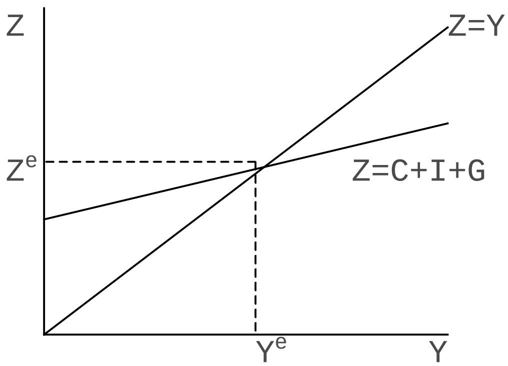
PROBLEM SET TWO SOLUTIONS

1.

a. We have the following two curves:

$Z = c_0+c_1(1-\tau)Y+I+G$ (Aggregate Demand)

$Z = Y$ (Equilibrium)



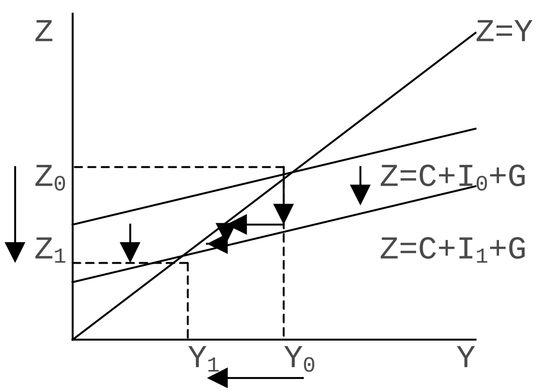
b. $Y^e = (c_0+I+G)/(1-c_1(1-\tau))$

c. We have the following three curves:

$Z = c_0+c_1(1-\tau)Y+I_0+G$ (AD where $r=r_0$)

$Z = c_0+c_1(1-\tau)Y+I_1+G$ (AD where $r=r_1>r_0$ so $I_0>I_1$)

$Z = Y$ (Equilibrium)



The economy initially starts in equilibrium at (Y_0,Z_0). The decline in investment demand shifts Aggregate Demand down by the amount of the change in investment. At Y_0 it is now true that $Z'<Y_0$ so there is an excess supply of goods produced. Producers reduce production until $Y=Z'$, but this decline in production reduces income, which reduces demand, resulting again in excess supply. This excess supply is smaller than the previous one. Producers again reduce

production, which again reduces income, and the process repeats until the new equilibrium is reached at (Z_1,Y_1)

d. $T = \tau Y^e = \tau(c_0+I+G)/(1-c_1(1-\tau))$

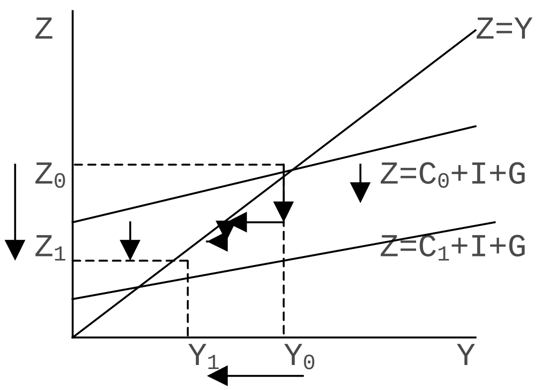
$d(G-T) = -dT = -\tau dY^e = -\tau dI/(1-c_1(1-\tau)) > 0$

e. We have the following three curves:

$Z = c_0+c_1(1-\tau_0)Y+I_0+G$ (AD)

$Z = c_0+c_1(1-\tau_1)Y+I_1+G$ (AD where $\tau=\tau_1>\tau_0$)

$Z = Y$ (Equilibrium)



The graph is similar to the increase in interest rates, in that it shifts Aggregate Demand down, but also there is a reduction in the slope as higher taxes reduce the "multiplier" part of equilibrium income. Equilibrium income falls.

Mathematically,

$dY^e/d\tau = -c_1\tau(c_0+I+G)/(1-c_1(1-\tau))^2 < 0$

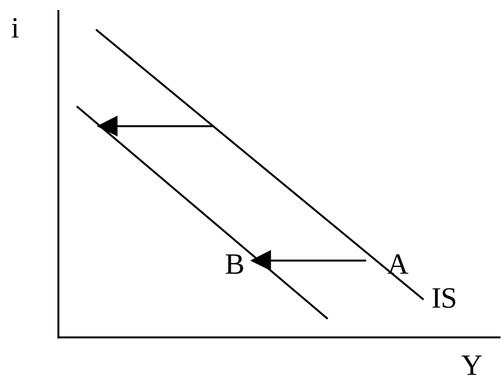
$$\begin{aligned} d(G-T)/d\tau &= -d(\tau Y^e)/d\tau \\ &= -Y^e-\tau dY^e/d\tau \\ &= -(c_0+I+G)/(1-c_1(1-\tau))+c_1\tau(c_0+I+G)/(1-c_1(1-\tau))^2 \\ &= Y^e[c_1\tau/(1-c_1(1-\tau))-1] \\ &= Y^e[c_1\tau-1+c_1-c_1\tau]/(1-c_1(1-\tau)) \\ &= Y^e[c_1-1]/(1-c_1(1-\tau)) < 0 \end{aligned}$$

Higher tax rates reduce the budget deficit (by increasing tax revenues) as long as $c_1 < 1$, the marginal propensity to consume out of disposable income is less than one.

f.

Reducing interest rates increases investment, which increases equilibrium output. This increase is by more than the increase in investment due to mulitplier effects. Higher investment creates more demand, more demand creates more output by the equilibrium condition, more output is more income which creates more consumption demand, which also creates more output, and so on…)

Fix an interest rate $i = i_0$, which fixes a level of investment corresponding to that interest rate, fixes the level of output given (c_0,G,c_1, and τ), denoted by A in the picture above. Consider an increase in the tax rate given the interest rate, which clearly reduces output from A to B. As the initial choice of interest rate was arbitrary, this must be true for all interest rates, and the whole curve shifts to the left.



2.

a. $\mathbf{(K/N)^{\alpha}(N/N)^{1-\alpha} = k^{\alpha} = Y/N = y}$

b. $\mathbf{(dK/dt) = I-\delta K = sY-\delta K = sK^{\alpha}N^{1-\alpha}-\delta K}$

c.

$$(dk/dt)/k = (dK/dt)/K - (dN/dt)/N$$
$$(dK/dt) = K[(dk/dt)/k+n]$$
$$= K[(dk/dt)N/K+n]$$
$$(dK/dt) = N(dk/dt)+nK$$

$$N(dk/dt)+nK = sK^{\alpha}N^{1-\alpha}-\delta K$$
$$(dk/dt) = (sK^{\alpha}N^{1-\alpha}-\delta K)/N$$

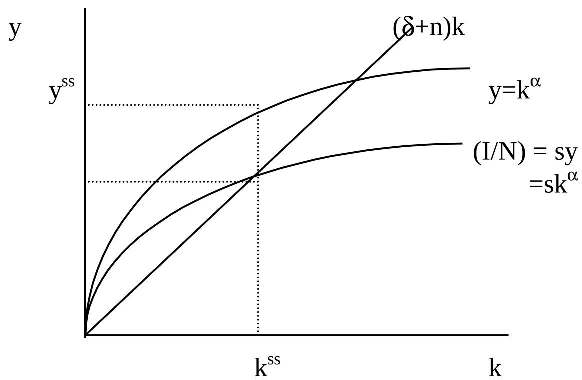
$$\mathbf{(dk/dt) = sk^{\alpha}-(\delta+n)k}$$

d.

$$(I/N) = sY/N = sy = sk^{\alpha}$$
$$d(I/N)/dk = \alpha sk^{\alpha-1}$$
$$d^2(I/N)/d^2k = \alpha(1-\alpha)k^{\alpha-2} < 0$$

as $k > 0$ and $0 < \alpha < 1$.

The concavity of this curve is being driven by diminishing returns to capital. Note $dY/dK = \alpha(N/K)^{1-\alpha}$ and it follows that $\lim_{K\to\infty} dY/dK = 0$, as capital is accumulated, the marginal product of capital goes to zero. As investment is a linear function of output, this implies that the marginal contribution to output of an additional unit of capital is going to zero as the capital stock is accumulated to infinity.



e.

When investment is greater than depreciation we have the following: $sk^\alpha > (\delta+n)k$ implying $(dk/dt) = sk^\alpha-(\delta+n)k > 0$, that capital is being accumulated. Alternatively where investment is less than depreciation we have the following: $sk^\alpha < (\delta+n)k$ implying $(dk/dt) = sk^\alpha-(\delta+n)k < 0$, that capital is being decumulated. This implies that capital will accumulate to the point where $(dk/dt) = 0$, where investment equals depreciation, illustrated by the intersection of the two curves above.

In the long-run, the growth rate of k is zero as it converges to a fixed point. As y is a simple function of k, it also converges and has no long run growth rate. Using the equation above, $(dk/dt)/k = (dK/dt)/K - (dN/dt)/N$ implying if $dk/dt = 0$ that $(dK/dt)/K = n$. Capital accumulates at the same rate as the population grows. A similar argument can be made for aggregate output Y. $(dY/dt) = \alpha(dK/dt)/K+(1-\alpha)(dN/dt)/N = \alpha n+(1-\alpha)n = n$, so that aggregate output also grows at rate n.

f.

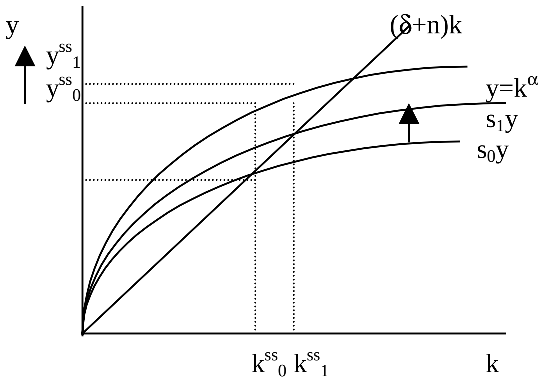
$$sk^\alpha = (\delta+n)k \text{ implies } k^{ss} = [s/(\delta+n)]^{1-\alpha}$$

$y^{ss} = (k^{ss})^{\alpha} = [s/(\delta+n)]^{(1-\alpha)\alpha}$

$dy^{ss}/ds = \alpha(1-\alpha)[s/(\delta+n)]^{(1-\alpha)\alpha-1} > 0$
as $0 < \alpha < 1$ and $s, \delta, n > 0$

g.

The ratio of K to N is immediately halved as the capital stock only adjusts slowly. Given the new ratio of capital to labor is much lower than steady-state, the marginal returns to capital are larger than depreciation, and capital begins to accumulate. The economy slowly returns



to steady-state k and y.
h.

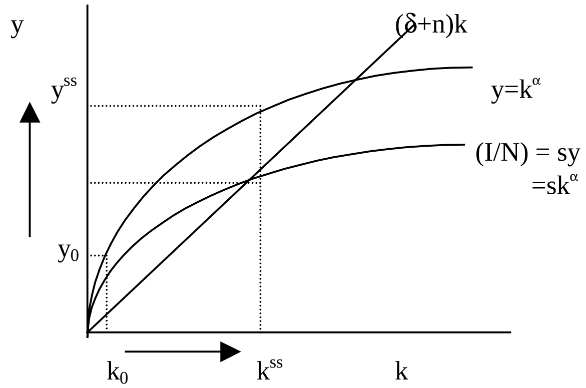
$(C/N) = c = (1-s)y = (1-s)y^{ss} = (1-s)[s/(\delta+n)]^{(1-\alpha)\alpha}$

Maximize this function with respect to the choice of s to yield $s^*(n,\delta,\alpha)$. The golden rule of capital is thus $k^* = [s^*/(\delta+n)]^{1-\alpha}$.

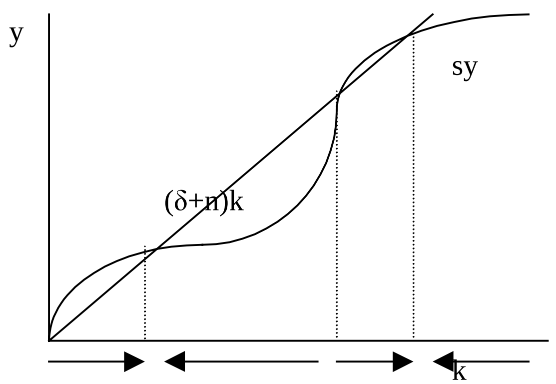
If $s < s^*$, using from part f that $dk^{ss}/ds > 0$, $k^{ss} < k^*$. This follows from the folowing. If $s = s^*$, think about what would happen to k^{ss} if s was reduced. As $dk^{ss}/ds > 0$, this implies k^{ss} must fall. Economies which don’t save enough endu up with steady-state ratios of capital to labor which are too low and thus per capital incomes which are too low (relative to the savings rates which maximize per capital consumption).

3.

a. As per capita income depends on only on per capita capital stock, a low k implies a low y. A big donation of capital from a rich country would not help this country in the long run, as the poor country will converge to steady-state over time. On the other hand, the donation would help the country get there faster.



b.



Dynamics of changes in k are driven by our equation of motion for capital per worker above. When investment per worker is higher than depreciation per worker, k is increasing. Note depreciation of k comes from two parts. In the numerator physical capital depreciates at rate δ, while in the denominator population grows at rate n, also “depreciating” k at rate n, for a sum of (δ+n).

There are three steady-states. The first and last are said to be stable in the sense that if you move away from them a little bit, you will return to them. The middle steady-state is unstable in the sense if you move away from it a little for some reason, you go to another steady-state.

The interesting thing about this case is dynamics between the two extremes.

c.

If a country is in the poorest steady-state, it is helpless to grow any further on its own. It could increase the savings rate and eliminate the middle dynamics, but this would require driving consumption per capita down to very low levels. A donation of capital to get the country just past the second steady-state would allow it to grow to the rest of the way on its own. This is similar to the migration question in part 2, an increase in capital immediately increases the ratio of K to N.