

14.06: Section Handout

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Topics to be covered today:

- Hall's Random Walk Hypothesis
- Precautionary Savings

1 Hall's Random Walk Hypothesis

Hall's result on the properties of consumption marked a clear challenge to the existing view of consumption. The early models of consumption had clear implications on the predictability of consumption. Hall's result challenged that based mostly on the basic intuition behind the permanent-income hypothesis, with some additional assumptions.

We shall review first a simple model of consumption with certainty and then we can move on to analyze the case with uncertainty.

1.1 The Certainty Case

Consider an agent with the following preferences:

$$U = \sum_{t=1}^T u(c_t), \quad u'(\cdot) > 0, \quad u''(\cdot) < 0, \quad (1)$$

where c_t is consumption in period t and we have assumed $\beta = 1$. The budget constraint of the agent is

$$\sum_{t=1}^T c_t \leq A_0 + \sum_{t=1}^T y_t \quad (2)$$

where we have also assumed that $r = 0$.¹ We can prove that under the assumption that $u'(\cdot) > 0$, then the budget constraint will be binding in equilibrium.

The Lagrangian for this problem is

$$\mathcal{L} = \sum_{t=1}^T u(c_t) + \lambda \left(A_0 + \sum_{t=1}^T y_t - \sum_{t=1}^T c_t \right) \quad (3)$$

¹Setting both the interest rate and the subjective discount rate equal to 0 will help us simplify the math a bit but the main result will still hold.

with FOCs

$$u'(c_t) = \lambda, \quad \forall t. \quad (4)$$

This is the basic idea of consumption smoothing, individuals will choose a consumption path so as to keep a constant marginal utility of consumption. Under our assumptions, the consumption level uniquely determines the marginal utility, then $c_1 = c_2 = \dots = c_T = \bar{c}$. Using (2) we obtain

$$\bar{c} = \frac{1}{T} \left(A_0 + \sum_{t=1}^T y_t \right). \quad (5)$$

Equation (5) has a very intuitive interpretation. The right hand side corresponds to the *permanent income*, and this is the basic result from the *permanent income hypothesis*, consumption is determined by the permanent level of income, not by the current level; savings in this model are equal to the difference between the current and the permanent income level. The *life-cycle hypothesis* relates connects the basic idea of consumption smoothing to the earnings profile, then an individual borrows when young, pays the debt and saves when adult (working age), and disaves when old (particularly after retirement).

1.2 The Uncertainty Case

Let us now deal with the uncertainty case. We will not use the same representation of uncertainty used in the lectures, instead we will write the individual's objective function as the expected intertemporal utility. All the assumptions made in section 1.1 will be maintained here, we will introduce the additional assumption that $u(c_t) = c_t - (a/2)c_t^2$ with $a > 0$. In this case we can write the objective function of the individual as follows

$$U = E_{t=1} \sum_{t=1}^T \left[c_t - \left(\frac{a}{2} \right) c_t^2 \right]$$

where $E_{t=1}$ is the expectations operator with the information available in time $t = 1$. The budget constraint is given by (2), so the consumption profile will satisfy it with equality. We can apply expectations to both sides of (2) to obtain

$$\sum_{t=1}^T E_{t=1}(c_t) \leq A_0 + \sum_{t=1}^T E_{t=1}(y_t). \quad (6)$$

Using the FOCs with respect to consumption we obtain the following result

$$1 - ac_1 = E_{t=1}(1 - ac_t), \quad \text{for } t = 2, 3, \dots, T. \quad (7)$$

Given the linearity of marginal utility we can obtain the following condition for consumption along the optimal path

$$c_1 = E_{t=1}(c_t), \quad \text{for } t = 2, 3, \dots, T. \quad (8)$$

So the individual expects to consume the same quantity in any subsequent period. Of course, this is based on the information she has access to in $t = 1$, so "news"

(innovations) in any relevant variable will induce changes in the consumption profile. Notice that if the individuals are rational, in the sense that they incorporate all the information available, then these changes cannot be predicted in $t = 1$, if they are predictable, then the individual should reoptimize in order to smooth consumption (as it is implied by the condition on the marginal utility of consumption, equation 7).

We can go a bit further and postulate an empirical model based on the basic results we have obtained so far.² Take equation (8) for $t = 2$, then we can clearly see that changes in consumption are unpredictable.

Under the assumption that agents incorporate all the available information when forming their expectations, we can write

$$c_t = E_{t-1}(c_t) + \varepsilon_t, \quad (9)$$

where ε_t is an expectational error and $E_{t-1}(\varepsilon_t) = 0$. Using (8) in (9)

$$c_t = c_{t-1} + \varepsilon_t \quad (10)$$

which is exactly Hall's famous result: *consumption follows a random walk*. The main intuition has already been explained: if the individual expects consumption to change, then she can do a better job by reallocating resources smoothing out those fluctuations according to (7), which implies that we expect no changes in consumption.

If we follow Romer (2001)³ we can also find the actual value of ε_t . the "change" in consumption. Take (8) and (6) to obtain

$$c_1 = \frac{1}{T} \left(A_0 + \sum_{t=1}^T E_1 [y_t] \right),$$

so consumption is given by the "average" value of expected lifetime resources. Take the same formula for $t = 2$

$$\begin{aligned} c_2 &= \frac{1}{T-1} \left(A_1 + \sum_{t=2}^T E_2 [y_t] \right) \\ &= \frac{1}{T-1} \left(A_0 + y_1 - c_1 + \sum_{t=2}^T E_2 [y_t] \right) \\ &= \frac{1}{T-1} \left(A_0 + y_1 - c_1 + \sum_{t=2}^T E_1 [y_t] + \underbrace{\left\{ \sum_{t=2}^T E_2 [y_t] - \sum_{t=2}^T E_1 [y_t] \right\}}_{\Delta \text{ in the expectation: new information}} \right) \end{aligned}$$

²Hall's paper presents a more general version with less assumption, but the main implications are the same. In particular, if utility is quadratic (so marginal utility is linear) and the discount rate equals the interest rate, then consumption follows a "random walk".

³See Section 7.2 of Romer's textbook.

$$\begin{aligned}
&= \frac{1}{T-1} \left(\underbrace{A_0 + y_1 + \sum_{t=2}^T E_1 [y_t]}_{Tc_1} - c_1 + \left\{ \sum_{t=2}^T E_2 [y_t] - \sum_{t=2}^T E_1 [y_t] \right\} \right) \\
c_2 &= c_1 + \frac{1}{T-1} \left(\sum_{t=2}^T E_2 [y_t] - \sum_{t=2}^T E_1 [y_t] \right). \tag{11}
\end{aligned}$$

Equation (11) tells us that the individual will adjust current consumption according to the change in the total expected value of income during lifetime, and will do it by a fraction equal to the inverse of the periods left before T . In simpler words (hopefully simplified by now), current consumption change in the same amount permanent income changed because of the news that arrived.⁴

Notice the following property, if we were to offer the individual to change his uncertain income stream for a certain one that is equal to the expected value, she would choose exactly the same consumption level. This is called *certainty equivalence*.

What's the fundamental assumption that delivers this result?

2 Precautionary Savings

Our previous model assumed that utility is quadratic. There are several problem with it, starting from the fact that marginal utilities can be negative for sufficiently high levels of consumption, and continuing from the fact that the cost of variations in consumption are independent of the income level. Why is this second part problematic?

The last comment should lead us to think about the third derivative of the utility function. So we have three options:

1. $u''' = 0$, which is the case we already analyzed;
2. $u''' < 0$, what does this mean?
3. $u''' > 0$.

We need some introspection here...

$u''' = 0$ implies that if you expect to face a more uncertain environment on next period, **your behavior today will not change as long as the expected value is the same**. Does this sound logical to you?

The main idea here is that when $u''' > 0$, marginal utility is convex, implying that $E[u'(c)] > u'(E[c])$, this result follows from Jensen's Inequality. Furthermore, if we increase the variance of consumption while keeping the mean constant,⁵ we would observe an even larger increase in $E[u'(c)]$.

⁴If you want to read a summary of the empirical literature on the random walk hypothesis, see Romer's textbook, section 7.3, pages 340-344.

⁵This is equivalent to introduce a mean-preserving spread.

What does this imply for savings? Well, it is easy to see that a positive third derivative plus uncertainty about future income will reduce current consumption, raising savings. This is known as *precautionary savings*.

Is this important in the data? There is evidence that suggesting precautionary savings are important indeed, but some other papers claim that the effect is quite limited in magnitude. For a brief review check Romer's textbook, page 357.

References

Hall, R. (1978); "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence", *Journal of Political Economy*, 86 (December): 971-987.