14.02 Principles of Macroeconomics
Problem Set 2
Fall 2005
Solution

Posted: Wednesday, September 21, 2005
Due: Wednesday, September 28, 2005

Please write your name AND your TA's name on your problem set. Thanks!

Exercise I. True/False/Uncertain? Explain

1. A decrease of the reserve ratio ϑ produces the same effect on the interest rate as an expansionary open market operation.

   Uncertain.
   For a given proportion of money that people want to hold in currency (c), a decrease in ϑ increases the money multiplier and therefore the money supply. The LM curve shifts down, as it would do if there were a monetary expansion.
   However, you may also think that a decrease in ϑ increases the proportion of money that people want to hold in currency (c). This could be justified by the lower warranty on their deposits. In this case, the combined effect of a decrease in ϑ and an increase in c has an uncertain effect on the money supply and a clear comparison with a monetary expansion cannot be made.

2. The safer and more efficient the banking sector is considered to be, the higher the money multiplier.

   True.
   For any level of $\vartheta < 1$ safer and more efficient financial markets makes it possible to hold a lower amount of money for a given amount of transactions. The proportion of money held in currency c decreases and the money multiplier $\frac{1}{c+\vartheta(1-c)}$ increases. (Please see also the Focus box on page 70 and 71 of the textbook for a discussion of this issue.)

3. The bond that offers the highest interest rate has to be the most expensive one.

   False.
   According to the pricing formula for bonds (here the example for a one year bond that pays \$100 next year) $\$P_B = \frac{\$100}{(1+i)}$, the higher the interest rate, the lower the price of the bond.

4. An increase in consumer confidence shifts the LM curve down and the IS curve to the right.

   False.
   An increase in consumer confidence corresponds to an increase in the value of the parameter c_0 . Autonomous spending increases and the IS curve moves right. However, the money market is not affected (beyond the effect on output), and the LM stays where it was (since the effect on output is reflected in a movement along the LM curve).

5. A monetary expansion is more effective in changing the interest rate when money demand is very sensitive to the interest rate.

   False.
   The sensitivity of money demand to the interest rate is represented by the slope of the money demand curve. The flatter the curve is, the more sensitive the demand is since a small change in the interest rate causes a big change in the amount of money demanded. When the money demand curve is flat, the same change in money supply has a smaller impact on the interest rate than when the money demand curve is steep. Intuitively, the interest rate has to change less to make money demand equal the new money supply since money demand is very sensitive to the change in interest rate.

Exercise II. The Money Market

Assume that the following is true about the economy:

Money Demand: $M^d = \$Y(0.4 - i)$
There is a one-year-bond that promises a payment of \$12, where *i* is the interest rate on the bond.

Nominal Income: $\$Y = 1100$
The only bank is the central bank.
Money Supply: $M^s = 220$

1. What is the money demand when the interest rate is 4%? When *i*=20%? For which interest rate is the money demand lower? Explain why.

   $M^d = 1100 * (0.4 - 0.04) = 1100 * 0.36 = 396$
   $M^d{}' = 1100 * (0.4 - 0.2) = 1100 * 0.2 = 220$

   The higher the interest rate is, the higher the cost of keeping money instead of bonds (the opportunity cost of money). Money demand is thus lower at the higher interest rate.

2. What is the price of the bond when $i = 4\%$? When $i = 20\%$? Explain.

If i=4%

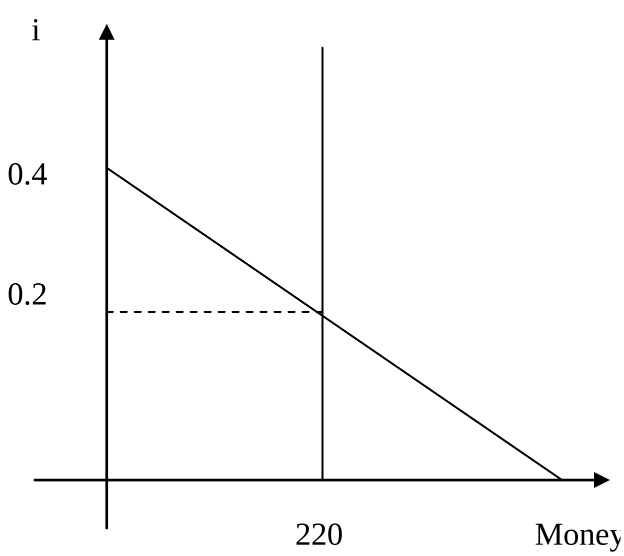
$$0.04 = \frac{12 - P_B}{P_B}$$
$$1.04 P_B = 12$$
$$P_B = 11.54$$

If i=20%

$$0.2 = \frac{12 - P_B}{P_B}$$
$$1.2 P_B = 12$$
$$P_B = 10$$

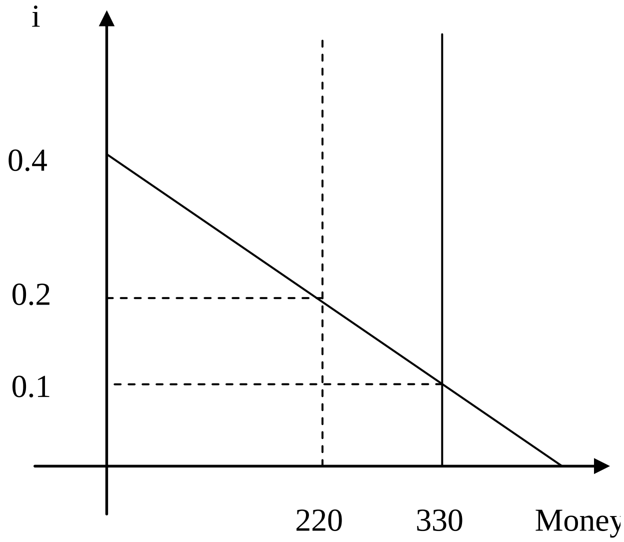
\$12 is the face value of the one year bond or, in other words, the amount of money paid when the bond expires. The general formula for one year bond is $\$P_B = \frac{\text{face value}}{1+i}$. The price of the bond decreases when the interest rate increases.

3. Draw a graph for money supply and the demand and calculate the equilibrium i.



$$M^s = M^d$$
$$220 = 1100 * (0.4 - i)$$
$$220 = 440 - 1100i$$
$$1100i = 220$$
$$i = 0.2$$
$$i = 20\%$$

4. Suppose the central bank increases the money supply by 110. Graph and calculate the new equilibrium *i*.



$$330 = 1100 * (0.4 - i)$$
$$330 = 440 - 1100i$$
$$1100i = 110$$
$$i = 0.1$$
$$i = 10\%$$

5. Keep $M^s = 330$. To what amount would nominal GDP have to change to have a money market equilibrium level consistent with an interest rate *i* equal to 20%? Explain.

$$M^s = M^d$$
$$330 = \$Y * (0.4 - 0.2)$$
$$330 = \$Y * 0.2$$
$$\$Y = 1650$$

In order to have a money market equilibrium with $i = 20\%$ and $M^s = 330$, Y should be quite high: \$1650. This depends on the sensitivity of the money demand to the interest rate. Kept constant the money supply and doubled the interest rate, equilibrium income increases by 50%.

Exercise III. Money Multiplier

Keep the same money demand and the nominal income as initially given in Exercise II.
Now imagine that there is a banking sector collecting deposits. The central bank requires a reserve ratio of $\vartheta = 50\%$.
People want to keep one third of their money demand as currency, and the rest as deposits.
The supply of central bank money is $H^s = 220$.

1) Calculate the money multiplier, CU^d, D^d, R^d, H^d (the demand for central bank money). Compare the equilibrium i with the one obtained in the previous exercise in part 3. Explain.

Money multiplier: $\dfrac{1}{c+\vartheta(1-c)} = \dfrac{1}{\frac{1}{3}+\frac{1}{2}*\frac{2}{3}} = \dfrac{3}{2} = 1.5$

$$M^d = M^s$$
$$cM^d + \vartheta(1-c)M^d = H^s$$
$$[c+\vartheta(1-c)]M^d = H^s$$
$$[c+\vartheta(1-c)]*1100*(0.4-i) = H^s$$
$$\frac{2}{3}*1100*(0.4-i) = 220$$
$$1100*(0.4-i) = 330$$
$$440 - 1100i = 330$$
$$110 = 1100i$$
$$i = 0.1$$
$$i = 10\%$$
$$M^d = 1100(0.4-0.1)$$
$$M^d = 330$$
$$CU^d = \frac{330}{3} = 110$$
$$D^d = \frac{2}{3}*330 = 220$$
$$R^d = \vartheta D^d = \frac{1}{2}*\frac{2}{3}*330 = 110$$
$$H^d = CU^d + R^d = 110 + 110 = 220$$

The equilibrium interest rate $i = 0.1$ is only half of the one founded in part 3 of the previous exercise. The reason is that the presence of a banking sector leads to a bigger multiplication of central bank money through the money multiplier. The money supply increases, and therefore the equilibrium interest rate (the reward that has to be given for holding bonds instead of currency) is lower.

2) Suppose the central bank money supply is decreased by 22. Calculate the new equilibrium *i*.

$$\frac{2}{3}*1100*(0.4-i)=198$$
$$1100*(0.4-i)=297$$
$$440-1100i=297$$
$$143=1100i$$
$$i=0.13$$
$$i=13\%$$

3) Keep $H^s=198$. What happens to *i* if the nominal GDP decreases by 275? Explain.

$$\frac{2}{3}*825*(0.4-i)=198$$
$$825*(0.4-i)=297$$
$$330-825i=297$$
$$33=825i$$
$$i=4\%$$

A decrease in nominal GDP lowers money demand, which for a given level of money supply leads to a decrease in the interest rate. The interest rate has to fall to give people an incentive to hold money instead of bonds, such that it raises money demand to the level where it is equal to money supply.

4) How can the central bank restore $\$Y=1100$ keeping H^s and *i* at the same levels used in part 3) of this exercise?

The Central Bank can change the reserve ratio as follows:

$$x*1100*(0.4-0.04)=198$$
$$x*1100*0.36=198$$
$$x*396=198$$
$$x=\frac{1}{2}=c+\vartheta(1-c)=\frac{1}{3}+\vartheta\frac{2}{3}$$
$$3=2+4\vartheta$$
$$\vartheta=\frac{1}{4}$$

Exercise IV. IS-LM Model

Assume

$C = 360 + 0.3 * Y_D$

$I = 200 + 0.2 * Y - 400i$

$T = 200$

$G = 300$

$\left(\frac{H}{P}\right)^s = 200$

$\frac{CU^d}{M^d} = c = 0.4$

$\vartheta = 0.25$

$(M / P)^d = 2.5Y - 7000i$

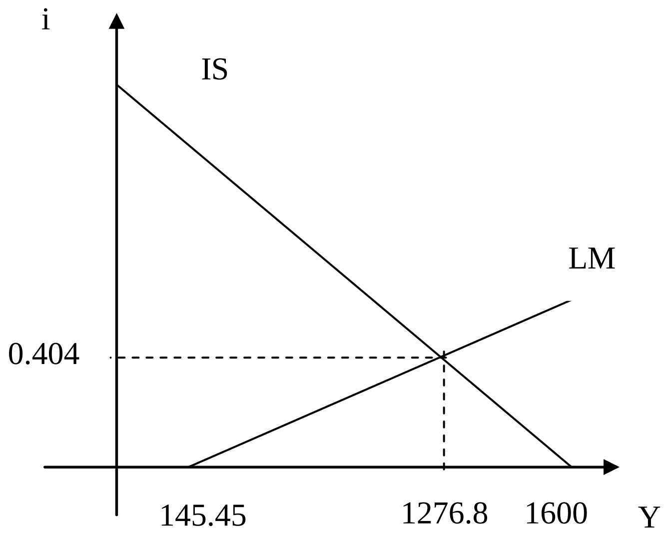
1) Derive the IS relation.

$$Y = 360 + 0.3 * (Y - 200) + 200 + 0.2Y - 400i + 300$$
$$Y = 800 + 0.5Y - 400i$$
$$Y = 1600 - 800i$$
$$i = (1600 - Y) / 800$$

2) Derive the LM relation.

$$M^s = M^d$$
$$\left(\frac{M}{P}\right)^s = \left(\frac{M}{P}\right)^d$$
$$H^s = (c + \vartheta(1 - c))\$YL(i)$$
$$\left(\frac{H}{P}\right)^s = (c + \vartheta(1 - c))YL(i)$$
$$200 = 0.55 * (2.5Y - 7000i)$$
$$363.64 + 7000i = 2.5Y$$
$$Y = 145.45 + 2800i$$
$$i = (Y - 145.45) / 2800$$

3) Solve for equilibrium real output, interest rate, C and I and graph the IS-LM diagram.



$$\begin{cases} Y = 145.45 + 2800i \\ Y = 1600 - 800i \end{cases}$$

$145.45 + 2800i = 1600 - 800i$

$3600i = 1454.55$

$i = 0.404$

$i = 40.4\%$

$Y = 1276.8$

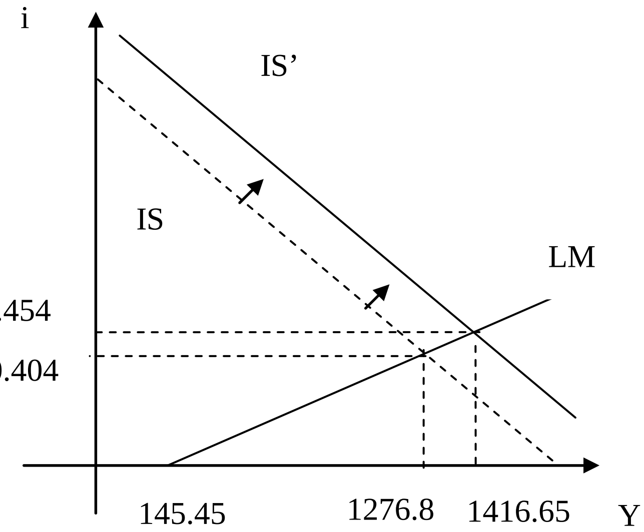
$C = 360 + 0.3 * (1276.8 - 200)$

$C = 683.04$

$I = 200 + 0.2 * 1276.8 - 400 * 0.4$

$I = 295.36$

4) The government decides to increase its deficit: G increases by 90 to 390. Draw a graph and describe what happens in the economy. Find equilibrium Y, *i*, C, and I.



The IS curve shifts to the right.

$$Y = 1780 - 800i$$
$$1780 - 800i = 145.45 + 2800i$$
$$3600i = 1634.55$$
$$i = 0.454$$
$$Y = 1416.65$$
$$C = 360 + 0.3*(1416.65 - 200)$$
$$C = 724.995$$
$$I = 200 + 0.2*1416.65 - 400*0.454$$
$$I = 301.73$$

5) What happens to our model if the investment is also a function of disposable income Y_D, instead of *Y*. Discuss this assumption and find the equilibrium *i* and Y before and after the fiscal expansion.

Assuming the investments depend on disposable income we allow for the taxes to affect the equilibrium outcome in two channels: consumption and investments. Since taxes also lower investment now, in addition to consumption, the IS curve is further to the left than in the model where taxes only affect consumption.

$$Y = 360 + 0.3*(Y - 200) + 200 + 0.2(Y - 200) - 400i + 300$$
$$Y = 760 + 0.5Y - 400i$$
$$Y = 1520 - 800i$$
$$i = (1520 - Y)/800$$

$$\begin{cases} Y = 145.45 + 2800i \\ Y = 1520 - 800i \end{cases}$$

$145.45 + 2800i = 1520 - 800i$
$3600i = 1374.55$
$i = 38.2\%$
$Y = 1215.05$

After

$$\begin{cases} Y = 145.45 + 2800i \\ Y = 1700 - 800i \end{cases}$$

$145.45 + 2800i = 1700 - 800i$
$3600i = 1554.55$
$i = 0.432$
$Y = 1355.05$

6) What happens in the standard setting, with I=I(Y,i), not I=I(Y_D,i), if the fiscal expansion is driven through a decrease in taxes T from 200 to 110?

$Y = 827 + 0.5Y - 400i$

$$\begin{cases} Y = 145.45 + 2800i \\ Y = 1654 - 800i \end{cases}$$

$145.45 + 2800i = 1654 - 800i$
$3600i = 1508.55$
$i = 0.419$
$Y = 1318.65$

7) Compare your answer to part 6 with that to part 4.

Even if $\Delta G = -\Delta T = 90$, the fiscal expansion produces different results if it is implemented through an increase of government spending or through a decrease in taxes. The effect of the decrease in taxes is mitigated by the fact that one dollar less of taxes does not mean one dollar more of consumption, but just 50 cents (since part of disposable income is saved). Instead one dollar more of public expense G increases the autonomous spending by exactly one dollar.