

Problem Set Four Solutions

1. Expectations and IS-LM answers

a) Consumption of durable goods is achieved by seeking credit in the financial markets. Therefore, consumption acts like investment in varying negatively with the real interest rate, r .

b)

$$r = i - \pi^e \quad (1)$$

$$Y = c(Y, i - \pi^e) + I(Y) + G \quad (2)$$

$$M = YL(i) \quad (3)$$

$$(4)$$

If expected inflation falls, the nominal interest rate must fall if real interest rates are to remain the same. In this example, the IS curve shifts to the left to indicate the new goods market equilibrium, output falls, the nominal interest rate falls, but not by the full amount in equilibrium, so the real interest rate rises a little.

Expected inflation does not enter the LM relation, so the LM curve does not shift.

c)

$$r = i - \pi^e \quad (5)$$

$$Y = c(Y, i - \pi^e) + I(Y) + G \quad (6)$$

$$M = YL(i - \pi^e) \quad (7)$$

$$(8)$$

Money demand depends on the difference between the nominal interest rate and the rate of return on money, which is equal in this question to the rate of inflation. This implies that money demand depends on the real interest rate.

d) If expected inflation falls, the nominal interest rate must fall, if real interest rates are to remain the same. The IS curve shifts to the left to indicate the new goods market equilibrium.

If expected inflation falls, the nominal interest rate must fall, if real interest rates are to remain the same. The LM curve shifts to the right to indicate the new money market equilibrium. Overall, output remains the same, the nominal

interest rate falls by the full amount that expected inflation does, so r remains the same.

2. Real Interest Rate answers

a) Draw the interest rate on new issues of three month T-bills from p.352, column one of "Economic Report..." (70- 6.458, 71- 4.348, 72- 4.071, 73- 7.041, 74- 5.838, 75- 4.989, 76- 5.265, 77- 7.221, 78- 10.041, 79- 11.506)

b) I get a $\lambda = 1.38$, however, it is an approximation, so answers close to this will be accepted. From p.335, column one, and using the formula $\pi_t^e = 1.38\pi_{t-1}$, the following estimates of expected inflation are computed. (70- 2.62, 71- 2.89, 72- 2.34, 73- 1.79, 74- 3.59, 75- 6.76, 76- 6.61, 77- 4.27, 78- 5.10, 79- 6.34, 80- 10.21)

c) $r = i - \pi^e$, then (70- 3.83, 71- 1.46, 72- 1.73, 73- 5.25, 74- 4.29, 75- -0.92, 76- -1.77, 77- 0.99, 78- 2.12, 79- 3.7, 80- 1.3)

Use these values and the values of expected inflation to draw the graph.

d) It was profitable to borrow in '75 or '76, as the real interest rate was negative. It was profitable to lend in '73 or '74, as the real interest rate was at the maximum for this time series.

e) Nominal interest rate changes do not follow exactly changes in the expected rate of inflation, as the static theory of real interest rate determination predicts. Some possible explanations: Excess demand or excess supply of capital causes the real interest rate to rise and fall. There are lags in the response of real interest rates to changes in expected inflation. There is a measurement error problem in the way we computed the expected inflation rate.

f) It is acceptable as an unbiased linear estimate on one lag of inflation. An adaptive estimate of the expected rate of inflation, based on a series of lagged measures of inflation, might produce a tighter fit.

g) One expects a negative correlation between changes in expected inflation and the real interest rate. From the first question, (1b) the change in expected inflation is not fully reflected in the nominal interest rate so the real interest rate changes, that is, expected inflation decreases, nominal interest rates decrease somewhat, so the real interest rate increases. (I think this is reflected in the data)

3. Present Discounted Value Answers

a) $V(T) = \frac{X(T)}{(1+r)^T}$

b) If the PDV was higher in another period, then it would be that period when the tree would be cut down.

c) It is equivalent to say $V(T + 1) = V(T)$ if $V(T) - V(T - 1) = 0$.

$$\frac{X(T+1)}{(1+r)^{T+1}} - \frac{X(T)}{(1+r)^T} = 0$$

$$\text{then } \frac{X(T+1) - X(T)(1+r)}{(1+r)^{T+1}} = 0$$

$$X(T)(1+r) = X(T+1)$$

$$r = \frac{X(T+1) - X(T)}{X(T)}$$

The growth of the tree will be equal to the interest rate at the time of the cutting. If the growth rate were greater than the interest rate, then the EPDV of the tree would still be increasing. If the growth rate was less than the interest rate, then the EPDV of the tree must be decreasing. In the first case, it is optimal to wait. In the second case, it is optimal to cut before the EPDV decreases.

d) A higher interest rate means payments today are worth more than payments tomorrow. The real interest rate is still equal to the growth rate at this higher rate of growth. The owner should cut the tree sooner, or equivalently, be less willing to wait.

e) The slope of the curve at time T is the interest rate.

4. Far-Sighted Consumer Answers

a) $V(T) = N(Y - T) = 45(40000 - 8000) = 1440000$
I would consume $1440000/63 = 22857$ each year in life.

b) Zero. In one given period, there is an equal amount of saving and dissaving, so the aggregate rate of savings is zero. We can think of consumption, saving, and income path of an individual over her lifetime as a population distribution at one point in time. The individual's total savings over life is zero; whatever is saved is later dissaved; likewise for the population: the saving of the young is dissaved by the old (and the students).

c) Income and Saving between three and forty-eight become non-linear, as they increase more rapidly in later years. Consumption is smooth. The amount of

saving and dissaving in aggregate still cancel each other out.

$$V(T) = .8(40000)(1 + (1.04) + (1.04)^2 + \dots + (1.04)^{44}) \quad (9)$$

$$C_t = V(T)/63 \quad (10)$$

$$(11)$$

$$d) V(T) = (Y - T) \frac{(1 - \frac{1}{(1+r)^N})}{1 - \frac{1}{1+r}}$$

This gives the present value when work is started. However, you're still a student, and need to discount back three more years, so multiply this EPDV by $1/(1+r)^3$

$$\text{let } Y = 40000, T = 8000, N = 45, r = 2$$

$$\text{then } V(T) = 890566 \text{ and } C = 890566/63 = 14136$$

$$V(T) = \frac{(.8)(40000)}{(1+r)^3} \left[\frac{1 - \frac{1}{(1+r)^{45}}}{1 - \frac{1}{1+r}} \right]$$

EPDV of Consumption equals the EPDV of Wealth.

$$C \left[1 + \frac{1}{1+r} + \dots + \frac{1}{(1+r)^{63}} \right] = V(T)$$

Real PDV of human wealth is \$2098136.

$$V(T) = \frac{(.8)(40000)}{(1+r)^3} \frac{1 - \frac{(1.04)^{45}}{(1.02)^{45}}}{1 - \frac{(1.04)}{(1.02)}}$$

Nominal PDV of human wealth is \$2010954.

$$V(T) = \frac{(.8)(40000)}{(1+r)^3} \frac{1 - \frac{(1.05)^{45}}{(1.03)^{45}}}{1 - \frac{(1.05)}{(1.03)}}$$

e) real interest rate equals two percent, inflation rate equals zero, growth equals zero.

$$V(T) = \frac{(.8)(40000)}{(1+r)^3} \frac{1 - \frac{1}{(1.02)^{45}}}{1 - (1.02)} + 48000 = 938566$$

$$C \frac{1 - \frac{1}{(1+r)^{63}}}{1 - \frac{1}{(1+r)}} = V(T) = 26071$$

$$\text{f) } 48000 = x \frac{1 - \frac{1}{(1.02)^{48}}}{1 - \frac{1}{1.02}}$$

$$x = 1548$$