PROBLEM SET EIGHT SOLUTIONS

Problem 1.

Part a.

The price-setting (PS) relationship requires that the price of one good be equal to the marginal cost of producing the good times $(1+\mu)$ where μ is a markup over cost, reflecting the absence of perfect competition in product markets.

$Y = A*N$ is the production function, so production of one good requires $1=A*N$ or $N = 1/A$ units of labor. The cost of labor is W per unit, so the marginal cost of producing one unit is W/A. Price-setting thus requires:

(PS) $P = (1+\mu)*W/A,$

 or equivalently,

(PS)' $W = P*A/(1+\mu)$

Equilibrium in labor markets is described by the pair (W_0,u_0) which is consistent with both price-setting and wage-setting. This is simply the solution to the system of equations:

(PS) $W = P*A/(1+\mu)$
(WS) $W = P^e*z*\ln(1/u)$

The equilibrium wage is simply given by the price-setting relationship. Inserting this wage into the wage-setting relationship implies the following,

$P*A/(1+\mu) = P^e*z*\ln(1/u)$
$P*A/[(1+\mu)*P^e*z] = -\ln(u)$
$u = \exp[-P*A/(1+\mu)*P^e*z]$

So the actual equilibrium is described by $W_0 = P*A/(1+\mu)$ and $u_0 = \exp[-P*A/(1+\mu)*P^e*z]$.

Recall the definition of the unemployment rate,

$u = U/L$
$u-1 = U/L-1$
$1-u = 1-U/L$
$1-u = (L-U)/L$
$L*(1-u) = N$ noting $U+N=L$
$L*(1-u) = Y/A$ noting $Y = A*N$
$Y = A*L*(1-u)$
$Y = A*(1-u)$ noting $L = 1$ (normalized labor force)

Thus actual output is described by $Y_0 = A*(1-u_0) = A*(1- \exp[-P*A/(1+\mu)*P^e*z])$.

The natural rate of unemployment and natural level of output are constructed by imposing $P = P^e$.

$u_n = \exp[-A/(1+\mu)*z]$
$Y_n = A*(1-\exp[-A/(1+\mu)*z])$

Aggregate Supply is simply the equation for actual output above. Invert this so P is a function of Y.

$Y = A*(1- \exp[-P*A/(1+\mu)*P^e*z])$
$\exp[-P*A/(1+\mu)*P^e*z] = 1-Y/A$
$-P*A/(1+\mu)*P^e*z = \ln(1-Y/A)$
$P = P^e*(1+\mu)*z*\ln[1/(1-Y/A)]$
$P = P^e*(1+\mu)*z*\ln[A/(A-Y)]$

This is alternatively (and equivalently) derived by plugging the wage-setting equation into the price-setting equation and inverting $Y = A*L*(1-u)$ for u as a function of Y.

To verify its slope, note $dY/dP = [A^2/(1+\mu)*P^e*z]*\exp[-P*A/(1+\mu)*P^e*z] > 0$

Part b.

(IS) $Y^d = C + I + G = c_0 + c_1(Y^d-T) + d_0 + d_1*Y^d - d_2*i + G$
(LM) $i = P*Y^d/M$

Insert LM into IS to yield,

$Y^d = c_0 + c_1(Y^d-T) + d_0 + d_1*Y^d - d_2*P*Y^d/M + G$

Solving for Y^d,

$Y^d = [c_0 - c_1T + d_0 + G]/[1-c_1-d_1+d_2P/M]$

$dY^d/dP = -1*(d_2/M)[c_0 - c_1T + d_0 + G]/[1-c_1-d_1+d_2P/M]^2 < 0$

Part c.

Just plug in $A_1 > A_0$ into the expressions for Y_n and u_n. It should be clear that equilibrium output and nominal wages increase while unemployment falls.

This period's AS curve shifts right due to the rise in technical progress, but not far enough to reach the new natural level of output because price expectations need to adjust. Recall that the natural level requires $P = P^e$. This period output increases and prices fall.

The AS schedule evaluated at equilibrium prices and output is as follows:

$A_1*P_1 = P^e*z*(1+\mu)*\ln[A_1/(A_1-Y_1)]$ where $P^e=P_0$

In the long-run $P=P^e$

$A_1 = z*(1+\mu)*\ln[A_1/(A_1-Y_n)]$

Note that $A_1P_1/P_0 < A_1$ as $P_1 < P_0$ so we have $Y_1<Y_n$ after some algebra.

Next period, expected prices fall, shifting the AS curve right again. This further increases output and reduces prices. The process repeats until $P^e = P$ and $Y = Y_n$, where output is permanently higher with prices and unemployment permanently lower.

Part d.

This policy is not sustainable. The shift in the AS curve reduces prices, which shifts the LM curve to the right, lowering the interest rate. The central bank must react by contracting the money supply, which shifts both the LM and AD curves to the left. This reduces both output and prices. The fall in prices again forces the central bank to reduce the money supply, shifting AD left until both output and prices are zero.

Obviously, the central bank will be forced to abandon its interest-rate peg this period or equivalently, depreciate the currency.

Part e.

This policy is sustainable. The shift in the AS curve reduces prices, which shifts the LM curve to the right. The central bank can prevent this by increasing the money supply, which shifts the LM curve to the right, and shifts the AD curve far enough right so that there is no change in the price level, magnifying the effect of output. As prices don't change, there will be no further shifts in the AS curve. Moreover, it is easy to show that the new level of output is the new natural level of output. Price expectations are equal to last periods prices, which haven't changed. Thus $P = P^e$ and the long-run is now. The fixed price policy of the central bank prevents deflation and quickens the adjustment process to the new long-run equilibrium.

Part f.

Similar to part d above. The shift in AS curve reduces prices, which shift the LM curve to the right, increasing output. The central bank must react by contracting the money supply to meet its output target, further reducing prices this period. Equilibrium this period will correspond to the target output level at a price where the new aggregate supply curve (at the new level of technology A) intersects the target level of output. Next period, price expectations adjust and shift the AS curve right (as prices have fallen), and the process repeats itself. Prices are quickly driven down to zero, so the policy is not sustainable.

Question 2.

The natural rate of unemployment depends on the degree of competition in goods markets, unemployment insurance and structural change in the labor market, and from the above technological progress. There is no mechanism through which demand can affect the natural rate.

Question 3.

The traditional Phillips curve relied on $\Pi^e = 0$. This was true in a period when sometimes inflation was positive and sometimes negative so on average expectations were correct. Low (but positive) inflation will thus not return us to the original Phillips curve because agents would make systematic (and easily correctable) mistakes in their predictions by assuming expected inflation is zero. Thus the U.S. would not return to the curve of the 1950s and 1960s.