PROBLEM SET 5
14.02 Principles of Macroeconomics
April 13, 2005

I. Answer each as True, False, or Uncertain, providing some explanation for your choice.

1. Fluctuations in stock prices may arise from changes in investors' attitudes towards risk.

TRUE - When investors are risk averse, the price of a stock incorporates a risk premium, and changes in investors' attitudes towards risk changes the risk premium and hence the stock price.

2. A permanent tax cut, all else equal, will have a quantitatively larger expansionary effect than a temporary one.

TRUE - Since consumption is a functon of lifetime wealth, a permanent tax cut will mean a larger increase in consumption than a temporary one.

3. Stock market bubbles may lead to "too much" investment.

TRUE - Since investment often responds to stock prices (according to Tobin's q-theory), it may happen that when stocks are "overpriced" due to a bubble, a firm responding blindly to stock prices in making its investment decisions will end up investing "too much" relative to what it should have had stock prices been priced correctly.

4. If the return on \$1 invested in stocks is greater than the return on \$1 invested in bonds (and investors are risk neutral), then arbitrage will ensure that the two returns are equalized through a decrease in the price of stocks.

FALSE - The arbitrage opportunity that the difference in returns gives rise to is to go long on (or buy) stocks and go short on (or sell) bonds. But this will only serve to raise the price of stocks relative to bonds until the point where the arbitrage opportunity no longer exists. Note that an increase in the relative price of stocks indirectly reduces the relative return on stocks.

5. The ability to borrow against your future (expected) income is a critical determinant of how closely your current consumption will track your current income.

TRUE - Economists call this kind of constraint a borrowing or liquidity constraint. If there were no borrowing constraints, then your consumption would depend only on your lifetime wealth, which is a function of but not equal to your current income.

II. Consumption

Suppose you are 22, about to graduate from MIT and take a job that promises an annual income of $\$Y$. You expect your salary to increase by $g\%$ each year. The annual nominal interest rate is $i\%$ and is expected to remain constant forever. Assume that you are paid once a year, and that you get your first paycheck at the end of the first year, the second one at the end of the second year and so on.

(i) Suppose you expect to live forever. Assume consumption is constant. When is this constant consumption finite? Explain what is going on when the condition (for it being finite) is not satisfied.

Your lifetime wealth is given by

$$\$W = \frac{\$Y}{1+i/100} + \frac{(1+g/100)\$Y}{(1+i/100)^2} + \frac{(1+g/100)^2\$Y}{(1+i/100)^3} + ...$$
$$= \frac{100.\$Y}{i-g}$$

So per-period consumption is finite if and only if lifetime wealth is finite. The condition for this is $i > g$. If this condition is violated, your income grows faster than the rate at which you discount it, so it must be that your lifetime wealth is infinite.

(ii) Assume that you expect to live for n years (i.e., your expected age at death will be $n+22$). When you sign your employment contract, you can choose one of two options - you can either work for all n years, or you can opt to retire at the age of 60 ($n+22 > 60$) in exchange for a single lumpsum payment of $\$L$ (at the end of that year). What condition would $\$L$ have to satisfy for you to choose the voluntary retirement option?

If you choose not to retire, your lifetime wealth is

$$\$W_1 = \frac{\$Y}{1+i/100} \frac{1-(\frac{1+g/100}{1+i/100})^n}{1-(\frac{1+g/100}{1+i/100})}$$

If you choose to retire, your lifetime wealth is

$$\$W_2 = \frac{\$Y}{1+i/100} \frac{1-(\frac{1+g/100}{1+i/100})^{38}}{1-(\frac{1+g/100}{1+i/100})} + \frac{\$L}{(1+i/100)^{38}}$$

Clearly you will choose to retire if and only if $\$W_2 > \$W_1$

Note that per-period consumption in the two cases are $\$W_1/n$ and $\$W_2/n$, so that comparing per-period consumption is equivalent to comparing lifetime wealth. This will not be the case in (iii)

(iii) Suppose you are once again faced with the option of choosing retirement at the age of 60, or never at all, and also that consumption is constant for your lifetime. If you choose to work, you expect to live for n years as in (ii). If you choose retirement at 60, however, you may expect to live for $N - n$ extra years (since working requires physical resources, and leisure partly renews them). Suppose the parameters in (ii) were such that you chose voluntary retirement. Is that sufficient to ensure that you choose voluntary retirement in this case as well? If not, why not, and what condition would need to be satisfied for you to choose voluntary retirement? (Note : this question is not about "the value of life", but merely about the distinction between consumption and wealth)

Now, lifetime wealth in the two cases is exactly the same as in (ii), but it is not enough to compare lifetime wealth. Rather one must compare per-period consumption. If you choose not to retire, your constant consumption every period is $\$C_1 = \$W_1/n$,while, if you choose to retire, it is $\$C_2 = \$W_2/N$. Now $\$W_2 > \$W_1$ is no longer sufficient for $\$W_2/N > \$W_2/n$, and it is the latter condition which determines whether you choose to retire or not. If this condition is met, you will choose to retire.

(iv) Now assume that retirement is no longer a matter of choice. You have to retire at the age of 60, and therefore there is no lump sum payment needed to induce you to retire. Assume that you expect to live for n years (so you expect to die at the age $n+22$, and $n+22>60$) and once again, that you smooth consumption over your lifetime.

(a) Find an expression for the constant annual consumption.

Your lifetime wealth is now $\$W = \frac{\$Y}{1+i/100}\frac{1-(\frac{1+g/100}{1+i/100})^{38}}{1-(\frac{1+g/100}{1+i/100})}$, which means that your constant annual consumption is $\$C = \$W/n$

(b) You may assume that this constant consumption is greater than your starting salary, so that in the early years you will have to borrow. Denote the number of years you will need to borrow as T. Find an expression that implicitly defines T (Ignore the possibility that T may not be a natural number). Discuss how T depends on n.

Clearly, you will borrow till the period when your income has grown just enough to equate current income with this constant consumption, so the condition that implicitly defines T is $\$C = (1+g/100)^T\$Y$, or equivalently,

$\frac{1}{1+i/100}\frac{1-(\frac{1+g/100}{1+i/100})^{38}}{1-(\frac{1+g/100}{1+i/100})} = (1+g/100)^T n$

If you were to plot consumption and income against time, then consumption would be a horizontal line, while income would increase upto the time you retire, and then drop off sharply to zero. So you would borrow till period T, save from T till you are 60, and dissave (i.e., run down your savings) from the age of 60 on. It follows that the longer your life span, the lesser will be your per-period consumption, and therefore, the quicker income will catch up with consumption, so that the smaller will be T. This is evident from the equation above. All else equal, a larger n must mean a smaller T.

III. Investment

A manufacturer is considering buying a machine which costs V_t in real terms, and which promises a profits stream (in real terms) of π_{t+1} at the end of the first year of operation, π_{t+2} at the end of the second year, and so on. The future is known with perfect certainty and a constant real interest rate of r is expected to prevail for the machine's life, which you may assume to be infinite. There is no depreciation.

(i) Find the condition under which the investment is worthwhile.

The cost of investment is V_t, while the benefit is $\frac{\pi_{t+1}}{1+r} + \frac{\pi_{t+2}}{(1+r)^2} + \frac{\pi_{t+3}}{(1+r)^3} + ...$

The condition required for the investment to be worthwhile is that the cost be lower than the benefit, or that the Net Present Value of the investment be positive.

(ii) Define i as the constant interest rate, which would make the present discounted value of future profits equal to the current cost of buying the machine (this rate is often called the Internal Rate of Return). Find a condition relating i and r such that the investment is worthwhile. Is this condition conceptually any different from the condition you derived in (i)? Why or why not?

i satisfies $V_t = \frac{\pi_{t+1}}{1+i} + \frac{\pi_{t+2}}{(1+i)^2} + \frac{\pi_{t+3}}{(1+i)^3} + ...$

The condition required for the investment to be worthwhile is that the internal rate of return be higher than the outside rate of return, which is r, because only then will $V_t < \frac{\pi_{t+1}}{1+r} + \frac{\pi_{t+2}}{(1+r)^2} + \frac{\pi_{t+3}}{(1+r)^3} + ...$

Thus the condition $i > r$ is conceptually the same as the condition you derived in (i)