

**PROBLEM SET 2 Solutions**  
**14.02 Principles of Macroeconomics**  
**February 23, 2005**  
**Due March 2, 2005**

**I. Answer each as True, False, or Uncertain, providing a few sentences of explanation for your choice.**

1. For a bond which promises a fixed payment in one year, the lower its price the higher the nominal interest rate.

**True.** By definition, the nominal interest rate is the rate of return of a bond of fixed nominal payment  $F$ . We can express the rate of return of this bond as  $i = \frac{(\$F - \$P_B)}{\$P_B}$ , which gives  $\$P_B = \frac{\$F}{1+i}$ . Thus, a lower bond price requires a higher nominal interest rate.

2. During recessions, governments can always choose to revive the economy through monetary expansion.

**False.** When the nominal interest rate is already 0 (liquidity trap), governments cannot increase money supply.

3. Investment decreases if government spending increases.

**Uncertain.** Given  $I = I(Y, i)$ ,  $\frac{\partial I}{\partial G} = \frac{\partial I}{\partial Y} \frac{\partial Y}{\partial G} + \frac{\partial I}{\partial i} \frac{\partial i}{\partial Y} \frac{\partial Y}{\partial G} = \frac{\partial Y}{\partial G} \left[ \frac{\partial I}{\partial Y} + \frac{\partial I}{\partial i} \frac{\partial i}{\partial Y} \right] \leq 0$  since  $\frac{\partial I}{\partial Y} + \frac{\partial I}{\partial i} \frac{\partial i}{\partial Y} \leq 0$ . Investment increases if  $\frac{\partial I}{\partial Y}$ , the increase in investment induced by an increase in output, is higher (through the goods market) than the decrease in investment induced by an increase in interest rate (through the money market). Investment decreases otherwise.

4. In equilibrium, an increase in money supply ( $M^s$ ) may lead to an increase in the interest rate. It is because the goods market can still be in equilibrium given that investment ( $I(Y, i)$ ) is responsive enough to an increase in output.

**False.** With an exogenous increase in money supply, interest rate has to decrease to clear the money market ( $M^S = M^d$ ).

5. According to the efficiency wage theories, a lower unemployment rate leads to higher real wages.

**True.** When the unemployment rate is low, workers know that it is easy to find jobs. Firms tend to increase wages to retain their employees.

6. The real wage decreases if the number of unemployed increases.

**Uncertain.** The wage-setting equation says that the real wage is negatively correlated with the unemployment rate. However, an increase in the number of unemployed may not mean an increase in the unemployment rate. For example,  $u = \frac{U}{L} = \frac{L-N}{L} = 1 - \frac{N}{L}$ , where  $L$  is the labor force,  $N$  is the number of employed. Suppose that  $U$  increase to  $U'$  and  $L$  increases to  $L'$ . If  $N$  increases proportionally less than  $U$ , but more than  $L$ , we have a lower unemployment rate but a higher number of employed.

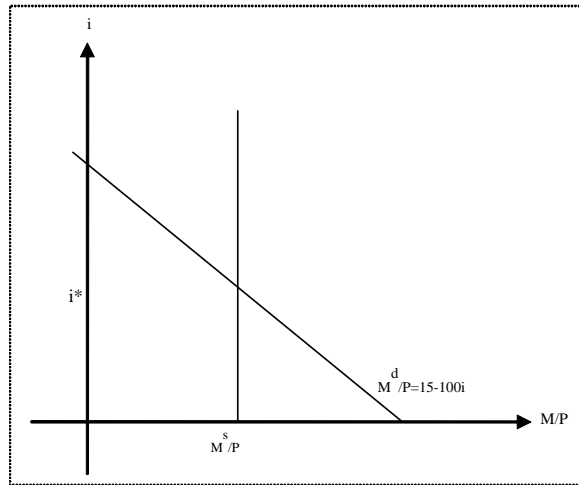


Figure 1: Money Market

## II. Short questions

### 1. Money Market

Consider an economy without private banks. Suppose that the money demand function is  $\left(\frac{M}{P}\right)^d = 15 - 100i$

The money supply  $M$  is 10 billion dollars and the price level  $P$  is 2.

a. Graph the supply and demand for real money balances.

Ans: See Figure 1.

b. What is the equilibrium interest rate?

Ans:

$$\begin{aligned} \frac{10}{2} &= 15 - 100i \\ 5 - 15 &= -100i \\ i &= \frac{10}{100} = 10\% \end{aligned}$$

c. Assume that the price level is fixed. What happens to the equilibrium interest rate if the money supply is raised from 10 to 24.

Ans:

$$\begin{aligned} \frac{24}{2} &= 15 - 100i \\ 12 - 15 &= -100i \\ i &= \frac{3}{100} = 3\% \end{aligned}$$

d. If the Fed wishes to stimulate business investment by lowering interest rate from that in (b) to 2%, what money supply should it set? How much bonds should it buy from the public.

*Ans:*

$$\begin{aligned}\frac{M'}{2} &= 15 - 100(0.02) \\ &= 15 - 2 \\ M' &= 26\end{aligned}$$

In the Fed's balance sheet,  $Bonds = Currency$ , so the Fed should buy  $M' - M = 26 - 10 = 16$  (billion).

## 2. ISLM

Consider the following stylized economy.

Good Market

Goods Demand:  $Z = C + I + G$

Consumption:  $C = 30 + 0.5Y_D$

Investment:  $I = 10 + 0.25Y - 200i$

Government Exp.:  $G = 30$

Financial Market

Money Supply:  $\frac{M^s}{P} = 36$

Money Demand:  $\frac{M^d}{P} = 0.2Y - 240i$

a. Assume a balanced budget. Derive the IS relation.

*Ans:*

Consider the goods market equilibrium ( $Y = Z$ ) and balanced budget ( $T = G = 30$ )

$$\begin{aligned}Y &= Z = \underbrace{30 + 0.5(Y - T)}_C + \underbrace{10 + 0.25Y - 200i}_I + \underbrace{30}_G \\ Y &= 0.75Y - 0.5T + 70 - 200i \\ 0.25Y &= -0.5G + 70 - 200i \\ &= -15 + 70 - 200i\end{aligned}$$

$$IS : Y = 220 - 800i$$

b. Derive the LM relation.

*Ans:*

Consider the money market equilibrium ( $\frac{M^d}{P} = \frac{M^s}{P}$ )

$$36 = 0.2Y - 240i$$

$$LM : Y = 180 + 1200i$$

c. Solve for the equilibrium interest rate, equilibrium output, equilibrium private consumption and equilibrium investment.

*Ans:*

For the two markets to clear at the same time, set  $IS = LM$  :

$$\begin{aligned}220 - 800i^* &= 180 + 1200i^* \\40 &= 2000i^*\end{aligned}$$

$$i^* = 2\%$$

From the LM relation:

$$\begin{aligned}Y^* &= 180 + 1200(0.02) \\&= 204\end{aligned}$$

From the given Consumption Function:

$$\begin{aligned}C^* &= 30 + 0.5Y_D^* \\&= 30 + 0.5(204 - 30) \\&= 30 + 87 \\&= 117\end{aligned}$$

From the given Investment Function:

$$\begin{aligned}I^* &= 10 + 0.25Y - 200i \\&= 10 + 0.25(204) - 200(0.02) \\&= 57\end{aligned}$$

### 3. Labor Market

Consider a small economy with total population 20 million, 16 million of them are noninstitutional civilians, 6 million of them are *not* looking for jobs and 9.2 million of them are employed.

a. Find the participation rate and the unemployment rate in this economy.

*Ans:*

Let  $\theta$  be the participation rate.

$$\theta = \frac{\text{labor force}}{\text{non-inst. civilians}} = \frac{16 - 6}{16} = 62.5\%$$

Let  $u$  be the unemployment rate.

$$\begin{aligned}
 u &= \frac{\# \text{ of unemployed}}{\text{labor force}} \\
 &= \frac{10 - 9.2}{10} = 8\%
 \end{aligned}$$

b. After a long recession (5 years), 0.2 million of them retired and 0.1 million gave up looking for jobs. Total number of employed decreased to 9 million. What is the unemployment rate now?

$$\begin{aligned}
 u &= \frac{(10 - 0.2 - 0.1) - 9}{10 - 0.2 - 0.1} = \frac{0.7}{9.7} \\
 &\approx 7.22\%
 \end{aligned}$$

c. The wage-setting relation is  $\frac{W}{P} = k - u$ , where  $0 \leq k \leq 1$  is a measure of the level of unemployment benefits. (Don't worry about  $\frac{W}{P}$  being negative.) The price-setting relation is  $\frac{P}{W} = 1 + \mu$ , where  $\mu$  captures the competitiveness of the goods market. Express the natural rate of unemployment as a function of  $\mu$  and  $k$ .

Ans:

$$\begin{aligned}
 k - u_n &= \frac{1}{1 + \mu} \\
 u_n &= k - \frac{1}{1 + \mu}
 \end{aligned}$$

d. What happens if unemployment benefits increase? Provide some intuition. Suppose that the government has to raise unemployment benefits because of political pressure, what else can it do in order to reduce  $u_n$ ?

Ans: The natural rate of unemployment increases. When unemployment benefits increase, it becomes less painful to be unemployed. The opportunity cost of working is higher.

The government can increase competition in the goods market (reduce markup). Examples include a more stringent enforcement of existing antitrust legislation to reduce monopoly power and collusion.

### III. Long question (Policy Mix)

Suppose the goods market is described as follows :

$c_0$  is the consumer confidence.

$c_1$  is the marginal propensity to consume.

$Y$  is the output or income.

$I$  is the investment, which is equal to  $K_0$  when the interest rate  $i = 0$  and output  $Y = 0$ . If  $i$  is positive,  $I$  decreases by  $a$  for each unit increase in  $i$ . If  $Y$  is positive,  $I$  increases by  $b$  for each unit increase in output.

$G$  is government spending, exogenously given at  $G_0$ .

There is no tax for this economy.

$\frac{M^s}{P}$  is the real money supply. Assume that  $P = 1$  and  $M^s = M_0$ .

$\frac{M^d}{P} = gY - hi$ .

Assume  $c_1 + b < 1$ .

1. Write down the equations for consumption, investment and aggregate demand in the economy.

*Ans:*

$$C = c_0 + c_1Y$$

$$I = K_0 - ai + bY$$

$$Z = c_0 + c_1Y + K_0 - ai + bY + G_0$$

2. Derive the equation for the IS curve.

*Ans:*

$$Y^* = Z = c_0 + (c_1 + b)Y^* + K_0 - ai + G_0$$

$$Y^* = \frac{1}{1 - c_1 - b} (c_0 + K_0 - ai + G_0)$$

Equivalently,

$$IS : i = \frac{1}{a} [(c_1 + b - 1)Y^* + G_0 + c_0 + K_0] \quad (1)$$

3. Derive the equation for the LM curve.

*Ans:*

$$\begin{aligned} \frac{M^s}{P} &= \frac{M^d}{P} \\ M_0 &= gY^* - hi \end{aligned}$$

$$LM : i = \frac{1}{h} (gY^* - M_0) \quad (2)$$

4. Draw IS and LM curves on an  $i$ - $Y$  space with correct labels. Also label the intercepts.

*Ans: See figure 2 below.*

Now you have a short-run framework for policy studies.

5. Assume that there is a decrease in business confidence in period 1 ( $K_0$  decreases to  $K_1 < K_0$ ). Assume that the government cares only about output, nothing else. If you were an economic advisor to the government, would you suggest the government to revive the economy in period 2 (restore  $Y_2^*$  back

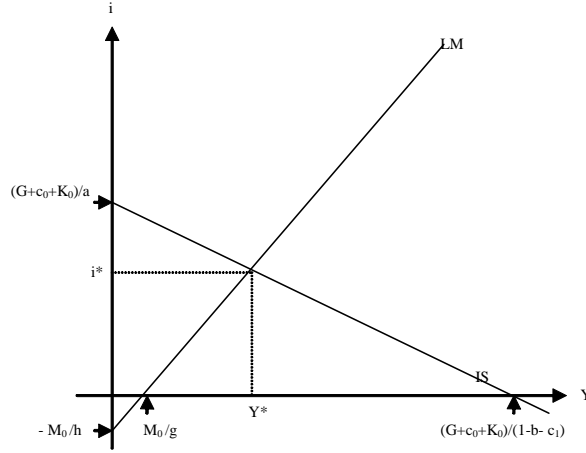


Figure 2: IS-LM for part 4.

to  $Y_0^*$ ) through fiscal expansion (an increase in  $G$ ), monetary expansion (an increase in money supply) or both? Show your results graphically.

Ans: See Figure 3 to 5 below. If the government cares only about output, either monetary or fiscal expansion (or both) can return the output level back to the original level.

6. If the government chooses monetary expansion, find an analytical expression for  $\Delta M_2 = M_2 - M_1$  such that  $Y_2^*$  is restored to  $Y_0^*$ .

Ans: Set  $LM = IS$  to find equilibrium  $Y^*$  which clears both goods and money markets.

$$\begin{aligned} \frac{1}{h}(gY_t^* - M_t) &= \frac{1}{a}[(c_1 + b - 1)Y_t^* + G_t + c_0 + K_t] \\ Y_t^* \left( \frac{g}{h} - \frac{c_1 + b - 1}{a} \right) &= \frac{1}{a}(G_t + c_0 + K_t) + \frac{M_t}{h} \\ Y_t^* &= \left( \frac{g}{h} - \frac{c_1 + b - 1}{a} \right)^{-1} \left[ \frac{1}{a}(G_t + c_0 + K_t) + \frac{M_t}{h} \right] \quad (3) \end{aligned}$$

What is the change of output due to a decrease in  $K_0$ ?

$$\begin{aligned} \Delta Y_1^* &= Y_1^* - Y_0^* \\ &= \left( \frac{g}{h} - \frac{c_1 + b - 1}{a} \right)^{-1} \left[ \frac{\Delta K_1}{a} \right] \end{aligned}$$

Using equation (3) again to derive the required increase in money supply to restore  $Y_2^*$  back to  $Y_0^*$

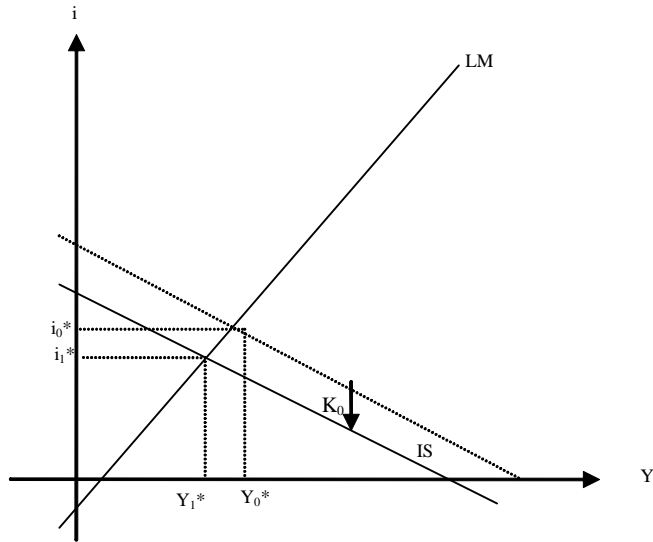


Figure 3: IS curve shifts down due to a decrease in  $K_0$

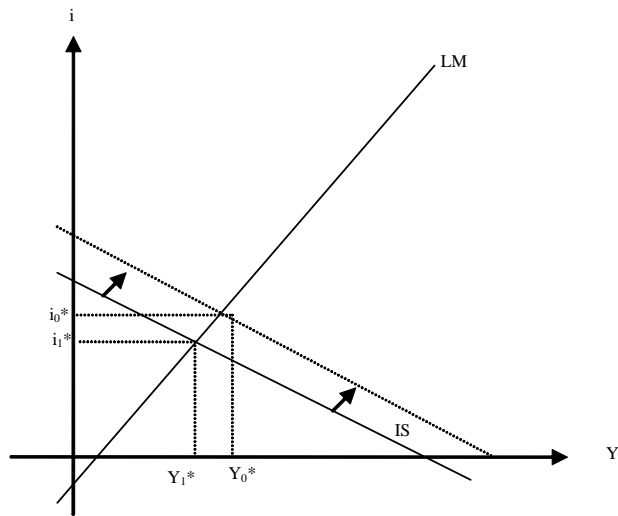


Figure 4: Increase  $G$ , IS shifts up



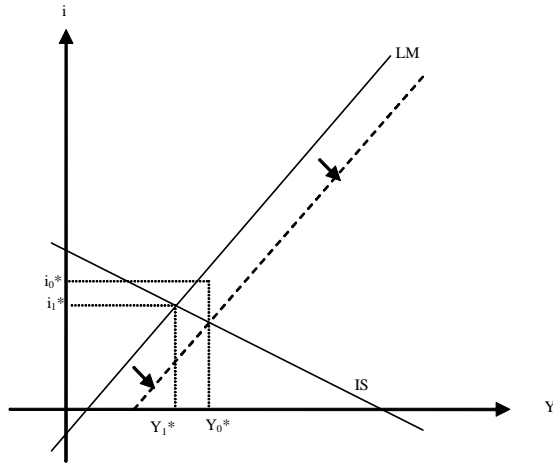


Figure 5: Increase  $M^s$ , LM shifts down

$$\begin{aligned} \left(\frac{g}{h} - \frac{c_1 + b - 1}{a}\right)^{-1} \left[\frac{-\Delta K_1}{a}\right] &= \Delta Y_2^* \\ &= \left(\frac{g}{h} - \frac{c_1 + b - 1}{a}\right)^{-1} \left(\left[\frac{\Delta M_2}{h}\right]\right) \\ \Delta M_2 &= \frac{-h \Delta K_1}{a} \end{aligned}$$

7. Suppose the government now cares about the composition of output as well as its level. Like before, it wants to restore the output level to  $Y_0^*$ . Can the government be indifferent between fiscal and monetary policy? Explain.

Ans: The government is not indifferent between the two policies if it cares about investment. Since investment is negatively related to the interest rate, increasing  $G$  is not the optimal choice for the government. It is because higher government spending increases aggregate demand and thus, shifts the IS curve up. A higher equilibrium interest rate depresses investment. In this case, the government prefers to employ monetary expansion which increases output and decreases equilibrium interest rate, raising investment.