OPTIONAL PROBLEM SET SOLUTIONS

1. $Z = C + I + G$
   $C = 20+0.5(Y-T)$
   $I = 25*(0.5-i)$
   $G = 20$ and $T = 20$
   $M_d = Y*(0.5-i)$
   $M_s = 32$

First derive the IS and LM for arbitrary G and M, because they are changed in the later parts of the question.

Goods Market equilibrium, $Z = Y$, implies $Y = 45 – 50*i +2*G$.
Asset Market equilibrium, $M_d = M_s$, implies $i = 0.5-M_s/Y$.

a. The IS curve (using $G = 20$) is $Y = 85 – 50*i$.
b. The LM curve (using $M_s = 32$) is $i = 0.5-32/Y$.
c. The equilibrium (Y,i) is found by solving the two-equation system above.

$$Y = 85-50*(0.5-32/Y) = 60+1500/Y$$

This is a quadratic in Y with two solutions. Choose the positive one so that approximately $Y^* = 79$ and, using the LM curve at Y^*, $i^* = 0.09$.
d. The IS curve is now $Y = 95 – 50*I$, and the equilibium (Y,i) is found as above.

$$Y = 95-50*(0.5-32/Y) = 70+1500/Y$$

This is a quadratic in Y with two solutions. Choose the positive one so that approximately $Y^* = 87$ and, using the LM curve at Y^*, $i^* = 0.13$.

e. The LM curve is now $i = 0.5-34/Y$, and the equilibrium (Y,i) is found as above.

$$Y = 85-50*(0.5-34/Y) = 60+1700/Y$$

This is a quadratic in Y with two solutions. Choose the positive one so that approximately $Y^* = 81$ and, using the LM curve at Y^*, $i^* = 0.08$.

No graphics.

2.

$d_1 = 1.05$
$d_2 = (1.05)*(1.07) = 1.1235$
$d_3 = (1.05)*(1.07)*(1.06) = 1.19091$

a. $P_b = 0.08*100/d_1+0.08*100/d_2+1.08*100/d_3 = 105.4266$

b. Solve for x the following equation:
$105.4266 = 8/(1+x)+8/(1+x)^2+108/(1+x)^3$.
The yields x = 0.05975 approximately through guess and check to arbitrary order of accuracy. The yield to maturity is the constant interest rate which equalizes the bond price and present value of income flows from the bond.

c. Now $d_1 = 1.05$, $d_2 = 1.11825$, $d_3 = 1.179754$ and the bond price is computed using the formula above as P = 106.3176 and x = 0.05225 approximately. Lower expected interest rates increase bond prices and reduce yields.

3. Z = C + I + G
C = 20+0.5(Y-T)
I = 25*(0.5-r)
G = 20 and T = 20
$M_d = Y*(0.5-r-p^e)$
$M_s = 32$

Only expected inflation, denoted p^e, and the money supply change in this question, so we derive the IS and LM curves for arbitrary values of M_s and p^e.

Goods market equilibrium requires Y = 85-50*r.
Asset market equilibrium requires $r = 0.5-p^e-M_s/Y$.

a. Stable prices require p = 0, returning us to the equilibrium above where $Y^* = 83$ and $i^* = r^* = 0.11$.

If p = 0.05 then the LM curve becomes r = 0.45-32/Y, and equilibrium income is derived as above.

Y = 85-50*(0.45-32/Y) = 62.5+1500/Y

The positive output equilibrium requires $Y^* = 81$ and $r^* = 0.054938$, and finally we have $i^* = r + p^e = 10.054938$. Output rises and nominal and real interest rates fall.

b. Solve as before but for arbitrary M_s and Y = 90. Combine goods market and asset market equilibrium conditions as above.

$Y = 85-50*(0.50-0-M_s/Y)$
$90 = 85-50*(0.50-0-M_s/90)$ implies $M_s = 54$.

Using the LM equation we have $i^* = r^* + p^e = r^* = 0.5 – 54/90 = -0.01$.
This cannot be achieved by a monetary expansion alone as nominal interest rates cannot be negative (agents would only hold money when nominal rates are zero or less).

c. The liquidity trap is a problem in which economies with very low nominal interest rates don't have the ability to use monetary expansions to increase output very much due to the existence a natural bound on nominal interest rates at zero. The example above nicely illustrates this proposition.