14.02 Principles of Macroeconomics, Spring 2000
5 pages

Problem Set 6
Answers

Question 1. (5 points each) True, False, Uncertain. Provide a brief explanation.

a) Suppose a person of Ukrainian descent works in Canada and regularly sends part of the earnings back home to her/his kin. Unlike export/import transactions of Canadian businesses, these earnings will be entered only in the current account, thus invalidating the *balance* of payments.

False. This transaction will be entered into the "Transfers" rubric of current account. However, either the bread-winner will convert the earnings into Ukrainian currency, or the recipients will do that themselves. Therefore, someone will buy Canadian dollars from them. This someone will either use them to invest in Canada or to buy Canadian merchandise. In either case the balance of payments will be restored.

b) As a financial investor choosing whether to channel funds to East Asia or Latin America to buy government bonds, you only care about interest rates in both places and expected shifts in nominal exchange rates vis-a-vis US$, regardless of expected inflation (provided there is no risk of default on bonds).

True. Once your bonds are paid off, you will not be spending the proceeds in the country where you invested. You will convert that money into US$. For those purposes it is sufficient to know what the nominal exchange rate vis-a-vis US$ will be (and the interest rate as well, in order to calculate how much you will receive in foreign currency). Inflation is therefore irrelevant for your decision.

c) Cynical politicians relying on unemployment-inflation trade-off to achieve temporary output gains would be severely disadvantaged by a move of the economy to wage indexation.

True. Introduction of indexed wage contracts would reduce the effectiveness of money creation in dragging the economy away from natural unemployment (read section 8-3 of the textbook). For a given decrease in unemployment rate, a much larger surprise inflation would be needed. Thus, the trade-off facing cynical politicians would worsen: for the same temporary gain in employment they would have to inflict more inflationary suffering. That would tie their hands.

d) Modern incarnation of the Phillips curve implies that it is still possible to *permanently* reduce unemployment by as much as one wants at the cost of very reasonable inflation rates: by inflating a bit, then waiting until public calms down, inflating again, waiting, and so forth.

False. When the public calms down, the level if inflation will be *permanently* higher at the restored natural unemployment rate. Trying to keep unemployment permanently below that natural rate will mean pushing inflation to infinity which is far from reasonable.

e) Which of the following trade policy measures is likely to cause a symmetric response (retaliation) from other governments, and hence makes countries unwilling to embark on the policies in the first place (explain in words, do not just give a single character answer):

1. (a) increase in import tariffs
   (b) fiscal expansion
   (c) barriers to capital mobility
   (d) devaluation (under fixed exchange rate regime only)
   (e) all of the above
   (f) a, b, and d
   (g) only a and d

Answer: g. Increases in import tariffs and devaluations cause retaliation because these policies benefit the country at the expense of the other economies. However, fiscal expansions do good to the other countries; there is no need to retaliate. Countries are unwilling to embark unilaterally on fiscal expansions because they worsen their balance of payments. If a country, attracting investment, closes for foreign funds, it does not care if other countries restrict capital mobility, since it is already insulated from capital movements.

Question 2 (points) **Opening goods markets and financial markets.**

Consider 2 adjacent economies which only trade with each other, described by the following equations (starred variables refer to the foreign country):

$C = 10 + 0.6\,(Y - T)$	$C^* = 5 + 0.3\,(Y^* - T^*)$
$G = 10$	$G^* = 5$
$T = 10$	$T^* = 10$
$Q = Y/10$	$Q^* = Y^*/10$
$X = 0.1Y^*$	$X^* = 0.1Y$

There is no investment. For this reason, assume also that there are no capital movements between these two countries.

a) (points) Compute the equilibrium outputs in the two countries (you may assume that each country takes the other country's output as given, not taking into account the influence of imports on that). Compute the trade balance of each country.

$$\begin{cases} Y = 10 + 0.6\,(Y - 10) + G + 0.1Y^* - Y/10 \\ Y^* = 5 + 0.3\,(Y^* - 10) + G^* + 0.1Y - Y^*/10 \end{cases} \Leftrightarrow \begin{cases} 0.5Y = 14 + 0.1Y^* \\ 0.8Y^* = 7 + 0.1Y \end{cases}$$

$$\begin{cases} Y \approx 30.513 \\ Y^* \approx 12.564 \end{cases}$$

$$X - Q = 0.1\,(Y^* - Y) = 0.1\,(12.564 - 30.513) = -1.7949$$

Since trade balances of the two countries add up to identical zero, $X^* - Q^* = 1.7949$.

b) (points) By how much should countries increase their spending G, G^*, to achieve target levels of output at \$40 and \$*15, if they decided to act separately?

In equilibrium, country 1 perceives its output as $0.5Y = G + 4 + 0.1Y^* = G + 4 + 1.2564 = G + 5.2564 \Rightarrow Y = 2\,(G + 5.2564)$. To achieve $Y = 40$, $G = 14.744$.

In the same fashion,

$$Y^* = \frac{G^* + 2 + 0.1Y}{0.8} = \frac{G^* + 5.0513}{0.8}$$

This gives desired spending at $G = 6.9487$.

c) (points) What if the governments could coordinate their actions? Would your answer to b) change?

Combine the two equations: $\begin{cases} 0.5Y = G + 4 + 0.1Y^* \\ 0.8Y^* = G^* + 2 + 0.1Y \end{cases}$. Setting $Y = 40$, $Y^* = 15$ gives the required spending levels at $G = 14.5$, $G^* = 6$. Both are lower than in b) because in coordinating their efforts, countries take into account positive influences on each other.

For the remainder of the question, assume that there is also investment in both countries. Inflation never happens in this world, so there is no difference between nominal and real exchange rate, denoted by E. Finally, international financial markets open up, exchange rates are flexible. The modified equations are given by (disregard all numbers from above in what follows, use abstract formulae)

$$Y = C(Y - T) + I(Y, i) + G + NX(Y, Y^*, E) \quad (1)$$
$$M/P = YL(i) \quad (2)$$

d) (points) Write down the condition on E, E^e, i, i^*, that ensures that foreign exchange markets are in equilibrium. Modify it so that E is on the left hand side. Substitute this expression for E into (1). Observe that you have just given an equation for the IS curve under openness.

Use the uncovered interest parity condition (UIP): $i - i^* = \frac{E^e - E}{E}$. Solving this in terms of E gives

$$E = \frac{E^e}{1 + i - i^*}. \tag{3}$$

Substituting into (1):

$$Y = C(Y - T) + I(Y, i) + G + NX\left(Y, Y^*, \frac{E^e}{1 + i - i^*}\right)$$

e) (points) Analyze the effects of a tax cut on Y, i, E, I, NX. Did the change in exchange rate amplify or reduce the effect of the cut on output?

As always, the tax cut shifts out the (modified) IS curve. This pushes up the interest rate and increases output. Higher interest rate implies that E should go down (3). This is the appreciation of the currency. Intuitively, higher interest rates make the country attractive for foreign capital, increase the demand for domestic currency and strengthen the exchange rate. Investment goes up due to higher level of aggregate activity ($Y \uparrow$) but there is offsetting influence of the interest rate as well. The overall effect depends on the relative strengths of these two effect. Net exports decrease due to both increase in Y (larger imports) and lower E (lower exports and larger imports).

Summary: $Y \uparrow, i \uparrow, E \downarrow, I?, NX \downarrow$.

Appreciation has a dampening effect on the output because of the worsened trade balance.

f) (points) How would your analysis in e) change if the exchange rate were fixed??

If exchange rates were fixed, $E^e = E$, so $i = i^*$ in equilibrium. That means that after the tax cut higher interest rate will cause capital inflow, that translates into increase in money supply under a fixed exchange rate. This shifts the LM curve out until the interest rate declines back to the international level. The output will rise even more than under the flexible regime, there will be no effect on i, no effect on E, positive effect on investment, and a worsening of the balance of payments (Q goes up due to higher Y).

Question 3 (points) Interest parity conditions.

Consider the price schedule for government bonds and foreign exchange in the United States and Russia. Both government bonds are one-year securities. The current exchange rate E stands at \$0.2/1R (Russian currency is R for *rouble).*

Bond	Face Value	Price	Currency
US	10,000	9,708.74	US$
Russia	100,000	61,349.69	*Rouble*

a. Calculate the nominal interest rate on each of the bonds and the expected exchange rate next year consistent with Uncovered Interest Parity. Note whether this signifies an expected appreciation or depreciation of the *rouble.*

Answer. The nominal return on each of the bonds

$$\begin{aligned} i_{US} &= \frac{FV_{US}}{P_{US}} - 1 = \frac{10,000}{9,708.74} - 1 = 0.03 \\ i_{RU} &= \frac{FV_{RU}}{P_{RU}} - 1 = \frac{100,000}{61,349.69} - 1 = 0.63 \end{aligned}$$

Also recall that uncovered interest parity is stated as $i_{US} - i_{RU} = \frac{E^e - E}{E}$

It easily follows that one can solve for E^e, the expected nominal exchange rate in one year: $E^e = E(i_{US} - i_{RU} + 1) = 0.2(0.03 - 0.63 + 1) = 0.08$

Since $E^e < E$ we have an expected APPRECIATION of the dollar relative to the rouble.

b. Assume you exchange dollars for roubles and purchase the Russian bond, but one year from now it turns out that E is actually \$0.05/1R. What is your actual nominal return compared to the return if you had just purchased the US bond? Are these differences in returns consistent with arbitrage?

Answer: The actual nominal return on the US bond remains 3%, but purchase of the Russian bond required a position in roubles, so the unexpected appreciation of the US dollar affects your return. Since you have to buy dollars when your bond matures and the price of dollars has unexpectedly risen, the nominal return from purchase of the Russian bond ex post is lower than the US bond. Mathematically, we can write this nominal return as follows:

$i_{RU} + \frac{E_{t+1} - E_t}{E_t} = 0.63 + \frac{0.05-0.2}{0.2} = -.12 < 0.03 = i_{US}$

(Note: you would get a substantially different answer if you used the "precise" UIP condition and not the approximate one. For a), it would give $E^e = 1.03/1.63 \cdot 0.2 = 0.12$; the nominal return in b) would be $(1+i_{ru})(1+\frac{E^e-E}{E}) = 0.40$, a nominal return of -0.6!! This is because $i_{ru} = 0.63$ is not a small percentage and so the approximation is quite imprecise.)

Looking backwards one year from now, the nominal return falls short of that on the US bond. This does not mean, however, that no one should have bought Russian bonds given their information one year ago. It is also possible for there to be lower appreciation of the dollar than expected, which would turn your position in Russian currency into a fabulous opportunity.

The point of this question is that the type of arbitrage necessary for uncovered interest parity conditions to hold is arbitrage between *expected* returns. Although the use of the term "arbitrage" suggests free money, there is nothing riskless about uncovered positions in foreign currency.

We tend to think identical goods should have the same prices. If identical goods have different prices, it is possible to buy at the low price and sell at the high price and make lots of money. Arbitrage refers by actions by agents taking advantage of these kinds of price differentials. You should discern between two types of arbitrage. There is *expected* arbitrage, which is taking advantage of differences in *expected* return. This means you expect to make money, but will sometimes do worse and other times do better, so you will make money on average. On the other hand there is *riskless* arbitrage, which means you will always make money regardless of the course of future events. The uncovered interest parity condition is an equilibrium condition equivalent to **no expected arbitrage opportunities** while the *covered* interest parity condition below is equivalent to **no riskless arbitrage opportunities**.

c. Assume that there exists a market for buying and selling foreign exchange one-year in the future, but fixing the price of the transaction today. Denote the forward price of one rouble in terms of dollars by F. In other words, you can enter into a contract today to sell one rouble for F dollars one year in the future. Derive the following approximation to the Covered Interest Parity as stated below:

$$i_{US} = i_{RU} + \frac{(F-E)}{E}$$

Answer. The nominal gross return on the US bond is simply the nominal interest rate as before. On the other hand, instead of taking an uncovered position in foreign currency, consider a strategy of selling roubles forward in the amount you expect to have one year from today. One US dollar converts into 1/E rouble, which, invested in the Russian bond, will yield $\frac{1+i_{RU}}{E}$ roubles next year. Sell this amount forward today at a price of F \$/R so the that nominal return on this investment strategy is $\frac{F(1+i_{RU})}{E}$. If nominal returns on each strategy are equal, then we have the following interest parity condition:

$$1 + i_{US} = \frac{F(1+i_{RU})}{E}$$

We can derive an approximation to covered interest parity:

$$i_{US} - i_{RU} = \frac{F-E}{E}$$

d. What is the forward price of 1 rouble consistent with Covered Interest Parity? Compare actual nominal returns between the two strategies if next year E is actually 0.05 as above. Is Covered Interest Parity between the two 1-year bonds really riskless arbitrage?

Answer. Solving for F in the interest parity condition above, it follows: $F = (1 + i_{US} - i_{RU}) E = 0.2 (1 + 0.03 - 0.63) = .08$.

Next years' exchange rate has no effect on the nominal returns of either strategy, because we have covered our position in Russian currency by selling roubles forward. For one-year bonds, departures from covered interest parity are opportunities for riskless arbitrage.

Question 4 (points)

Consider a production function $Y = AN$, where A is a technological index

a) How should the basic price setting equation $P = (1 + \mu) W$ be adjusted to reflect varying productivity?

Answer. $P = \dfrac{(1 + \mu) W}{A}$

b) Suppose that workers make forecasts of future productivity when making their wage demands, so that the wage-setting equation is as follows:

$$W = P^e A^e F(u, z).$$

We start with $A_0 = 1$. Suddenly, due to indefatigable R&D effort, A increases to $A_1 > 1$. Combine the two equations to find the new natural unemployment rate in the long run under the assumption that technological growth stops (hint: long run $\Leftrightarrow A = A^e$, $P = P^e$). What about the full-employment output?

Answer. $P = P^e \dfrac{A^e}{A} (1 + \mu) F(u, z)$. Using $A = A^e$, $P = P^e$, $1 = (1 + \mu)(1 - \alpha u + z)$ in the long-run equilibrium. Therefore, the natural rate is given by $u^N = \dfrac{z + \mu + \mu z}{\alpha + \mu\alpha} \approx \dfrac{z + \mu}{\alpha}$ if we ignore the second-order terms. Natural level of output is given by $Y^N/N = A_1$. Note that $Y^N/L = A_1 (1 - u) = A_1 \frac{\alpha - z - \mu}{\alpha} > A_0 \frac{\alpha - z - \mu}{\alpha}$. Natural rate of unemployment is unchanged but output is higher.

c) Do the same Phillips curve derivation as in ch. 8-1, but adjusted for possibly discrepant A and A^e. Denote rates of technological progress by a, a^e.

Answer: As in one of our first lectures and in the appendix to chapter 8,

$$\begin{aligned}
P_t &= P_t^e \frac{A_t^e}{A_t} (1 + \mu)(1 - \alpha u_t + z) \\
\frac{P_t}{P_{t-1}} &= \frac{P_t^e}{P_{t-1}} \frac{A_t^e}{A_{t-1}} \frac{A_{t-1}}{A_t} (1 + \mu)(1 - \alpha u_t + z) \\
1 + \pi_t &= (1 + \pi_t^e) \frac{(1 + a_t^e)}{(1 + a_t)} (1 + \mu)(1 - \alpha u_t + z) \\
1 + \pi_t &\approx (1 + \pi_t^e)(1 + a_t^e - a_t)(1 + \mu)(1 - \alpha u_t + z) \\
&\approx 1 + \pi_t^e + a_t^e - a_t - \alpha u_t + \mu + z
\end{aligned}$$

Removing the approximate equality sign and assuming as in the book, that $\pi_t^e = \pi_{t-1}$, the Phillips curve is given by

$$\pi_t - \pi_{t-1} = (a_t^e - a_t) + (\mu + z) - \alpha u_t.$$

d) Now suppose that in period t, the labor market reached the equilibrium in c) but imagine that workers are expecting the output growth to be continued in period $t + 1$. What will happen to unemployment and inflation in periods $t + 1$? Upon observing no progress, workers revise their wage demands for $t + 2$ and stop expecting progress in the future. What will happen in periods $t + 2$, $t + 3$, ... ?

Answer. Period $t : 0 = \mu + z - \alpha u^N$ (since expectations of productivity growth were correct $a_t = a_t^e$).

Period $t + 1 : \pi_{t+1} - \pi_t = a_{t+1}^e - 0 + \mu + z - \alpha u_{t+1}$. Since the Central Bank keeps on with the policy of *no* surprise inflation, $\pi_{t+1} - \pi_t = 0 = a_{t+1}^e - 0 + \mu + z - \alpha u_{t+1}$ implying that $u_{t+1} = \dfrac{1}{\alpha} \left(a_{t+1}^e + \mu + z\right) > u^N$.

Period $t + 2$: Workers have revised their expectations of productivity to the existing level A_1. This means $a_{t+2}^e = 0 = a_{t+2}$,

$$\pi_{t+2} - \pi_{t+1} = a_{t+2}^e - a_{t+2} + \mu + z - \alpha u_{t+2} = \mu + z - \alpha u_{t+2} = 0.$$

$u_{t+2} = \dfrac{1}{\alpha} (\mu + z) = u^N$. Unemployment is back to natural rate because the productivity expectations, and hence, wage demands have been adjusted.