}{\tilde{A}N}$ and $y = \frac{Y}{\tilde{A}N}$.

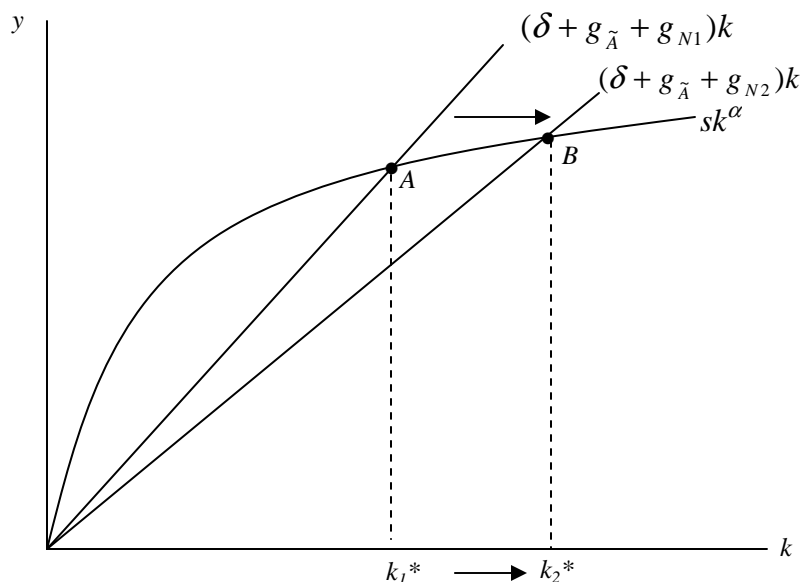
Define f so that $f(k) \equiv F(k,1)$.

Then, $y = f(k) = k^\alpha$.

7. Draw the diagram that plots required investment and investment for this economy. Label the steady-state level of capital per effective worker k_1^* . Label the steady state as point A on the diagram.

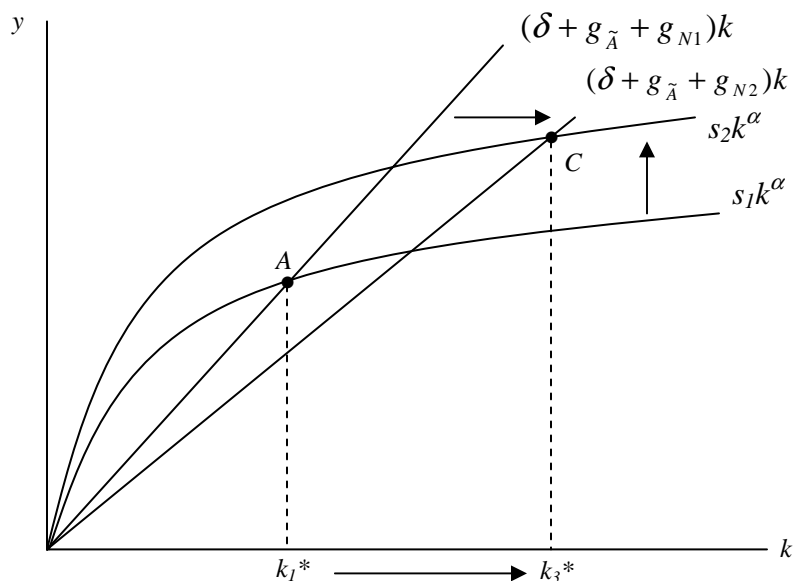


8. Suppose the rate of population growth falls. Use a diagram to analyze what happens in Solowakia. Label the new steady state capital per effective worker k_2^* . Label the new steady state as point B.



As g_N decreases ($g_{N2} < g_{N1}$), less capital is required per effective worker, and therefore we achieve a higher steady state capital per effective worker k_2^* .

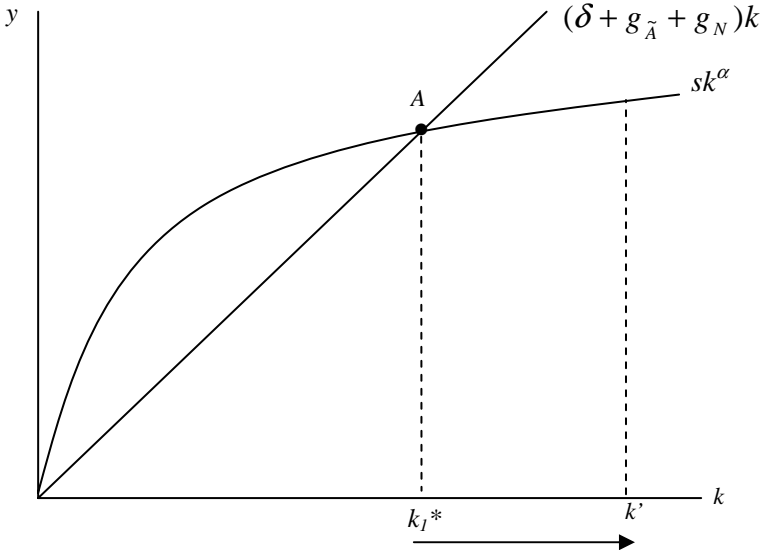
9. Starting at the steady state in part 7 (point A), suppose the saving rate increases and the depreciation rate decreases at the same time. Use a diagram to analyze what happens in Solowakia. Label the new steady state capital per effective worker k_3^* . Label the new steady state as point C.



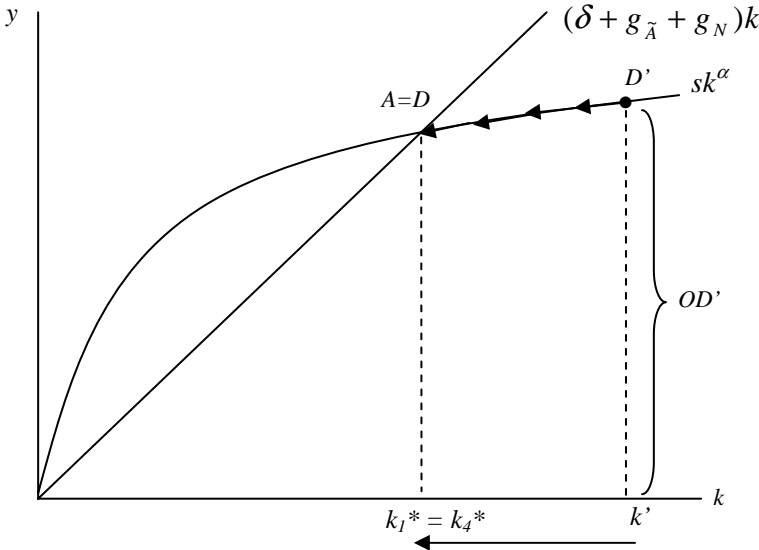
Here we have a double shift. A higher saving rate shifts the investment relation up. It follows that the steady-state level of capital per effective worker increases. A lower depreciation rate shifts out the required investment relation, which also leads to a higher steady-state capital per effective worker.

10. Starting at the steady state in part 7 (point A), suppose a neighboring country gives Solowakia a gift of new capital equipment. Use a diagram to analyze what happens in the long run. Label the new steady state capital per effective worker k_4^* . Label the new steady state as point D.

If we were given a gift of capital, capital would (immediately) jump up to a level such as k' in the short-run.



However, in the long run, the dynamics will bring the economy back to the original steady-state level of capital per effective worker, k_1^ . Why? When we receive the gift of capital, we*



end up at point D', where investment per effective worker equals the vertical distance OD'. The amount of investment required to maintain that level of capital per effective worker is clearly greater than the amount OD'. (Note that for all $k > k_1^$ the line $(\delta + g_A + g_N)k$ is above the curve sk^α .) Because investment that is required to maintain the existing level of capital*

per effective worker exceeds actual investment at D', k decreases. Hence, starting from k' , the economy moves to the left, with the level of capital per effective worker decreasing over time. This continues until investment per effective worker is just sufficient to maintain the existing level of capital per effective worker, that is until we return to the initial steady-state, A. (See page 248.) So, the effect of the gift will only be temporary (because nothing happened to investment or required investment).

Part III. Expected Present Discounted Values

1. Suppose that the interest rate is 5% today and is expected to stay at 5% for the next three years. Calculate the present discounted value of a security that pays \$10,500 in one year, \$11,025 in two years, \$23,152.5 in three years, and nothing thereafter.

$$PDV = \frac{10,500}{1+0.05} + \frac{11,025}{(1+0.05)^2} + \frac{23,152.5}{(1+0.05)^3} = 40,000$$

It is not $10,500 + 11,025/1.05 + \dots$ because the security pays \$10,500 not today, but a year from now.

Note also that the fact that this security stops making payments after the first three years means that its maturity is 3 years.

2. Suppose the security in part 1 sells for \$38,342.08 today. Would its discount rate need to be smaller or higher than 5%? Answer without computing it first. Then calculate the discount rate. Assume again that the interest rate is expected to remain the same for the next three years.

If the security is sold at \$38,342.08 (which is less than \$40,000), the discount rate is higher than 5%. Recall that the price of a bond (which is basically the same as present discounted value) and the interest rate move in opposite directions.

$$\$V = \frac{10,500}{1+i} + \frac{11,025}{(1+i)^2} + \frac{23,152.5}{(1+i)^3} = \$38,342.08$$

$$i=7\%$$

Note that this discount rate is called yield to maturity. It is the rate of return on cash flow of a security that is held until it matures (3 years).

3. What is the constant payment required to achieve the same present discounted value over three years as in part 2 (i.e. \$38,342.08), at the same constant interest rate as you found in part 2?

$$\$38,342.08 = \$z \left(\frac{1}{1+0.07} + \frac{1}{(1+0.07)^2} + \frac{1}{(1+0.07)^3} \right)$$

$$\$z \approx 14,610.31$$

4. Suppose that the security pays the annual payment you calculated in part 3 forever at the constant interest rate from part 2. Calculate the present discounted value of this security.

$$\$V = \frac{14,610.31}{0.07} = \$208,718.71$$