PROBLEM SET TWO   **DUE 26 FEBRUARY**

*You will legibly write both your full name and section
on your completed assignment.*

**From Chapter 3 (This question should complement the material presented
in lecture on 16 February)**

1.   The Keynesian Model of Output

Assume aggregate demand (Z) is the sum of consumption (C), government
spending (G), and investment (I) such that $Z = C+I+G$. Further assume
consumption is linear in disposable income ($Y^d=(1-\tau)Y$) such that $C = c_0+c_1Y^d$, where (τ) represents the tax rate on income paid by all agents.
Define an equilibrium by the condition that production (Y) equals
demand such that $Y = Z$.

   a.   Graph in (Z,Y) space curves for aggregate demand (Z) and
         the equilibrium condition.
   b.   Solve for equilibrium output (Y) as a function of I, G, τ,
         and the parameters c_0 and c_1.
   c.   Assume that the interest rates increases, which increases
         the cost of borrowing for firms, and consequently reduces
         the level of investment (I). Graphically illustrate the
         new equilibrium, and describe in words the transition to
         this new equilibrium.
   d.   Define the government budget deficit as the excess of
         government spending over tax revenues, G-T. What are tax
         revenues for this economy? What is the impact of the
         increase in interest rates from part c on the government's
         budget deficit?
   e.   Consider a very small increase in the tax rate (small
         enough to take a derivative). What is the impact of the
         higher tax rate on equilibrium income? Illustrate both
         graphically and mathematically. What is the impact of the
         higher tax rate on the government's budget deficit?
   f.   Graph in (i,Y) space equilibrium output versus the interest
         rate. Describe in words what is happening as you move
         along the curve, reducing interest rates as you move.
         Consider the increase in the tax rate from part e, and
         illustrate what this does to your curve.

**From Chapters 22-23 (Please read pp. 457-462 and pp. 466-476 for some
context as this material will probably depart from that presented in
lecture on 17 February)**

2.   The Solow Growth Model

   a.   Consider the following aggregate production function,
         mapping the aggregate stocks of capital (K) and labor (N)
         into the aggregate level of output (Y): $Y = K^\alpha N^{1-\alpha}$. You
         may assume throughout this problem that $0<\alpha<1$. Using the
         constant-returns to scale property of this production
         function, write per capita output (y=Y/N) as a function of
         per capita capital stock (k=K/N), and α.

b.   Assume aggregate savings (S) is simply a constant fraction of output: S=sY. Further assume that aggregate investment (I), is always equal to aggregate savings. Finally assume that a fraction of the capital stock $\delta>0$ is destroyed each year through wear and tear. Write an equation in continuous time relating the instantaneous change in the stock of capital (dK/dt) to the stock of capital (K), the stock of labor (N), and the parameters s, δ, and α. Note that investment represents additions to the stock of capital. Your equation will be slightly different from that in the book, which is derived in discrete time.

c.   Assume the population grows at rate n>0, implying (dN/dt)/N = n. Rewrite the above equation, now relating the instantaneous change in the per capita stock of capital (dk/dt) to the per capita stock of capital (k) and the parameters s, n, δ, and α. Hint: convince yourself that (dk/dt)/k = (dK/dt)/K-n is true and then replace dK/dt with this expression. All this really says is that $\%\Delta(A/B)$ = (d(A/B)/dt)/(A/B) = $\%\Delta A$-$\%\Delta B$ = (dA/dt)/A-(dB/dt)/B.

d.   Graph the solution your equation in k using two curves in (Y,k) space as follows. For one curve, graph "investment" (I/N) versus (k). Verify that this curve should be concave by checking $(d^2(I/N)/dk^2)<0$. Discuss what economic assumption is driving the concavity of this curve. Hint: look back to page 459. For the other curve, graph "depreciation" $(n+\delta)k$ versus (k).

e.   Consider a level of k such that investment is larger than depreciation. What is happening to k and y? Alternatively consider a level of k such that investment is smaller than depreciation. What is happening to k and y? Where are k and y going to end up, regardless of initial conditions? What is the growth rate of k, y, K, and Y in the long-run?

f.   Solve your equation for the steady state level of k and y. Simply set (dk/dt)=0 and solve for k. Verify that a higher savings rate increases steady-state k and y, but does not affect the growth rate of k or y. Illustrate this graphically.

g.   Assume the economy is in steady state, but there is a sudden migration that doubles the labor force. Describe the time path of k and y as the economy adjusts to this shock.

h.   Note per capital consumption (c=C/N) in this economy is simply (1-s)y. Using your equation for steady-state y, form a relation for c as a function of only s and the parameters n, δ, and α. Describe how you might solve for the rate of savings which maximizes per capita consumption. Call this savings rate s^*. Define the golden-rule level of capital k^* by your equation for steady-state capital evaluated at $s=s^*$. Use your result from part f to predict whether or not an economy where $s<s^*$ also has $k<k^*$.

3.   **Economic Growth and Development**

a.   Consider the Solow growth model described above, and define a poor country as one where k is much less than the steady-state level k. What does this imply about per capita

income? Would a big donation of capital (K) from a rich country help this economy in the long-run? Explain using your graph.

b. Consider a different production function than the one above. In particular, assume Y=F(K,L) such that y=f(k) is concave for low values of k, turns convex for intermediate values of k, but again returns to a concave function for high values of k. Demonstrate graphically that there can be three steady-states in this economy. Do so by determining the dynamics of k (dk/dt) at different points along the k-axis.

c. Assume a country is in the "poorest" steady-state, using the same definition above. What are the prospects for this economy's future? Would a big donation of capital (K) from a rich country help this economy in the long-run?