PROBLEM SET 3 SOLUTIONS
14.02 Introductory Macroeconomics
March 16, 2005

I. Answer each as True, False, or Uncertain, providing some explanation for your choice.

1. The aggregate demand curve is downward sloping because people demand fewer goods at higher prices.

FALSE. The AD curve is downward sloping because an increase in prices reduces real money supply, which induces an interest rate increase (for the money market to be in equilibrium) which in turn induces an output decrease (through the goods market).

2. The neutrality of money implies that monetary policy cannot affect the output level.

FALSE/UNCERTAIN. Money is neutral in the medium run. In the short run, monetary policy can affect the output level.

3. The Phillips curve relation implies that when output is below its natural level, the inflation rate increases.

FALSE. When output is below its natural level, the unemployment rate is above the natural rate, so the Phillips curve relation implies that the inflation rate should decrease.

4. The same Phillips curve relation describes equally well episodes of positive and negative inflation.

FALSE. During deflationary episodes, the Phillips curve relation appears to weaken, as described at the end of Chapter 8.

II. AS-AD

Consider the following economy :

1. Consumption function : $C = c_0 + c_1(Y - T_0)$ (taxes are some constant lump sum amount)
2. Investment function : $I = I_0 + bY - ai$
3. Government spending : $G = \overline{G}$ (a constant)
4. Money demand function : $M^d/P = hY - ki$
5. Money supply : $M^s/P = M_0/P$ (M_0 is a constant)
6. Wage setting : $W/P^e = \alpha - \beta u$
7. Production function : $Y = AN$
8. Price setting : $P = (1 + \mu)W/A$

Note : $c_0, c_1, b, a, h, k, \alpha, \beta, A, \mu, c_0, I_0$ are all positive parameters

(a) Derive the AS and AD curves (assume the labor force is L). Verify that they are upward and downward sloping respectively.

To derive the AS curve, we use equations 6 through 8, and eliminate W and u from them, so

$AP/(1 + \mu) = P^e(\alpha - \beta(1 - Y/AL))$

which rearranged reads $P = P^e(\alpha - \beta(1 - Y/AL))(1 + \mu)/A$ (AS curve)

with derivative $\frac{\partial P}{\partial Y} = P^e\beta(1 + \mu)/A^2L > 0$

To derive the AD curve, we use equations 1 through 5 to obtain goods and money market equilibrium conditions, and then combine these to eliminate i

The goods market equilibrium condition is $Y = C + I + G = c_0 + c_1(Y - T_0) + I_0 + bY - ai + \overline{G}$

while the money market equilibrium condition is $M_0/P = hY - ki$

Substituting $i = \frac{1}{k}(hY - M_0/P)$ from the latter equation into the former we get

$Y(1 - c_1 - b + \frac{ah}{k}) = c_0 - c_1T_0 + I_0 + \frac{a}{k}M_0/P + \overline{G}$

which rearranged reads $P = \frac{a}{k}M_0/[Y(1 - c_1 - b + \frac{ah}{k}) - (c_0 - c_1T_0 + I_0 + \overline{G})]$ (AD curve)

with derivative $\frac{\partial P}{\partial Y} = -\frac{a}{k}\frac{M_0}{[Y(1-c_1-b+\frac{ah}{k})-(c_0-c_1T_0+I_0+\overline{G})]^2}(1 - c_1 - b + \frac{ah}{k}) < 0$

(b) If workers never made expectational errors, what would the AS curve look like? Why?

The AS curve would be vertical at Y_n, the natural level of output, since the unemployment rate would always be equal to the natural rate.

(c) Suppose the economy described by these equations is in medium run equilibrium, and then it is perturbed by the following policy mix : an increase in taxes (say from T_0 to T_1) and an increase in money supply (say from M_0 to M_1). This policy mix may give rise to more than one outcome in the short run, and associated with each of the possible outcomes would be a unique dynamic adjustment process. Using the equations given above, derive the conditions under which

      (i) the dynamic adjustment would involve rising prices

      (ii) the dynamic adjustment would involve falling prices

In order to solve this question, make the following assumptions about the parameters : $1 - c_1 - b + ah/k > 0$, $\alpha > \beta$, and $(\alpha - \beta)(1 - c_1 - b + \frac{ah}{k}) > (c_0 - c_1T_1 + I_0 + \overline{G})\beta/(AL)$

First compute the initial equilibrium level of output. This must be the natural level of output since the economy is in medium run equilibrium. The natural rate of unemployment is given by setting $P = P^e$ in the WS and PS equations, so

$\alpha - \beta u_n = A/(1 + \mu)$

which gives $u_n = \frac{\alpha - A/(1+\mu)}{\beta}$

which gives $Y_n = AL(1 - \frac{\alpha - A/(1+\mu)}{\beta})$

Second, consider the effects of this policy mix. Its immediate impact is only on the AD curve. The increase in taxes shifts it down and to the left while the increase in money supply shifts it up and to the right. So there are three possible outcomes - the AD curve on net shifts down and to the left, OR up and to the right, OR not at all. If it shifted down and to the left, then we would have a short run equilibrium with output below the natural level, so the subsequent adjustment to the new medium run equilibrium (with output equal to the natural level but a lower price) would involve falling prices. On the other hand, if it shifted up and to the right, then we would have a short run equilibrium with output above the natural level, so the subsequent adjustment to the new medium run equilibrium (with output equal to the natural level but

a higher price) would involve rising prices. So essentially, the question asks you to write down the mathematical conditions under which the new short run equilibrium output is above (this would correspond to (ii)) or below (this would correspond to (i)) the natural level of output.

To do this, first write down the equilibrium equation for the economy, i.e., eliminate P from the AS and AD curves

$$\frac{a}{k} M_1 / [Y(1-c_1-b+\frac{ah}{k})-(c_0-c_1 T_1+I_0+\overline{G})] = P^e(\alpha-\beta(1-Y/AL))(1+\mu)/A$$

Note that I use M_1 and T_1 in this equilibrium equation, since I am computing the immediate-post-shock short run equilibrium output. Next solve this for Y. This is complicated algebra - we have a quadratic equation in Y that can be written as follows :

$$\theta_2\theta_5 Y^2 + (\theta_4\theta_2+\theta_3\theta_5)Y + (\theta_4\theta_3-\theta_1) = 0$$

where $\theta_1 = aM_1/k > 0$

$\theta_2 = (1-c_1-b+\frac{ah}{k}) > 0$

$\theta_3 = -(c_0-c_1 T_1+I_0+\overline{G}) < 0$

$\theta_4 = P^e(\alpha-\beta)(1+\mu)/A > 0$

$\theta_5 = P^e\beta(1+\mu)/(A^2 L) > 0$

Given the assumptions on the signs, the discriminant is positive, and one root must be positive and one negative. We consider the positive root, which is

$$Y^* = \frac{-(\theta_4\theta_2+\theta_3\theta_5)+\sqrt{(\theta_4\theta_2+\theta_3\theta_5)^2-4(\theta_4\theta_3-\theta_1)\theta_2\theta_5}}{2\theta_2\theta_5}$$

Then, the condition required for

(i) is $\dfrac{-(\theta_4\theta_2+\theta_3\theta_5)+\sqrt{(\theta_4\theta_2+\theta_3\theta_5)^2-4(\theta_4\theta_3-\theta_1)\theta_2\theta_5}}{2\theta_2\theta_5} > AL(1-\frac{\alpha-A/(1+\mu)}{\beta})$

(ii) is $\dfrac{-(\theta_4\theta_2+\theta_3\theta_5)+\sqrt{(\theta_4\theta_2+\theta_3\theta_5)^2-4(\theta_4\theta_3-\theta_1)\theta_2\theta_5}}{2\theta_2\theta_5} < AL(1-\frac{\alpha-A/(1+\mu)}{\beta})$