PROBLEM SET 5
14.02 Principles of Macroeconomics
April 6, 2005
Due April 13, 2005

I. Answer each as True, False, or Uncertain, providing some explanation for your choice.

1. Fluctuations in stock prices may arise from changes in investors' attitudes towards risk.

2. A permanent tax cut, all else equal, will have a quantitatively larger expansionary effect than a temporary one.

3. Stock market bubbles may lead to "too much" investment.

4. If the return on $1 invested in stocks is greater than the return on $1 invested in bonds (and investors are risk neutral), then arbitrage will ensure that the two returns are equalized through a decrease in the price of stocks.

5. The ability to borrow against your future (expected) income is a critical determinant of how closely your current consumption will track your current income.

II. Consumption

Suppose you are 22, about to graduate from MIT and take a job that promises an annual income of $\$Y$. You expect your salary to increase by $g\%$ each year. The annual nominal interest rate is $i\%$ and is expected to remain constant forever. Assume that you are paid once a year, and that you get your first paycheck at the end of the first year, the second one at the end of the second year and so on.

(i) Suppose you expect to live forever. Assume consumption is constant. When is this constant consumption finite? Explain what is going on when the condition (for it being finite) is not satisfied.

(ii) Assume that you expect to live for n years (i.e., your expected age at death will be $n+22$). When you sign your employment contract, you can choose one of two options - you can either work for all n years, or you can opt to retire at the age of 60 ($n + 22 > 60$) in exchange for a single lumpsum payment of $\$L$ (at the end of that year). What condition would $\$L$ have to satisfy for you to choose the voluntary retirement option?

(iii) Suppose you are once again faced with the option of choosing retirement at the age of 60, or never at all, and also that consumption is constant for your lifetime. If you choose to work, you expect to live for n years as in (ii). If you choose retirement at 60, however, you may expect to live for $N - n$ extra years (since working requires physical resources, and leisure partly renews them). Suppose the parameters in (ii) were such that you chose voluntary retirement. Is that sufficient to ensure that you choose voluntary retirement in this case as well? If not, why not, and what condition would need to be satisfied for you to choose voluntary retirement? (Note : this question is not about "the value of life", but merely about the distinction between consumption and wealth)

(iv) Now assume that retirement is no longer a matter of choice. You have to retire at the age of 60, and therefore there is no lump sum payment needed to induce you to retire. Assume that you expect to live for n years (so you expect to die at the age $n + 22$, and $n + 22 > 60$) and once again, that you smooth consumption over your lifetime.

(a) Find an expression for the constant annual consumption.

(b) You may assume that this constant consumption is greater than your starting salary, so that in the early years you will have to borrow. Denote the number of years you will need to borrow as T. Find an expression that implicitly defines T (Ignore the possibility that T may not be a natural number). Discuss how T depends on n.

III. Investment

A manufacturer is considering buying a machine which costs V_t in real terms, and which promises a profits stream (in real terms) of π_{t+1} at the end of the first year of operation, π_{t+2} at the end of the second year, and so on. The future is known with perfect certainty and a constant real interest rate of r is expected to prevail for the machine's life, which you may assume to be infinite. There is no depreciation.

(i) Find the condition under which the investment is worthwhile.

(ii) Define i as the constant interest rate, which would make the present discounted value of future profits equal to the current cost of buying the machine (this rate is often called the Internal Rate of Return). Find a condition relating i and r such that the investment is worthwhile. Is this condition conceptually any different from the condition you derived in (i)? Why or why not?