
14.02 Principles of Macroeconomics
Problem Set 4
Fall 2005
*****Solutions*****

Posted: Wednesday, October 26, 2005
Due: Wednesday, November 2, 2005

Please write your name AND your TA's name on your problem set. Thanks!

Exercise I. True/False? Explain

- 1) The more indexed wages are, the lower is the sacrifice ratio.

True. If all wages are indexed, $\pi_t^e = \pi_t$. Disinflation can be obtained without any costs in terms of higher unemployment.

- 2) In the medium run equilibrium the economy reaches its natural inflation level.

False. There is no a natural level of inflation.

- 3) A change in inflation expectations does not have any effect in the medium run.

True. In the medium run equilibrium $\pi_t^e = \pi_t$. All else equal, a change in inflation expectations has to be temporary and does not have any medium run effect in the economy. If we think about the change in inflation expectations in terms of variations in P^e , we can represent this change as a shift of the AS curve. In the medium run the AS curve has to move back to its original position.

- 4) Inflation per se does not affect investment choices.

True. Investment choices are affected by the real interest rate, which is equal to the nominal interest rate minus inflation. Uncertainty about inflation can affect investment decisions since this uncertainty makes it difficult for firms to forecast future prices and profits. Not inflation per se.

- 5) The Fisher Hypothesis can be interpreted as a restatement of the neutrality of money.

True. The Fisher hypothesis states that, after a monetary policy operation, any change in nominal interest rate is fully reflected in a change in inflation in the medium run. This means that monetary policy can not affect real interest rates.

- 6) For the Central Bank, committing to keeping a low nominal interest rate or committing to keeping a low inflation rate is equivalent.

False. After a positive temporary demand shock, for example, the Central Bank may have to raise the nominal interest rate a lot in order to keep inflation under control.

Exercise II. Inflation, Activity, and Nominal Money Growth

Assume that the economy is described by the following equations:

$$u_t - u_{t-1} = -0.5(g_{yt} - \bar{g}_y) \text{ (Okun's Law)}$$

$$P_t = P_t^e(1 + \mu)(1 - 2u_t + z) \text{ (Aggregate Supply relation)}$$

$$g_{yt} = g_{mt} - \pi_t \text{ (Aggregate Demand relation)}$$

- 1) Derive the Phillips curve $\pi_t - \pi_t^e = -\alpha(u_t - u_n)$ from the Aggregate Supply relation.

Assume $\mu = z = 0.04$. What is the value for α ? Calculate the natural level of unemployment u_n .

$$P_t = P_t^e(1 + \mu)(1 - 2u_t + z)$$

$$\frac{P_t}{P_{t-1}} = \frac{P_t^e}{P_{t-1}^e}(1 + \mu)(1 - 2u_t + z)$$

$$\text{Given that } \frac{P_t}{P_{t-1}} = (1 + \pi_t) \text{ and } \frac{P_t^e}{P_{t-1}^e} = (1 + \pi_t^e)$$

$$(1 + \pi_t) = (1 + \pi_t^e)(1 + \mu)(1 - 2u_t + z)$$

After taking logs, the expression can be approximated by

$$\pi_t = \pi_t^e + \mu + z - 2u_t$$

$$\text{Rearranging terms we obtain } \pi_t - \pi_t^e = -2\left(u_t - \frac{\mu + z}{2}\right)$$

$$\text{Where } \alpha = 2 \text{ and } u_n = \frac{\mu + z}{2} = 4\%.$$

Consider the case where $\pi_t^e = \pi_{t-1}$. Assume $\bar{g}_y = 0.04$ and $g_{mt} = \bar{g}_m = 0.14$ is constant over time.

- 2) Calculate the medium run equilibrium for the economy (g_{yt} and π_t).

In equilibrium we have $u_t - u_{t-1} = 0$, because $u_t = u_n = 0.04$ for any t .

From the Okun's Law we obtain $g_{yt} = \bar{g}_y = 0.04$.

From the Aggregate Demand relation it follows that:

$$0.04 = 0.14 - \pi_t, \text{ which implies } \pi_t = 0.1 \text{ in equilibrium.}$$

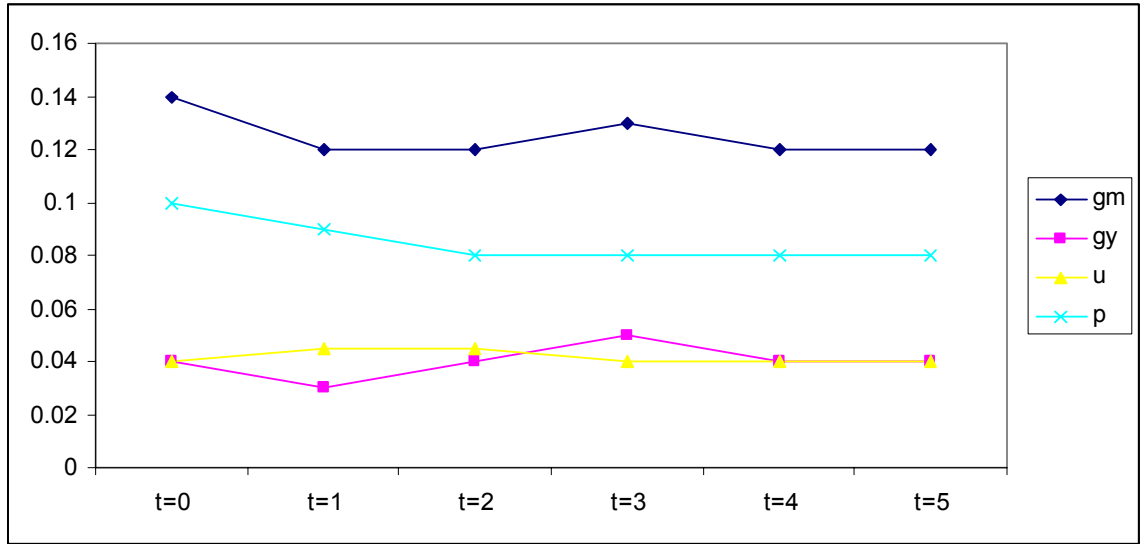
The Central Bank decides to decrease inflation by 2 percentage points. To achieve this goal, they plan to do a monetary contraction as follows: The nominal money supply is announced to be decreased by 2 percentage points at $t=1$, increased by one percentage point at $t=3$, decreased again by one percentage point at $t=4$. Then it will stay constant.

Note: For the following problems, you should submit a printout of your spreadsheet and the graph.

- 3) Use a spreadsheet to describe the dynamics to the new medium run equilibrium. That is, find g_{mt} , g_{yt} , u_t , π_t , and the sacrifice ratio for each year from $t=1$ on until $t=5$. Start from the equilibrium for $t=0$ which you calculated in part 2). Graph all the variables over time.

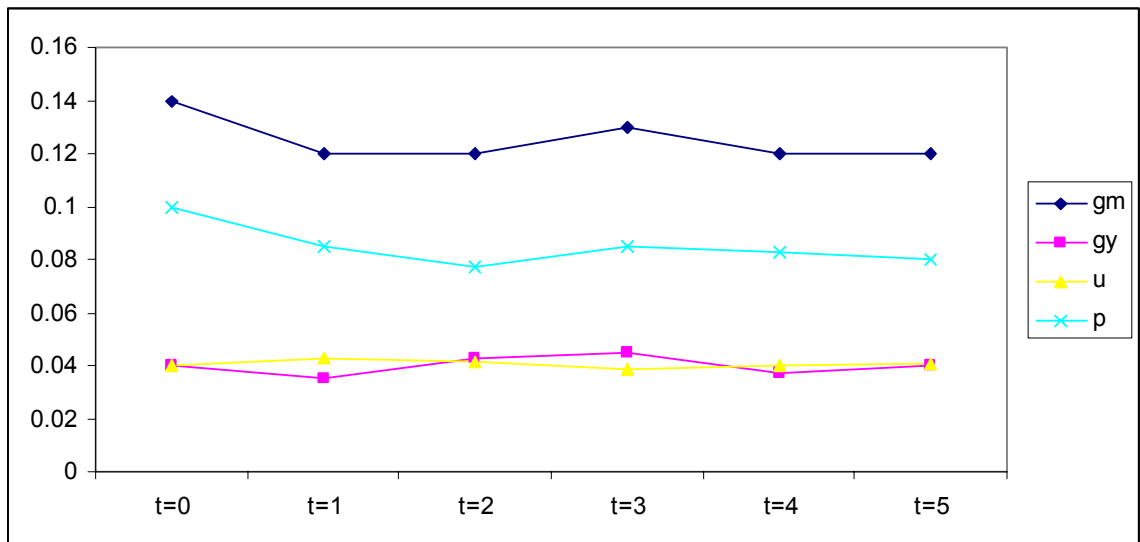
The sacrifice ratio at time t is $\frac{\sum_t (u_t - u_n)}{\pi^{\text{old equilibrium}} - \pi_t}$.

	t=0	t=1	t=2	t=3	t=4	t=5	
gm		0.14	0.12	0.12	0.13	0.12	0.12
gy		0.04	0.03	0.04	0.05	0.04	0.04
u		0.04	0.045	0.045	0.04	0.04	0.04
p		0.1	0.09	0.08	0.08	0.08	0.08
sacriface ratio			0.5	0.5			



- 4) (Lucas Critique) Consider now the case where 2/3 of the wage setters take the monetary policy into account when they form their expectations and set $\pi_t^e = \pi_t$. The remaining 1/3 of wage setters keep $\pi_t^e = \pi_{t-1}$. Use a spreadsheet to calculate the dynamics to the new medium run equilibrium as in part 3). Compare the sacrifice ratios you obtained in the two scenarios and comment.

	t=0	t=1	t=2	t=3	t=4	t=5	
gm		0.14	0.12	0.12	0.13	0.12	0.12
gy		0.04	0.035	0.0425	0.045	0.036875	0.039688
u		0.04	0.0425	0.04125	0.03875	0.040313	0.040468
p		0.1	0.085	0.0775	0.085	0.083125	0.080312
sacrifice ratio			0.166667	0.166667	0.166667	0.166667	0.166477



The sacrifice ratio in this scenario is much lower than the one in the previous setting. This change is due to the fact that the major part of the population forms its inflation expectation correctly. In the limit, when the whole population sets $\pi_t^e = \pi_t$, the sacrifice ratio becomes zero.

- 5) (Nominal rigidities, wage indexation). Imagine that each year 2/3 of the wage contracts are adjusted for inflation ($\pi_t^e = \pi_t$) and the remaining 1/3 keep $\pi_t^e = \pi_{t-1}$. Use a spreadsheet to calculate the dynamics to the new medium run equilibrium as in part 3) and 4). Compare your results to the ones you obtained in part 4). What are the differences between the two scenarios?

	t=0	t=1	t=2	t=3	t=4	t=5	
gm		0.14	0.12	0.12	0.13	0.12	0.12
gy		0.04	0.035	0.0425	0.045	0.036875	0.039688
u		0.04	0.0425	0.04125	0.03875	0.040313	0.040468
p		0.1	0.085	0.0775	0.085	0.083125	0.080312
sacrifice ratio			0.166667	0.166667	0.166667	0.166667	0.166477

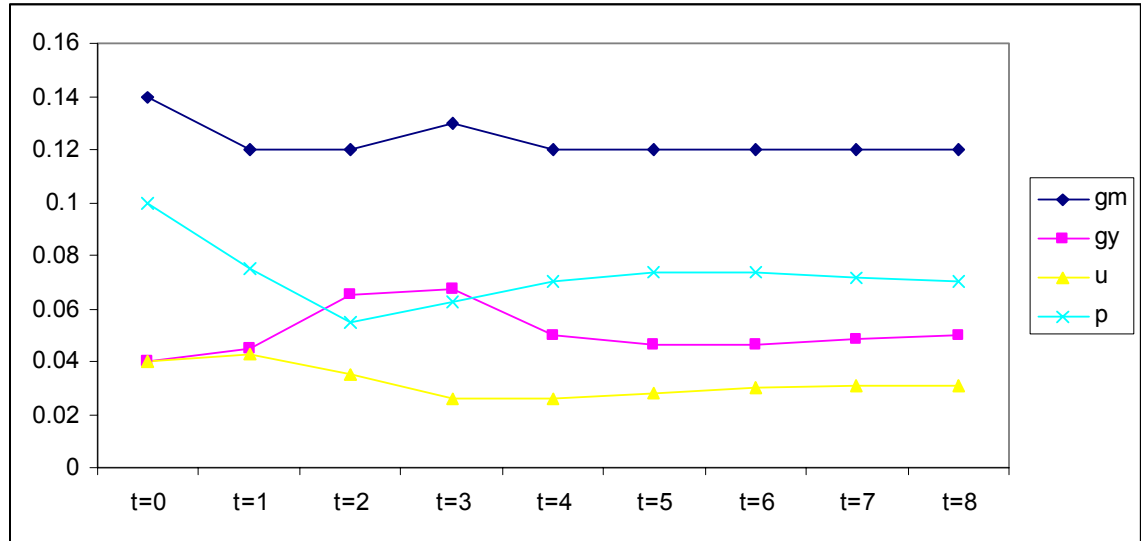
The Lucas critique about the expectation formation process and the Stanley Fischer/John Taylor argument about nominal rigidities lead to the same conclusion: the sacrifice ratio can be different in different settings. In the example, given our choice of parameters, the results are the same for both settings. However, conceptually the settings are very different. In the first case we are assuming that all contracts are being revised every year and that wage-setters inflation expectations play a crucial role. In the second case we are assuming that all wage-setters inflation expectations are $\pi_t^e = \pi_t$, but at any period t only a fraction of all contracts are revised, and they last for more than one period.

- 6) Now assume that $\pi_t^e = \pi_{t-1}$ always and for everybody. Imagine that at $t=1$ the economy becomes more competitive forcing μ to drop to 0.02. Assume that this changes u_n and \bar{g}_y in the same proportion. The Central Bank does not become aware of this change and keeps to its initial plan (the one described above). How does the new medium run equilibrium change? Calculate the new dynamics.

$$u_n = \frac{\mu + z}{2} = \frac{0.02 + 0.04}{2} = 0.03$$

By assumption we have also that $\bar{g}_y = 0.05$.

	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	
gm		0.14	0.12	0.12	0.13	0.12	0.12	0.12	0.12	0.12
gy		0.04	0.045	0.065	0.0675	0.05	0.04625	0.04625	0.048125	0.05
u		0.04	0.0425	0.035	0.02625	0.02625	0.028125	0.03	0.030938	0.030938
p		0.1	0.075	0.055	0.0625	0.07	0.07375	0.07375	0.071875	0.07
sacrifice ratio			0.5	0.388889	0.366667	0.333333	0.309524	0.309524	0.322222	0.333333



Exercise III. Financial Markets

Consider an economy where two different stocks and one bond are the only financial instruments. The bond pays \$10 each year forever starting from next year. Its price is \$40. The first stock is expected to pay a constant dividend of \$5, until $t=4$. The second stock is expected to pay a constant dividend of \$10, until $t=3$.

- 1) Assume that the nominal interest rate in the economy is constant and that the arbitrage law holds. Calculate the ex dividend prices of the two stocks.

Using the information about the bond, we can calculate the interest rate: $40 = \frac{10}{i}$
gives $i = 0.25$

$$P^A = 5 \left[\frac{1 - \left(\frac{1}{1+i}\right)^5}{1 - \left(\frac{1}{1+i}\right)} - 1 \right] = 11.808$$

$$P^B = 10 \left[\frac{1 - \left(\frac{1}{1+i}\right)^4}{1 - \left(\frac{1}{1+i}\right)} - 1 \right] = 19.52$$

- 2) Imagine that in the market there is one trader with more information. She is a small trader which means that her trades don't change the market price. At $t=1$ she gets insider information that the second stock will pay a higher dividend, namely \$20.5 at $t=3$. At $t=2$ the information is released to the whole market. The insider information should allow the small trader to earn extra money. If she owns \$640 at $t=1$ and cannot borrow, how much money can she end up with at $t=2$ if she exploits her information advantage? How much money would she have had at $t=2$ if she didn't get the insider information?

Note that

$$P_{t=1}^B = \frac{10}{1+0.25} + \frac{10}{(1+0.25)^2} = 14.4$$

Given the insider information, the insider will be not indifferent between the three assets. At $t=1$ she will put all her money on asset 2, buying $\frac{640}{14.4} = 44$ stocks (with the rest of 6.4 dollars).

At $t=2$ the market gets the new information and update its price for the second stock:

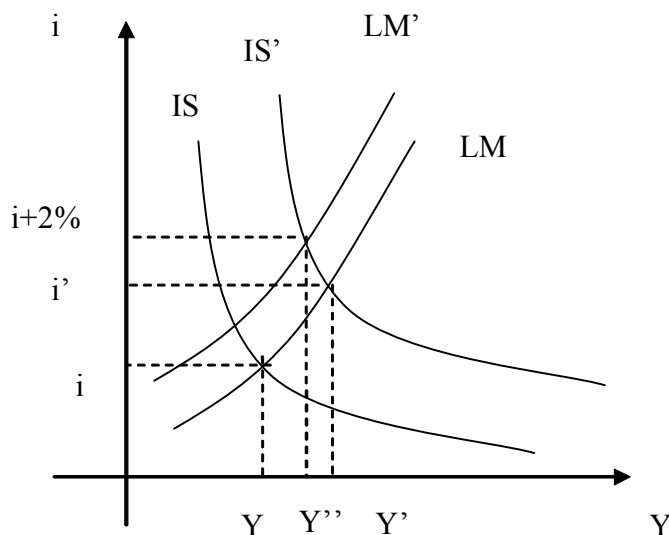
$$P_{t=2}^B = \frac{20.5}{1+0.25} = 16.4.$$

The trader can sell all her shares at $t=2$ and get $44 \cdot 16.4 = 721.6$. She gets also all the dividends matured at $t=1$: $10 \cdot 44 = 440$.

The gain from this operation is $(721.6 + 440 - 633.6) = \528 .

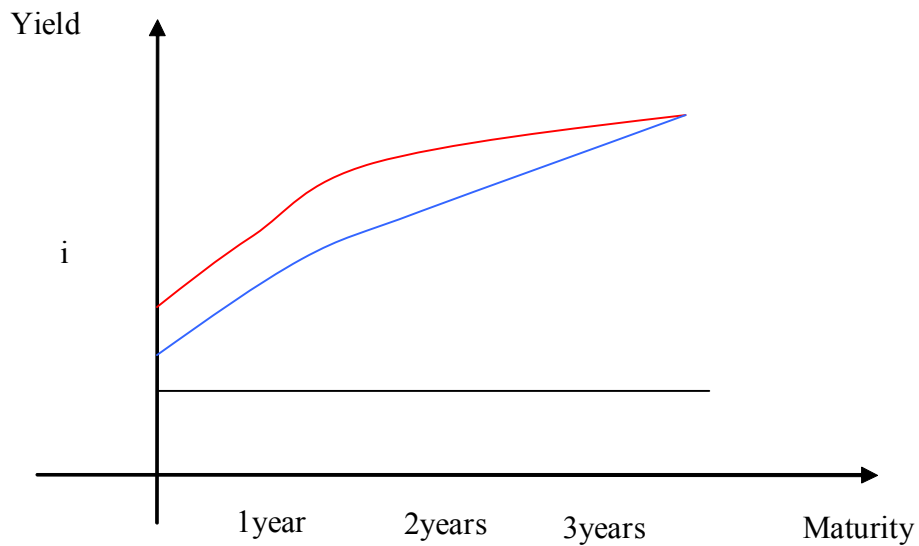
If she did not have any insider information, she would have been indifferent between the three assets. Indeed the arbitrage law would have held also for her. In the worst case she could have decided to not invest in the second stock buying just the bond and the first asset. In this case, she would have gained just \$160.

- 3) Now assume that the scenario in part 2) never happened and that all traders share the same information. At $t=1$ the economy is hit by a positive shock in consumer confidence. The market immediately makes the prediction that at $t=2$ the Central Bank will intervene by increasing the nominal interest rate by 2 percentage points. How do the prices of the three financial assets change at $t=1$? (Be careful: do you have enough information to calculate exact amounts? What is missing?) Use IS-LM graphs in explaining your reasoning.



Given that the interest rate is expected to increase, the bond price has to decrease. We now have $i = 0.27$ which implies that the bond price is about \$37. However, the changes in the stock prices are difficult to predict. There are two opposing effects: the interest rate is expected to increase and dividends are expected to decrease. The first factor causes stock prices to decrease and the second factor causes stock prices to increase. The combined effect is ambiguous, given that we do not know how the dividends change reflecting the increase in output.

- 4) Imagine that at $t=2$ the Central Bank intervention happens to be less severe than was expected. Draw and explain three yield curves that can help you describe the evolving situation at $t=0$, at $t=1$ and $t=2$.



The black line is the yield curve at $t=0$. The economy is not expected to be affected by any shock: all short term interest rates are at their medium run equilibrium.

The red line is the yield curve at $t=1$. The market is affected by a shock that moves the IS curve to the right and increase the short term interest rate. In the future the market expects the Central Bank to increase the interest rate. In the long run, the output level is expected to go back to the natural level of output and, assuming that the shock to the consumer confidence is permanent, the interest rate will have to increase even more.

The blue line represents the yield curve at $t=2$. The Central Bank increases the interest rate less than what was expected. In any case, the future interest rates are expected to increase until reaching their medium run equilibrium level.