

## Optional Problem Set 2 Solutions

As  $M = 10 \cdot t$  and  $P = M/10$  we have  $P = t$  and  $dP = 1$ .

Recall  $\varepsilon = EP^*/P$ . We have  $E = 1$  fixed. Thus  $\Delta\varepsilon/\varepsilon = \Delta E/E + \Delta P^*/P^* - \Delta P/P$ . Assume for simplicity that foreign prices are fixed and normalize to one so  $\Delta P^* = 0$  and  $\varepsilon = 1/P$ .

### Part 1.

$$\Delta\varepsilon/\varepsilon = -\Delta P/P = -1/P < 0.$$

The percent change in the real exchange rate is simply minus the rate of domestic inflation. Consequently, we have a real appreciation in every period.

### Part 2.

Note  $\delta NX/\delta\varepsilon = 1/\varepsilon > 0$  so the Marshall-Lerner conditions hold.

$$NX = \log(\varepsilon) \text{ so } \Delta NX = \Delta\varepsilon/\varepsilon = -1/P < 0.$$

As net exports are falling and the real exchange rate is rising, the country is losing competitiveness.

### Part 3.

If the country uses reserves to pay for the trade deficit, a crisis will occur when the central bank runs out of foreign currency. It is possible to solve for the time when this actually happens in continuous time,

$$10 = R_1 = \int_1^T NX_s ds = \int_1^T \log(1/s) ds$$

Solve for  $T$ , the time when reserves are fully depleted.

Alternatively, assume that foreign currency reserves can be used as collateral for short-term loans and that every unit of reserves can secure  $C$  units of loans. Thus the crisis is postponed until the private sector requires more loans for which the central bank has collateral. Since the nominal interest rate is zero the country does not need to compound interest, and the change in the stock of debt is simply the trade deficit. This happens according to the formula below,

$$10C = R_1 * C = \int_1^T NX_s ds = \int_1^T \log(1/s) ds$$

It should be clear that as long as  $C > 1$  borrowing from foreign banks delays the crisis longer than the first strategy using only central bank reserves.

A final note concerns expectations. In the description above, the crisis does not occur until reserves are depleted, but if agents actually realize that there will be a time in the near future where this will happen, a crisis will occur immediately.