

## PROBLEM SET SIX SOLUTIONS

### Question 1.

- a. The nominal return on each of the bonds  
 $i_{us} = FV_{us}/P_{us} - 1 = 10000/9615.38 - 1 = 0.04$   
 $i_{ge} = FV_{ge}/P_{ge} - 1 = 13,333/12698.10 - 1 = 0.05$

Also recall that uncovered interest parity is stated as:

$$i_{us} = i_{ge} + (x - E)/E \text{ where } x = E^e$$

It easily follows that one can solve for  $x$ , the expected nominal exchange rate in one year:

$$x = E(i_{us} - i_{ge} + 1) = 0.75(0.04 - 0.05 + 1) = 0.75(1.09) = 0.8175$$

Since  $x < E$  we have an APPRECIATION of the dollar relative to the DM.

- b. The actual nominal return on the US bond remains 4%, but purchase of the German bond required a position in DM, so the unexpected depreciation of the US dollar affects your return. Since you have to buy dollars when your bond matures and the price of dollars has unexpectedly fallen, one would expect the nominal return from purchase of the German bond ex post to be higher than the US bond. Mathematically, we can write this nominal return as follows:  
 $i_{ge} + (x - E)/E = 0.05 + (1.25 - 0.75)/0.75 = 0.05 + 0.6667 = 0.7167 > 0.04$

Looking backwards one year from now, the nominal return far exceeds that on the US zero of similar maturity and default risk. This does not mean, however, that everyone should have bought German bonds given their information one year ago. It is also possible for there to be an unexpected appreciation of the dollar (more than the markets expected) which would hammer your position in German currency.

The point of this question is that the type of arbitrage necessary for uncovered interest parity conditions to hold is arbitrage between expected returns. Although the use of the term arbitrage suggests free money, there is nothing riskless about uncovered positions in foreign currency.

We tend to think identical goods should have the same prices. If identical goods have different prices, it is possible to buy at the low price and sell at the high price and make lots of money. Arbitrage refers to actions by agents taking advantage of these kinds of price differentials. You should discern between two types of arbitrage. There is expected arbitrage, which is taking advantage of differences in expected return. This means you expect to make money, but will sometimes do worse and other times do better, so you will make money on average. On the other hand there is riskless arbitrage, which means you will always make money regardless of the course of future events. The uncovered interest parity condition is an equilibrium condition equivalent to no expected arbitrage opportunities while the covered interest parity condition below is equivalent to no riskless arbitrage opportunities (at least in one-year zero coupon bonds).

- c. The nominal gross return on the US zero-coupon bond is simply the nominal interest rate as before. On the other hand, instead of taking an uncovered position in foreign currency, consider a strategy of selling DM forward in the amount you expect to have one year from today. One US dollar converts into  $1/E$  DM, which invested in the German bond will yield  $(1 + i_{ge})/E$  DM next year. Sell this amount forward today at a price of  $F$  \$/DM so the nominal return on this investment strategy is  $(1 + i_{ge})F/E$ . If nominal returns on each strategy are equal, then we have the following interest parity condition:  
 $1 + i_{us} = (1 + i_{ge})F/E$   
Using  $\log(1 + z) = z$  we can derive an approximation to covered interest parity:  
 $i_{us} = i_{ge} + (F - E)/E$

- d. Solving for  $F$  in the interest parity condition above, it follows:

$$F = E[(i_{us} - i_{ge}) + 1] = 0.75[0.04 - 0.05 + 1] = 0.7425$$

Next year's exchange rate has no effect on the nominal returns of either strategy, because we have covered our position in German currency by selling DM forward. For one-year zeros, departures from covered interest parity are opportunities for riskless arbitrage. See below for other bonds.

- e. The forward exchange market only allows you to get rid of risk inherent in taking positions in foreign currency. For the two-year German bond that you want to sell in one-year, there is interest rate risk. This is because the amount that you will be able to sell the bond for in one year will depend on interest rates (not the case above because you were receiving fixed cash flows). An unexpected increase in German interest rates will reduce bond prices, reducing your cash flows in DM. The second effect is you are obligated to sell DM based on what you thought bond prices would be under the old interest rates. Since bond prices have fallen you have fewer DM than you thought, and you will have to buy DM to make good on your forward contract. The bad news is that higher German interest rates generally mean a depreciation of the dollar, so buying DM becomes more expensive. In the end, what was supposed to be easy money is now a bad situation. This is not a special case. There are a number of "arbitrage" mutual funds and investment banks which attempt to exploit departures from covered interest parity. Occasionally, these institutions take big losses, for the reason mentioned above.

## Question 2.

- a. Equilibrium corresponds to  $Y = Z$  AND  $Y^* = Z^*$ .

$$Y = C + I + G + X - M = 30 + 0.6(Y - 10) + 0.1Y^* - 0.1Y$$

$$Y^* = C^* + I^* + G^* + X^* - M^* = 30 + 0.6(Y^* - 10) + 0.1Y - 0.1Y^*$$

$$Y = 2(14 + G + 0.1Y^*) = 48 + 0.2Y^*$$

$$Y^* = 2(14 + G^* + 0.1Y) = 48 + 0.2Y$$

$$Y = Y^* = 48 / 0.8 = 60$$

- b. Assume the domestic country does not expect the foreign country to change fiscal policy. This implies the following system of equations describing equilibrium:

$$Y = 2(14 + G + 0.1Y^*) = 28 + 2G + 0.2Y^*$$

$$Y^* = 48 + 0.2Y$$

Assume domestic fiscal policy is successful, implying

$$Y^* = 48 + 0.2Y^P = 48 + 0.2 \cdot 80 = 64$$

Given foreign income after the fiscal expansion, one can solve for the necessary increase in G:

$$Y = Y^P = 80 = 28 + 2G^P + 0.2 \cdot 64 \text{ implies } G^P = (80 - 0.2 \cdot 64) / 2 = 19.6$$

The domestic country must increase government spending by 9.6. The domestic budget deficit increases by the same amount given it was previously balanced. The domestic trade balance deteriorates as it was previously balanced and is now  $NX = 0.1(Y^* - Y) = 0.1(64 - 80) = -1.6$ .

The domestic fiscal expansion raises foreign output by 4, improves their trade balance by 1.6, and has no effect on the foreign budget deficit.

- c. Denote the common level of government spending by  $x$  and assume the coordinated fiscal expansion is successful in both countries so  $Y = Y^* = Y^P$ . The system of equations that follows is presented below:

$$Y = Y^P = 80 = 2(14 + x + 0.1Y^P) = 28 + 2x + 0.2 \cdot 80 = 44 + 2x$$

$$Y^* = Y^P = 80 = 2(14+x+0.1Y^P) = 28+2x+0.2*80 = 44+2x$$

Solving for  $x$  in either of the equations yields  $G=G^*=18$ . So the coordinated action intuitively requires a smaller budget deficit than for a country acting alone, and there is no change in the trade balance which is zero since the countries are identical with identical output. In practice it is difficult for countries to credibly commit to the fiscal policy. It is possible for one country to say it will raise government spending by 8, but then not do it and reap the benefits of the other country's fiscal expansion. Even without promises, a troubled country may wait for others to be the engine of growth (maybe tells a story of Japan's "inability" to increase  $G$  and the US finally cutting interest rates despite low unemployment)

### Question 3.

- a. Credibly fixed exchange rates imply  $E^e=E$ . As this is believed to be a one-time depreciation, the story is that  $E$  increases but the new  $E^e$  will be equal to the new level of  $E$ . Graphically, this corresponds to a shift right in the IS curve (as net exports increase with the depreciation). There will be no change in the uncovered interest parity condition as financial markets expect no future depreciation, so the domestic interest rate must remain unchanged. The nominal depreciation and no expected depreciation require the expected exchange rate rise, however, so the UIP curve shifts right far enough for the new exchange rate to be consistent with the same old interest rate. The central bank must accomplish this through expansionary open market operations, shifting the LM curve to the right. Equilibrium output unambiguously increases and interest rates do not change.
- b. If financial markets expect some probability of a depreciation, it seems reasonable then the  $E^e > E$ , even if only by a little bit. A devaluation hurts foreign investors expecting to sell dollars in the future, so as this becomes more likely, domestic interest rates must increase, corresponding to a shift up in the interest parity curve. If the central bank does nothing to nominal interest rates, the currency will begin to devalue, because at the old domestic interest rate, a larger  $E$  is required for uncovered interest parity to be met. In response, the central bank must increase interest rates by reducing the money supply to save the fixed exchange rate. In this case interest rates increase and output falls. Note that to defend the currency, interest rates must rise and output must fall. Countries near recession or with large debts might be reluctant to take their medicine and simply devalue, even though they did not plan to in the first place.

There is a second effect worth mentioning. For domestic agent believing the currency to depreciate, they will reduce their money holdings and purchase foreign currency and foreign bonds, speculating that the domestic country will depreciate. This corresponds to a reduction the money supply as the money multiplier falls, shifting the LM curve up and to the left, similar to the contraction required by the central bank to maintain the exchange rate. This implies at least part the shift in the LM curve happens automatically as domestic agents lose faith in the central bank.